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Taylor iteration downward continuation method for gravity gradient tensor data

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Abstract Gravity gradient tensor downward continuation can highlight the anomaly characteristics of the shallow, reflect the characteristics of abnormal body horizontal density variation. Nowadays, most gradient tensor downward continuation studies are focusing on spatial domain, and the current obstacles are slow computation speed and small downward continuation height in frequency domain research. In this study, we showed a new method to downward continuation of gravity gradient data using Taylor iteration method. The general Taylor iteration method formula was deduced, and we proved the convergence of the approach. Modeling experiment showed a good fit to large point spacing. The analysis of the iterations radio showed that the Taylor expansion item is correlated with convergence speed. Furthermore, the application to the actual data of America Vinton, LA demonstrated that Taylor iteration method has a good effect to actual data.

Keywords Gravity gradient tensor · Taylor iteration downward continuation · Convergence · Iterations radio

1 Introduction

In recent years, with the development of airborne gravity and magnetic survey technology (Guan et al. 2002) and satellite gravity technology (Sun 2002; Zheng et al. 2010), plenty has been achieved by using gravitational tensor data (Robert and Pavel 2013; Klees et al.

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2000; Pail et al. 2011; Janak et al. 2009; Drinkwater et al. 2007). However, the level of these gravitational tensor data is generally much higher than the ground gravity survey surface. Therefore, the gravitational tensor data downward continuation is necessary.

The continuation of potential field data can highlight the anomaly characteristics of the deep or shallow source; thereby improve the resolution of anomaly and the reliability of the data interpretation. Continuation is composed of upward continuation and downward continuation. Although a rigorous theoretical model has been built for upward continuation, there is no suitable model for downward continuation, which is an ill-posed problem, amplifying the high frequency noise of data. Since regularization is a key technique that can solve the downward continuation ill-posed problem, choosing of appropriate regularization parameters is crucial (see e.g. Xu 1992, 2009).

In 1960s, Strakhov et al. put forward the idea of using iterative method to potential field continuation with no application (Strakhov and Devisyn 1965). Fedi and his colleagues then used ISVD method to study the downward continuation of potential field (Fedi and Florio 2002). A new quality-method for truncation was later proposed by Xu (1998). In Xu's method, the quality-based TSVD estimator with the basic ridge estimate of x as its initial values had outperformed F-statistical-based and L-curve TSVD approaches in terms of solution stability, bias and mean square error. Xu (2006), Xu and Yu (2007) derived the potential field iteration method formula and the interpolation-iteration method for potential field continuation from undulating surface to plane in detail, compared the effect of iteration downward continuation method and FFT downward continuation method, and applied the iteration method to the potential field data analysis. Wang et al. studied the derivative-iteration method for downward continuation and Taylor-iteration in potential field, which increased the potential field continuation height (Wang et al. 2011, 2012). The former studies provided the basic of gravitational tensor data downward continuation. Liu et al. introduced the spherical interior Dirichlet method, the Poisson integral iteration method and the spectral method of airborne gravity data downward continuation into the downward continuation of satellite gravity tensor data, achieved the gravity tensor downward continuation in spatial domain (Liu et al. 2011; Liu and Wang 2012). Jiang et al. analyzed the gravity tensor data upward and downward continuation problem in frequency domain, but the downward continuation height was only two times point spacing (Jiang et al. 2013).

In this paper, a new tensor research method, Taylor Iteration downward continuation method, is proposed for gravity gradient tensor data. It can largely increase the downward continuation point spacing. We first changed downward continuation factor into Taylor series expansion in frequency domain, using the sum of first N terms in series instead of downward continuation factor approximately. Then we did the downward continuation for the initial values and calculated the upward continuation values in the initial level from the first downward continuation values. Finally, after calculating the errors of the initial values and the upward continuations, the iteration procedure was stopped until the error of the measured values and N times iteration values were less than the given error.

