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Multiobjective Optimal Design of Welded Box Beams

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Abstract: *The cost, mass, and maximal deflection are selected as objective functions. In the cost function, the material and fabrication costs are included. The variables are the four plate dimensions of a symmetrical welded box beam. The design constraints relate to the bending stress and local buckling of plate elements. The shear-stress constraint and size limitations are also considered. The optimal beam dimensions are computed using several single- and multiobjective optimization methods. The results of an illustrative numerical example show the effect of yield stress of steel and that of the weighting coefficients.*

1 INTRODUCTION

The multiobjective optimization gives designers aspects for selection of the most suitable structural version. Some applications have been treated, e.g., in refs. 1, 10, and 11. Our aim is to show the application of multiobjective optimization technique in an illustrative numerical example of a simple welded box beam.

It has been shown[4,5] that the fabrication cost affects the optimal dimensions of welded structures. Therefore, we use not only the mass but also the cost as an objective function, which contains the material and fabrication costs. The deflection of a beam is often limited to fulfill the serviceability requirements. For example, in *Eurocode 3*,[2] the beam deflection is limited to $L/200$ to $L/500$, where L is the span length. Therefore, our aim is to find structural solutions that minimize the maximal deflection.

The design constraints on stresses and local buckling of plate elements are formulated according to *Eurocode 3*. Several single- and multiobjective optimization methods are used to show their suitability for the solution of the defined nonlinear programming problem.

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2 THE OBJECTIVE FUNCTIONS

The cost function is defined by

$$K = K_m + K_f = k_m \rho V + k_f \Sigma T_i \qquad (1)$$

where K_m and K_f are the material and fabrication costs, respectively, k_m and k_f are the respective cost factors, ρ is the material density, V is the volume of the structure, and T_i are the times corresponding to the fabrication parts.

According to the cost-calculation method proposed by Pahl and Beelich[12] and modified by Farkas and Jármai,[6] the times can be calculated as follows: Time (in minutes) for assembly and tacking is

$$T_1 = C_1 \Delta \sqrt{\rho V} \sqrt{\kappa} \qquad C_1 = 1.0 \text{ min/kg}^{0.5} \qquad (2)$$

where Δ is a difficulty factor expressing the complexity of the fabrication of various kinds of structures, and κ is the number of structural elements to be assembled and prepared for welding. Time (in minutes) for welding is

$$T_2 = \Sigma C_{2i} a_{wi}{}^{1.5} L_{wi} \qquad (3)$$

where, for manual arc welding, $C_2 = 0.8 \times 10^{-3}$ min/mm$^{2.5}$, for automatic CO_2 welding, $C_2 = 0.4 \times 10^{-3}$ min/mm$^{2.5}$, a_w is the weld dimension, and L is the weld length in millimeters.

Additional time for changing of electrodes, deslagging, and chipping is

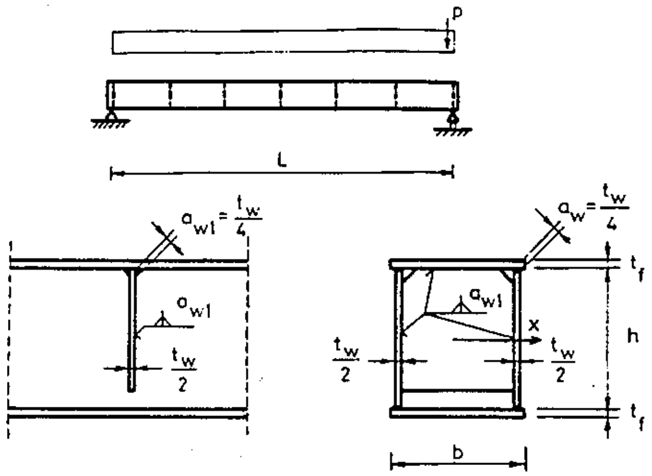
$$T_3 = \Sigma C_{3i} a_{wi}{}^{1.5} L_{wi} \qquad (4)$$

where, for manual arc welding, $C_3 = 0.24 \times 10^{-3}$ min/mm$^{2.5}$, and for automatic CO_2 welding, $C_3 = 0.12 \times 10^{-3}$ min/mm$^{2.5}$.

