MEASUREMENT-INDUCED NONLINEAR DYNAMICS IN QUANTUM PROTOCOLS

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Early examples of nonlinear quantum state transformations are quantum state purification protocols [1–4]. They rely on identically prepared quantum systems which are subjected to an entangling unitary transformation and a subsequent selective measurement performed on parts of the system. The iterative application of these operations typically results in a strong dependence of the final state on the initial conditions and in measurement-induced complex chaos [5, 6]. Recently, it has been demonstrated [7] that the resulting strong sensitivity to initial conditions can in principle be used to amplify small initial differences of quantum states thus realizing a Schrödinger microscope [8] capable of distinguishing nonorthogonal quantum states.

Here we propose a cavity quantum electrodynamical scenario for implementing a Schrödinger microscope capable of amplifying differences between nonorthogonal atomic quantum states. The scheme (shown in Fig.1) involves an ensemble of identically prepared two-level atoms interacting pairwise with a single mode of the radiation field as described by the Tavis-Cummings model.

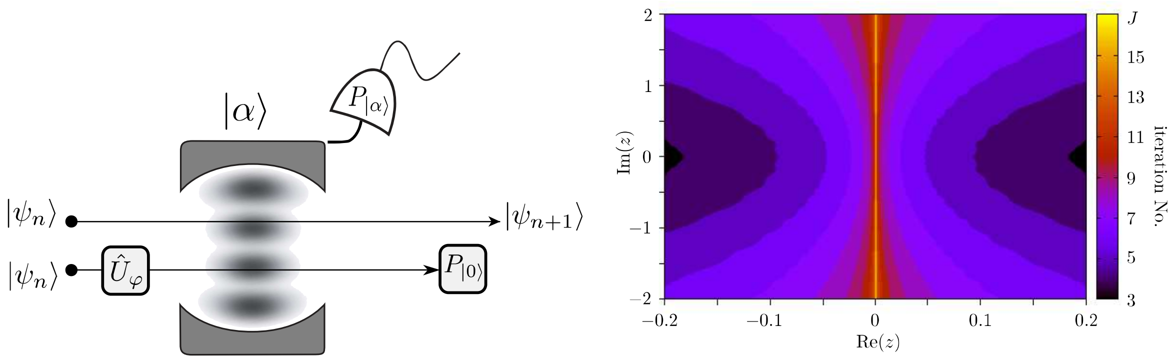

Figure 1: (left) Scheme for implementing a Schrödinger microscope. (right) The plane of initial states colored according to the number of iterations of $f_{\varphi=0}$ needed for a complex number z to reach the fixed points 1 (when $\mathrm{Re}(z) > 0$) or -1 (when $\mathrm{Re}(z) < 0$) with a precision of 0.1.

The two atoms, initially in the same state $|\psi_n\rangle = \mathcal{N}(|0\rangle + z|1\rangle)$, ($z \in \mathbb{C}$) interact with the cavity field prepared in a coherent state $|\alpha\rangle$. Before the interaction, the gate $\hat{U}_\varphi = e^{i\varphi}|0\rangle\langle 0| - e^{-i\varphi}|1\rangle\langle 1|$ is applied to one of the atoms and after the interaction and the projection of the field onto the initial coherent state, this same atom is projected onto its ground state. Finally, the other atom is left in the state $|\psi_{n+1}\rangle = \mathcal{N}(|0\rangle + f_\varphi(z)|1\rangle)$, where $f_\varphi(z)$ is a quadratic rational function of z dependent on the parameter φ introduced by the unitary $\hat{U}_\varphi$. Due to $f_\varphi(z)$ the atomic state is transformed in a nonlinear way. The dynamical properties of this nonlinear quantum transformation can be analyzed for different values of the parameter φ, and we find that e.g. in the special case when $\varphi = 0$ the resulting $f_{\varphi=0}(z) = 2z/(1 + z^2)$ transformation allows approximate orthogonalization of atomic states already after a few iterations of the protocol (see Fig.1), and thus the application of the scheme for quantum state discrimination [9].

The above type of Schrödinger microscope is useful for orthogonalizing qubit states that are initially situated on different hemispheres of the Bloch sphere (i.e., in regions separated by a great circle). We propose a further type here, which can approximate initial unknown states to any given pair of orthogonal states even if they are situated on the same hemisphere of the Bloch sphere, separated by any circle. This is equivalent to the quantum informational task of *quantum state matching*, i.e., deciding whether an unknown qubit state falls in a prescribed neighborhood of a reference state (that can be considered as one member of the mentioned orthogonal pair). We show that the prescribed neighborhood together with the given orthogonal pair of states unambiguously determines a quadratic rational map (a nonlinear transformation) which has the same iterative behavior (see details in Fig.2). We analyze which one- and two-qubit operations would physically realize the scheme. It is possible to find a single two-qubit unitary gate for each map or, alternatively, a universal special two-qubit gate together with single-qubit gates in order to carry out the task. We note that it is enough to have a single physical realization of the required gates due to the iterative nature of the scheme [10].

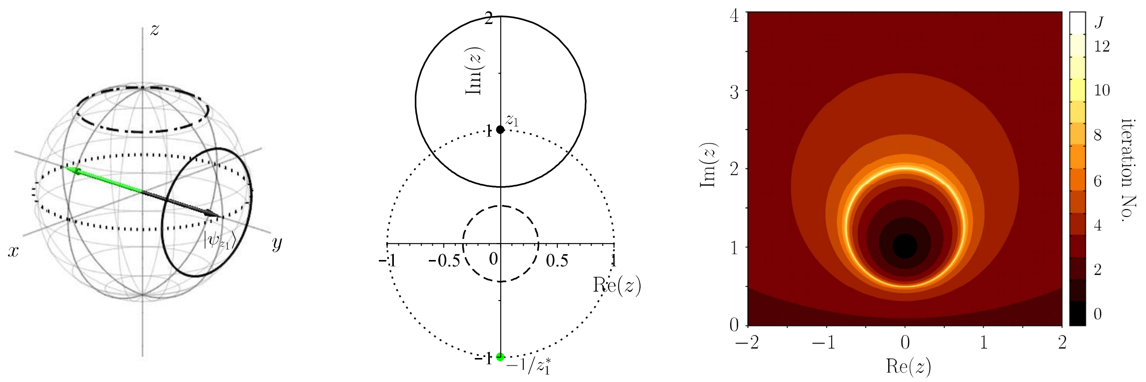

Figure 2: The nonlinear map which matches qubit states with state $|\psi_{z_1}\rangle = (|0\rangle + i|1\rangle)/\sqrt{2}$. (left) The prescribed neighborhood (black circle) can be transformed into a great circle. The transformed circles can be easily corresponded to nonlinear maps by considering the circle as the border between the two convergence regions of the map. This procedure can be used to find the map corresponding to the prescribed circle. The iterative application of this map will then approximate states to the reference state $|\psi_{z_1}\rangle$ (or to its orthogonal pair) if they are inside the black circle (or outside). (center) The same idea shown on the complex plane. (right) The complex plane after iterations of the map. The prescribed neighborhood corresponds to an initial overlap of $|s_\varepsilon|^2 = 0.9$.

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