2 Theory

Gravity tensor data meets the Laplace equation in the passive space outside the source, Xue derivated the solution (Xue 1978). As is well-known, the downward continuation formula is given by

$$V_{ij}(x,y,z)=\frac{-z}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{V_{ij}(x,y,0)}{\left[(\zeta-x)^2+(\eta-y)^2+z^2\right]^{3/2}}d\zeta d\eta \tag{1}$$

Assuming $R(x,y,z)=\frac{-z}{2\pi\left[(\zeta-x)^2+(\eta-y)^2+z^2\right]^{3/2}}$, then (1) can be rewritten as

$$V_{ij}(x,y,z)=V_{ij}(x,y,0)*R(x,y,z) \tag{2}$$

We do Fourier transform for both sides of (2), then get

$$\widetilde{V}_{ij}(u,v,z)=\widetilde{V}_{ij}(u,v,0)*\widetilde{R}(u,v,z) \tag{3}$$

where u, v are wavenumbers in x and y direction, respectively $z>0$ represents the downward continuation height, '$\sim$' represents spectrum and '$*$' is convolution operator. In practice, their own defined factors, instead of $\widetilde{R}(u,v,z)$, are usually used.

We make $\psi(u,v,z)=e^{2\pi\sqrt{u^2+v^2}z}$, where we change $\psi(u,v,z)$ into Taylor series at $z_0=0$, and selected the first N terms:

$$\varphi(u,v,z)=\sum_{n=0}^{N}\frac{(2\pi\sqrt{u^2+v^2}z)^n}{n!} \tag{4}$$

We do downward continuation for $V_{ij}^{A}(x,y,0)$ at level A to get the $V_{ij}^{B(0)}$ at level B, that is

$$\widetilde{V_{ij}^{B(0)}}(u,v,z)=\widetilde{V_{ij}^{A}}(u,v,0)\varphi(u,v,z) \tag{5}$$

where $\widetilde{V_{ij}^{B(0)}}(u,v,0)$ is the approximate spectrum at continuation level B, $\widetilde{V_{ij}^{A}}(u,v,0)$ is the spectrum at level A. Both sides of (5) times $\phi(u,v,z)=e^{-2\pi\sqrt{u^2+v^2}z}$, which will generate the approximate spectrum at the continuation level A that is upward continuation from $\widetilde{V_{ij}^{B(0)}}(u,v,0)$, that is

$$\widetilde{V_{ij}^{A(1)}}(u,v,0)=\widetilde{V_{ij}^{B(0)}}(u,v,z)\phi(u,v,z) \tag{6}$$

Then, the first-order residual spectrum is

$$\delta\widetilde{V_{ij}^{A(1)}}=\widetilde{V_{ij}^{A}}(u,v,0)-\widetilde{V_{ij}^{A(1)}}(u,v,0) \tag{7}$$

We do downward continuation of the first-order $\delta\widetilde{V_{ij}^{A(1)}}$ to the level B, which will get the first-order residual spectrum at level B, that is

$$\delta\widetilde{V_{ij}^{B(1)}}=\delta\widetilde{V_{ij}^{A(1)}}\varphi(u,v,z) \tag{8}$$

Then the first-order approximate spectrum at level B is represented as

$$\widetilde{V_{ij}^{B(1)}}(u,v,z)=\widetilde{V_{ij}^{B(0)}}(u,v,z)+\delta\widetilde{V_{ij}^{B(1)}} \tag{9}$$

Repeat the steps above, finally we get

$$\begin{cases} \widetilde{V_{ij}^{B(m)}}(u,v,z) = \varphi(u,v,z)\widetilde{V_{ij}^{A}}(u,v,0) + (1-\varphi(u,v,z)\phi(u,v,z))\widetilde{V_{ij}^{B(m-1)}}(u,v,z) \\ \widetilde{V_{ij}^{B(0)}}(u,v,z) = \widetilde{V_{ij}^{A}}(u,v,0)\varphi(u,v,z) \end{cases} \quad (10)$$

where $V_{ij}^{A}(x,y,0)$ is the gravity gradient tensor at level A, and $V_{ij}^{A(m)}(x,y,0)$ is the gravity gradient tensor at level A after m times iteration. The iteration will not stop until $\max\left|V_{ij}^{A}(x,y,0) - V_{ij}^{A(m)}(x,y,0)\right| < \varepsilon$. Finally, we do inverse Fourier transformation for $\widetilde{V_{ij}^{B(m)}}(u,v,z)$ to get the gravity gradient tensors.