Equation (1) can be rewritten in the form

$$K/k_m \text{ (kg)} = \rho V + k_f/k_m (T_1 + T_2 + T_3) \qquad (5)$$

To give internationally acceptable results, wide ranges of values for k_m and k_f are considered, that is, $k_m = 0.5 - 1.2$



8.6

Fig. 1. Welded box beam with transverse diaphragms.

\$/kg, $k_f = 15 - 45$ \$/manhour $= 0.25 - 0.75$ \$/min, so the ratio k_f/k_m varies in the range of 0 to 1.5 kg/min. The case of $k_f = 0$ corresponds to the minimum weight design. Prices are in U.S. dollars.

In order to stiffen the box beam against the torsional deformation of the cross-sectional shape, some transversal diaphragms should be used. As shown in Fig. 1, in our example we use seven diaphragms, so $\kappa = 11$. Note that the mass of these diaphragms is neglected. For the four longitudinal fillet welds, we consider the constants $C_2 = 0.4 \times 10^{-3}$ and $C_3 = 0.12 \times 10^{-3}$ min/mm$^{2.5}$; for manual arc-welded transverse fillet welds connecting the diaphragms to the box beam, we use $C_2 = 0.8 \times 10^{-3}$ and $C_3 = 0.24 \times 10^{-3}$ min/mm$^{2.5}$.

For the difficulty factor, we take $\Delta = 2$, so the final formula of the cost function as the first objective function is

$$f_1 = K/k_m\ (\text{kg}) = \rho AL + k_f/k_m\, x \left[2\sqrt{\rho AL}\,\sqrt{11} + 0.52 \times 10^{-3} \times 4L\left(\frac{t_w}{4}\right)^{1.5} + 1.04 \times 10^{-3} \times 7 \times 2(2h + b)\left(\frac{t_w}{4}\right)^{1.5} \right] \quad (6)$$

Disregarding the fabrication costs, i.e., taking $k_f = 0$, we obtain the mass function as the second objective function:

$$f_2 = \rho AL \quad (7)$$

The third objective function to be minimized is the maximal deflection of the beam due to the uniformly distributed normal static load ρ_0 neglecting the self mass:

$$f_3 = \frac{5p_0L^4}{384EI_x} \quad (8)$$

where $E = 2.1 \times 10^5$ MPa is the modulus of elasticity for steels, and I_x is the moment of inertia, that is,

$$I_x = \frac{h^3t_w}{12} + \frac{2bt_f^3}{12} + 2bt_f\left(\frac{h + t_f}{2}\right)^2 \quad (9)$$

3 THE DESIGN CONSTRAINTS

The constraint on bending stress, according to *Eurocode 3*, can be expressed as

$$\sigma_{\max} = \frac{\gamma_1 M_{\max}}{W_x} \le \frac{f_y}{\gamma_{m0}} \quad (10)$$

where the safety factors are $\gamma_1 = 1.5$ and $\gamma_{m0} = 1.1$. The bending moment is

$$M_{\max} = \frac{pL^2}{8} \quad (11)$$

Considering also the self mass,

$$p = p_0 + \rho Ag \quad (12)$$

where $g = 9.81$ m/s^2 is the gravitational acceleration.

Furthermore, f_y is the yield stress; for steel Fe 360, $f_y =$ 235 MPa, and for steel Fe 510, $f_y = 355$ MPa. The section modulus is

$$W_x = \frac{I_x}{\frac{h}{2} + t_f} \quad (13)$$

Note that we consider the cross section of class 3, which means that the stress distribution is linear elastic and the yield stress is reached in extreme fiber without any local buckling of plate elements.

The local buckling constraint for the compressed upper flange is

$$t_f \ge \delta b \quad \frac{1}{\delta} = 42\epsilon \quad \epsilon = \sqrt{\frac{235}{\sigma_{\max}}} \quad (\sigma_{\max} \text{ in MPa}) \quad (14)$$

and for bent webs is

$$\frac{t_w}{2} \ge \beta h \quad \frac{1}{\beta} = 124\epsilon \quad (15)$$

The shear constraint can be expressed according to *Eurocode 3* for $1/\beta = 124\epsilon$ as

$$Q_{\max} = \frac{\gamma_1 pL}{2} \le 0.627 \times 0.5Q_b \quad (16)$$

where

$$Q_b = \frac{ht_w}{\gamma_{m1}} \frac{f_y}{\sqrt{3}}$$

With $\gamma_{m1} = 1.1$, Eq. (16) takes the form

$$\frac{\gamma_1 pL}{2} \le 0.1645ht_wf_y \quad (17)$$

since the deflection minimization leads to maximization of the beam dimensions, size constraints should be defined as follows:

$$h \le h_{\max} \quad t_w \le t_{w,\max} \quad b \le b_{\max} \quad t_f \le t_{f,\max} \quad (18)$$

