QUANTUM STATE DISCRIMINATION OF THREE-LEVEL SYSTEMS VIA A NONLINEAR PROTOCOL

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Discrimination of nonorthogonal quantum states is an important task in modern quantum computation and quantum control [1]. Various protocols have been proposed for effective quantum state discrimination (QSD) (see reviews [2, 3]). A crucial ingredient of these methods is to have an ensemble of identical quantum systems for implementing unambiguous QSD [4, 6, 7]. Measurement-induced nonlinear dynamics is experimentally feasible in quantum optics [5], and as it has been shown [6, 7] that nonlinear quantum transformations could be a possible way for implementing QSD of two-level quantum systems. In this report we propose a scheme which can be used for QSD of three-level quantum systems.

Let us assume that we have an ensemble of identically prepared quantum systems in the state

$$|\psi_0\rangle = N(|0\rangle + z_1|1\rangle + z_2|2\rangle), \tag{1}$$

where $N = (1 + |z_1|^2 + |z_2|^2)^{-1/2}$ is the norm of $|\psi_0\rangle$. The problem is to distinguish two possibilities: (i) $|z_1| > |z_2|$ and (ii) $|z_1| < |z_2|$. We propose to use M steps of the nonlinear transformation shown in Fig.1.

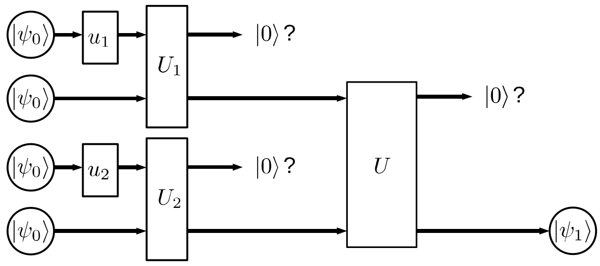

Figure 1: Scheme of the non-linear transformation $|\psi_0\rangle \rightarrow |\psi_1\rangle$.

We take two pairs of the system in initial state $|\psi_0\rangle$. Then one member of each pair is transformed by a single-system unitary u_j ($j = 1, 2$), after which U_j acts on a whole pair, and then we perfom selective projective measurements $P = |0\rangle\langle 0|$ on the first system of each pair. If both results are “yes” then we take the unmeasured systems from each pair and apply a joint unitary operator U on them. Then we perfom again a projective measurement P on the first system in the pair, and if the result is again “yes” then the unmeasured system transforms into the state $|\psi_1\rangle$. We propose to use the following unitary operators:

$$u_1 = |0\rangle\langle 2| + |2\rangle\langle 0| + |1\rangle\langle 1|, \quad U_1 = |01\rangle\langle 11| + |11\rangle\langle 01| + |00\rangle\langle 00| + \cdots + |22\rangle\langle 22|, \tag{2}$$

$$u_2 = |0\rangle\langle 1| + |1\rangle\langle 0| + |2\rangle\langle 2|, \quad U_2 = |02\rangle\langle 22| + |22\rangle\langle 02| + |00\rangle\langle 00| + \cdots + |21\rangle\langle 21|, \tag{3}$$

$$U = |01\rangle\langle 10| + |10\rangle\langle 01| + |00\rangle\langle 00| + |02\rangle\langle 02| + |11\rangle\langle 11| + \cdots + |22\rangle\langle 22|, \tag{4}$$

where by dots we denote all other possible diagonal elements. This procedure corresponds to the following nonlinear transformation of the initial state of Eq. (1)

$$|\psi_1\rangle = N' \left(|0\rangle + \frac{z_1}{z_2} z_1 |1\rangle + \frac{z_2}{z_1} z_2 |2\rangle \right). \tag{5}$$

Therefore, if $|z_1| \neq |z_2|$ we will have for some relatively large M: $|\psi_M\rangle \approx |1\rangle$ in the case of $|z_1| > |z_2|$, and $|\psi_M\rangle \approx |2\rangle$ in the case of $|z_1| < |z_2|$. We can distinguish these two cases by perfoming the projective measurement $|1\rangle\langle 1|$ on the system in the state $|\psi_M\rangle$, and then draw conclusion about the initial state $|\psi_0\rangle$. In Fig.2 (a,b) we show the probability of measuring state $|1\rangle$ after one step ($M = 1$) and three steps ($M = 3$), respectively. The border between regions with high and low probability corresponds to the $|z_1| = |z_2|$ condition, and this border becomes sharper with increasing M. Thus, the effectiveness of QSD increases with increasing M.

Another question arises: can we distinguish states in which $|z_1| = |z_2|$? For example, in Eq.(1) we have $z_1 = \rho$ and $z_2 = \rho \exp(i\varphi)$ ($\mathrm{Im}\rho = 0$) and we wish to know whether $\varphi > 0$ or $\varphi < 0$. It can be shown that in this case we have to implement the unitary transformation R

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & -i/\sqrt{2} \end{pmatrix} \tag{6}$$

on each initial state before doing the nonlinear transformation of Fig.1. Due to this operation we can treat the problem as above, since

$$R|\psi_0\rangle = N(|0\rangle + z_1'|1\rangle + z_2'|2\rangle), \tag{7}$$

$$|z_1'| = \rho\sqrt{1+\sin\varphi}, \quad |z_2'| = \rho\sqrt{1-\sin\varphi}, \quad \varphi \neq 0 \rightarrow |z_1'| \neq |z_2'|. \tag{8}$$

Therefore, applying the nonlinear transformation on the state (7) we can distinguish the two different situations: $0 < \varphi < \pi \rightarrow |z_1'| > |z_2'|$, and $-\pi < \varphi < 0 \rightarrow |z_1'| < |z_2'|$. In Fig.2 (c,d) we show the probability of measuring state $|1\rangle$ as a function of the initial values of ρ and φ.

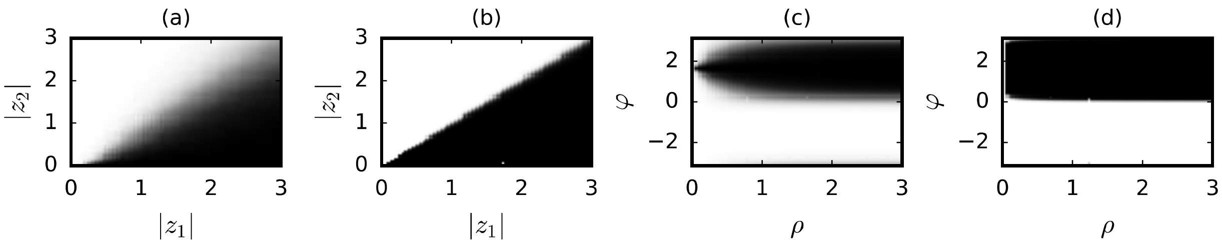

Figure 2: Probability of measuring state $|1\rangle$ after the nonlinear transformation as a function of the parameters of the initial state. $M = 1$ for subfigures (a) and (c), and $M = 3$ for subfigures (b) and (d). Black filling corresponds to probability 1, no filling corresponds to probability 0.

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