4 THE OPTIMIZATION PROCEDURE

In the optimization procedure, the optimal values of variables h, t_w, b, and t_f should be determined so as to minimize the objective functions and fulfill the design constraints. Note that for the single-objective problem to minimize the mass f_2, the following approximate formulas can be derived[3]:

$$h = \sqrt[3]{0.75W_0/\beta} \quad t_w/2 = \beta h \quad b = h\sqrt{\beta/\delta} \quad t_f = \delta b \tag{19}$$

where

$$W_0 = \frac{\gamma_1 M_{\max}}{f_y/\gamma_{m0}}$$

is the required section modulus.

For computer-aided optimal design, a decision support system has been developed[7–9] containing five single-objective and seven multiobjective optimization techniques. The single-objective optimization methods are as follows: Himmelblau's method of flexible tolerances, Weisman's direct random search method, Rosenbrock's hillclimb method, the complex method of Box, and the Davidon-Fletcher-Powell method. The multiobjective optimization methods are as follows: min-max, weighting min-max, global criterion types 1 and 2, weighting global criterion, pure weighting, and normalized weighting.

A multicriteria optimization problem can be formulated as follows:

Find x such that

$$\mathrm{f}_k(2\mathrm{x}) = \mathrm{opt}\ \mathrm{x} \tag{20}$$

such that

$$g_j(\mathbf{x}) \geq 0 \qquad j = 1, \ldots, m$$
$$h_i(\mathbf{x}) = 0 \qquad i = 1, \ldots, q$$

where $\mathbf{x}$ is the vector of decision variables defined in n-dimensional Euclidean space, and $f_k(\mathbf{x})$ is a vector function defined in r-dimensional Euclidean space. $g_j(\mathrm{x})$ and $h_i(\mathrm{x})$ are inequality and equality constraints.

The solutions of this problem are the Pareto optima. The definition of this optimum is based on the intuitive conviction that the point $\mathbf{x}^*$ is chosen as the optimal if no criterion can be improved without worsening at least one other criterion.

We have used the min-max, the weighting min-max, two types of global criterion, weighting global criterion, pure weighting, and normalized weighting techniques. They are described in details in refs. 1 and 11.

4.1 A brief description of the methods

The min-max optimum compares relative deviations from the separately reached minima. The relative deviation can be calculated from

$$z_i'(x) = \frac{|f_i(x) - f_i^0|}{|f_i^0|} \qquad \text{or} \qquad z_i''(x) = \frac{|f_i(x) - f_i^0|}{|f_i(x)|} \tag{21}$$

If we know the extremes of the objective functions, which can be obtained by solving the optimization problems for each criterion separately, the desirable solution is the one that gives the smallest values of the increments of all the objective functions. The point x* may be called the best compromise solution considering all the objective functions simultaneously and on equal terms of importance.

$$z_i(x) = \max\{z_i'(x), z_i''(x)\} \qquad i \in I \tag{22}$$

$$\mu(x^*) = \min\ \max\ \{z_i(x)\} \qquad x \in X,\ i \in I \tag{23}$$

where X is the feasible region.

One gets the weighting min-max method combining the min-max approach with the weighting method, and a desired representation of Pareto optimal solutions can be obtained:

$$z_i(x) = \max\ \{w_i\, z_i'(x),\ w_i\, z_i''(x)(x)\} \quad i \in I \tag{24}$$

The weighting coefficients w_i reflect exactly the priority of the criteria, the relative importance of them. We can get a distributed subset of Pareto optimal solutions.

The global criterion method means that a function that describes a global criterion is a measure of closeness of the solution to the ideal vector of $\mathbf{f}^0$. The common form of this function is (type 1)

$$f(x) = \sum_{i=1}^{r} \left[\frac{f_i^0 - f_i(x)}{f_i^0} \right]^P \tag{25}$$

It is suggested to use $P = 2$, but other values of P such as 1, 3, 4, etc. can be used. Naturally, the solution obtained will differ greatly according to the value of P chosen.

It is recommended to use relative deviations (type 2):

$$L_P(f) = \left[\sum_{i=1}^{r} \left| \frac{f_i^0 - f_i(x)}{f_i^0} \right|^P \right]^{1/P} \qquad 1 \leq P \leq \infty \tag{26}$$

If the weighting global criterion method is used, by introducing weighting parameters one could get a great number of Pareto optima with Eq. (25). If we choose $P = 2$, this means the Euclidean distance between Pareto optimum and ideal solution.[7] The coordinates of this distance are weighted by the parameters as follows:

$$L_P(f) = \left[\sum_{i=1}^{r} w_i \left| \frac{f_i^0 - f_i(x)}{f_i^0} \right|^P \right]^{1/P} \qquad 1 \leq P \leq \infty \tag{27}$$

The pure weighting method involves adding all the objective functions together using different weighting coefficients

Table 1
Characteristics of beams optimized using different single-objective techniques

Technique		h (mm)	t_w (mm)	b (mm)	t_f (mm)	f_1 (kg)	f_3 (mm)
Flexible tolerance	$f_{1,\min}$	1450	22	700	19	9,332	19.7
	$f_{3,\min}$	1800	32	1000	40	20,801	5.1
Direct random search	$f_{1,\min}$	1400	22	650	22	9,402	20.5
	$f_{3,\min}$	1800	32	1000	40	20,801	5.1
Hillclimb	$f_{1,\min}$	1300	20	550	32	9,343	21.9
	$f_{3,\min}$	1800	32	1000	40	20,801	5.1

Table 2
Characteristics of beams optimized using different multiobjective optimization methods and various weighting coefficients

Technique	h (mm)	t_w (mm)	b (mm)	t_f (mm)	f_1 (kg)	f_3 (mm)
Min-max	1800	20	750	39	13,762	7.3
Global 1, $P = 3$	1800	20	750	40	13,947	7.2
Global 2, $P = 5$	1800	20	900	33	13,910	7.2
Weighting min-max						
w_1/w_3						
0.9/0.1	1750	24	700	18	10,834	12.2
0.75/0.25	1800	22	950	20	11,974	9.5
0.5/0.5	1800	20	750	39	13,762	7.3
0.25/0.75	1800	18	1000	38	15,294	6.1
0.1/0.9	1800	24	1000	40	17,869	5.5
Weighting global						
0.9/0.1	1800	22	950	20	11,974	9.5
0.75/0.25	1800	20	900	28	12,799	8.2
0.5/0.5	1800	20	950	35	14,329	7.0
0.25/0.75	1800	18	950	40	15,284	6.1
0.10/0.9	1800	18	1000	40	15,786	5.9
Normalized weighting						
0.9/0.1	1800	26	500	13	10,136	14.0
0.75/0.25	1800	22	950	20	11,974	9.5
0.5/0.5	1800	18	950	40	15,284	6.1
0.25/0.75	1800	22	950	40	16,658	5.9
0.1/0.9	1800	32	1000	40	20,801	5.1

Table 3
Characteristics of optimized beams made of steel Fe 360 and Fe 510

Steel	Technique		h (mm)	t_w (mm)	b (mm)	t_f (mm)	f_1 (kg)	f_3 (mm)
Fe 360	Single-objective Optimization	$f_{1,\min}$	450	22	700	19	9,332	19.7
		$f_{3,\min}$	1800	32	1000	40	20,801	5.1
	Min-max method		1550	30	750	25	16,343	8.8
Fe 510	Single-objective Optimization	$f_{1,\min}$	1300	20	500	17	7,051	34.2
		$f_{3,\min}$	1800	32	1000	40	20,301	5.1
	Min-max method		1500	24	750	40	14,253	10.2

for each. It means that we transform our multicriteria optimization problem to a scalar one by creating one function of the form

$$f(\mathbf{x}) = \sum_{i=1}^{r} w_i f_i(\mathbf{x}) \tag{28}$$

where $w_i \geq 0$ and $\Sigma_{i=1}^{r} w_i = 1$. If we change the weighting coefficients, the result of solving this model can vary significantly and depends greatly on the nominal value of the different objective functions.

The normalized weighting method solves the problem of the pure weighting method; e.g., in the pure weighting method, the weighting coefficients do not reflect proportionally the relative importance of the objective, because of the great difference on the nominal value of the objective functions. In the normalized weighting method, w_i reflects closely the importance of objectives:

$$f(\mathbf{x}) = \sum_{i=1}^{r} \frac{w_i f_i(\mathbf{x})}{f_i^0} \tag{29}$$

where $w_i \geq 0$ and $\Sigma_{i=1}^{r} w_i = 1$. The condition $f_i^0 \neq 0$ is assumed.

5 THE RESULTS OF A NUMERICAL EXAMPLE

Data: $p_0 = 80$ kN/m, $L = 15$ m, $\rho = 7850$ kg/m^3, $h_{max} = 1800$, $b_{max} = 1000$, $t_{w,max} = 40$, and $t_{f,max} = 40$ mm.

Table 1 shows the results of the single-objective optimization using three different techniques. The differences between results are very small. All the techniques treat the variables as continuous ones and give unrounded optima. To give plate sizes available in the market, a secondary search is used for finding the discrete optima. The discrete steps for h and b are 50 mm for thicknesses $t_w/2$ and t_f 1 mm.

In Table 2, the multiobjective—Pareto—optima are given, obtained using five different techniques, for steel Fe 360 and for the ratio $k_f/k_m = 1.5$. Figure 2 shows the results in the coordinate system $f_1 - f_3$ for steels Fe 360 and Fe 510. The notation f_1^{510} means the optimum of f_1 for steel Fe 510.

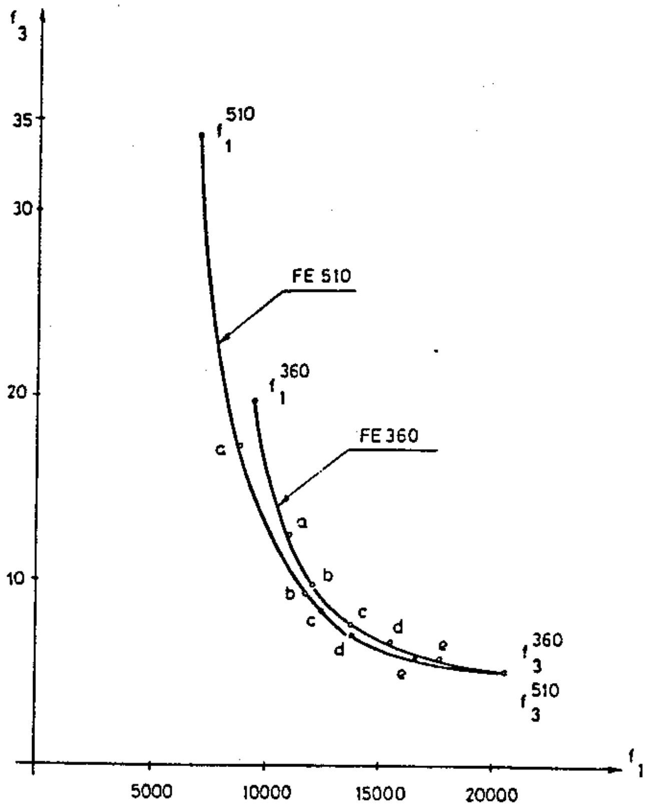


Fig. 2. Optima in a coordinate-system f_1-f_3 for various weighting coefficients, for steels Fe 360 and Fe 510, according to the weighting min-max technique. The points relate to the following weighting coefficients: (a) $w_1 = 0.90$, $w_2 = 0.10$, (b) $w_1 = 0.75$, $w_2 = 0.25$; (c) 0.50/0.50, (d) 0.25/0.75, and (e) 0.10/0.90.

The Pareto optima for various weighting coefficients of the weighting min-max technique can be seen between the limiting points of the single-objective optima. Note that the points c are the same also for the min-max technique. It can be seen that the single optimum of the deflection f_3 does not depend on the steel type, that is, $f_3^{360} = f_3^{510}$.

It can be seen from the Table 3 that cost savings of 24% may be achieved using Fe 510 instead of Fe 360, but the deflection will be nearly doubled.

Table 4 shows the results of the single-objective optimization for Fe 360 and $k_f/k_m = 1.5$ for the three objective

Table 4
Characteristics of beams optimized using single-objective optimization technique

Objective function	h *(mm)*	t_w *(mm)*	b *(mm)*	t_f *(mm)*	f_1 *(kg)*	f_2 *(kg)*	f_3 *(mm)*
f_1	1450	22	700	19	9,332	6,888	19.7
f_2	1500	24	700	16	9,577	6,876	19.0
f_3	1800	32	1000	40	20,801	16,202	5.1

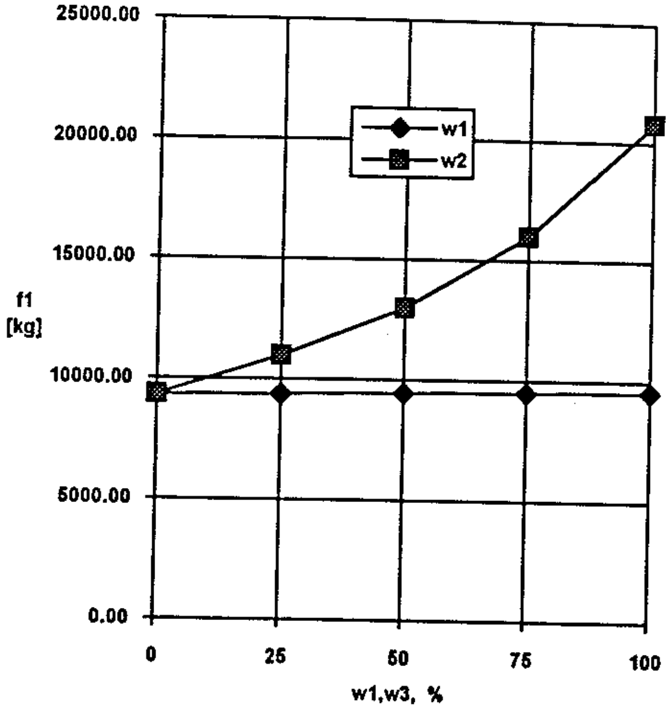


Fig. 3. Effect of weighting coefficients w_2 and w_3 on f_1.

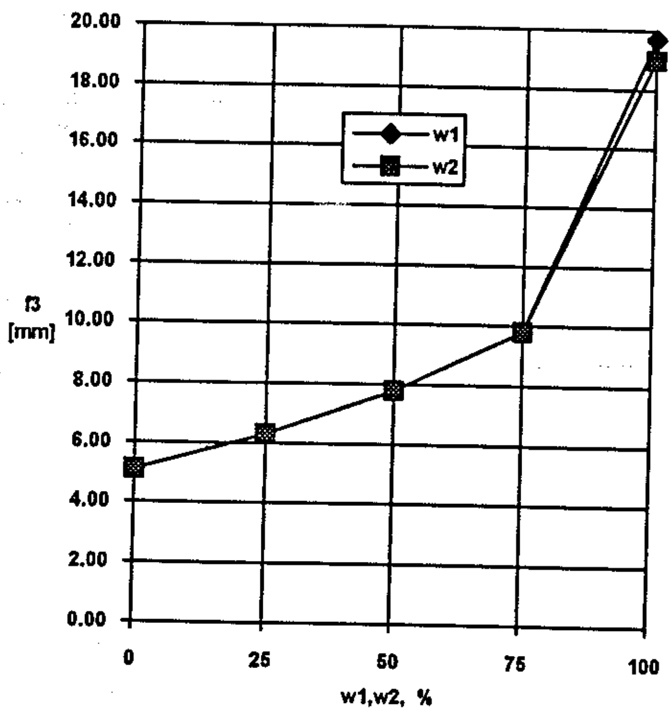


Fig. 4. Effect of w_1 and w_2 on f_3.

functions. Figures 3 and 4 show the effect of the relative importance of an objective function on the value of the other objective function.

6 CONCLUSIONS

The investigated numerical example illustrates the possibilities given for designers to select the most suitable structural version considering the cost, mass, and maximal deflection of a structure. It can be seen from Table 4 that the fabrication cost is about 26% of the total cost and therefore does not affect significantly the optima. In other words, the mass and the cost function are only slightly conflicting. Therefore, the mass f_2 is not shown in the figures. The effect of fabrication cost is much more significant in the case of a stiffened plate, as has been shown in another study.[5]

The deflection minimization leads to maximal prescribed sizes and to a significant increase in cost and mass. The use of steel Fe 510 instead of Fe 360 results in 24% cost savings without deflection minimization and no savings considering the deflection minimization. The multiobjective optimization gives structural versions for selected weighting coefficients according to Table 2 and Fig. 2.

ACKNOWLEDGMENTS

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