3 Convergence analysis

We do convergence analysis for this method to validate the stability of the Taylor iteration downward continuation and computation speed. From Eq. (10) we get

$$\frac{\widetilde{V_{ij}^{B(m)}}(u,v,z) - \widetilde{V_{ij}^{B(m-1)}}(u,v,z)}{[\widetilde{V_{ij}^{B(m-1)}}(u,v,z) - \widetilde{V_{ij}^{B(m-2)}}(u,v,z)]} = [1-\varphi(u,v,z)\phi(u,v,z)] \quad (11)$$

We do further processing for Eq. (11), then

$$\widetilde{V_{ij}^{B(m-1)}}(u,v,z) = \{1-[1-\varphi(u,v,z)\phi(u,v,z)]^{m}\}\psi(u,v,z)\widetilde{V_{ij}^{A}}(u,v,0) \quad (12)$$

We limit Eq. (12), then

$$V = \lim_{m\to\infty} \widetilde{V_{ij}^{B(m-1)}}(u,v,z) = \widetilde{V_{ij}^{B(m)}}(u,v,z) = \psi(u,v,z)\widetilde{V_{ij}^{A}}(u,v,0) \quad (13)$$

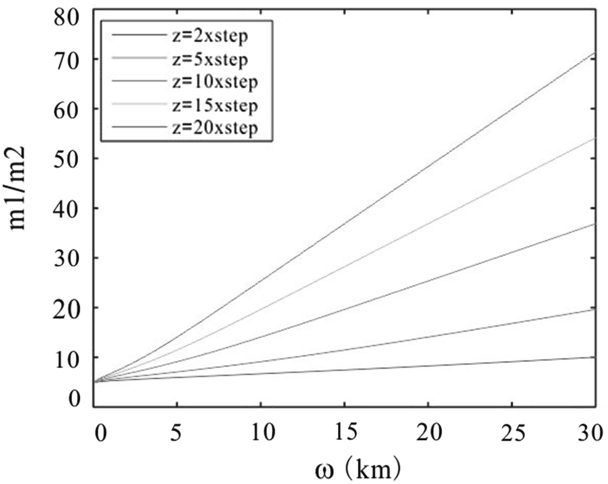

Fig. 1 Radio curve of iterations

When $m \to \infty$, the spectrum after m times iteration verges to the continuation spectrum at level B. As mentioned above, when $\max\left|V_{ij}^{A}(x,y,0) - V_{ij}^{A(m)}(x,y,0)\right| < \varepsilon$, the iteration will stop. That means the desired $\widetilde{V_{ij}^{B(m)}}(u,v,z)$ exist and it is a limit value. In other words, this method is convergent.

From Eq. (12) we can get the downward continuation value of level B, where we make the downward continuation values at N = 0 and N = 1 are equal, the iteration numbers are m1 and m2 respectively, make $\omega = 2\pi\sqrt{u^2 + v^2}$, then we get

$$\frac{m1}{m2} = \frac{\log(1 - e^{-\omega z} - \omega z e^{-\omega z})}{\log(1 - e^{-\omega z})} \tag{14}$$

For the different N in the Taylor series, Eq. (12) represents that the iteration number radio when the downward continuation values have the same accuracy.

Figure 1 shows the relationship between the radio and w at different downward continuation heights. We found that the value of m1/m2 presents the increasing trend with the downward continuation height from the graph. When the downward continuation height and error range are given, the iteration number of N = 0 is bigger than that of N = 1. When the downward continuation height is large, we just need small iteration number to achieve the given error range, which will save the computation time. However, if the truncation item N > 1, which will largely enlarge the high frequency information. Although the iteration number will reduce, the result is not expected. Because if N is big enough, Eq. (4) is equal to $\psi(u,v,z) = e^{2\pi\sqrt{u^2+v^2}z}$,this method is same to FFT method. Generally, N = 0 or N = 1 is appropriate.

In order to analyze the truncation error accuracy of Taylor iteration downward continuation, we brought in the relative mean square error and the average relative error to quantitatively describe the tensor continuation accuracy. The relative mean square error formula is

$$e_{ij} = \sqrt{\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left(V_{ij}^{theo}(m,n) - V_{ij}^{count}(m,n)\right)^2} \tag{15}$$

We use (13) to computer the square error between gravity tensor and the mean of them,

$$e_{ij}^{'} = \sqrt{\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left(V_{ij}^{count}(m,n) - V_{ij}^{mean}(m,n)\right)^2} \tag{16}$$

Then the average relative error of downward continuation formula is

$$\varepsilon = \frac{e_{ij}}{e_{ij}^{'}} \times 100\ \% \tag{17}$$

where $(V_{ij}^{theo}(m,n)$ is the theory value, $V_{ij}^{count}(m,n)$ is the calculated value, $V_{ij}^{mean}(m,n)$ is the mean of $V_{ij}^{count}(m,n)$, i and j represent x, y and z respectively, M is the survey line number, and N is the survey point number at every survey line.

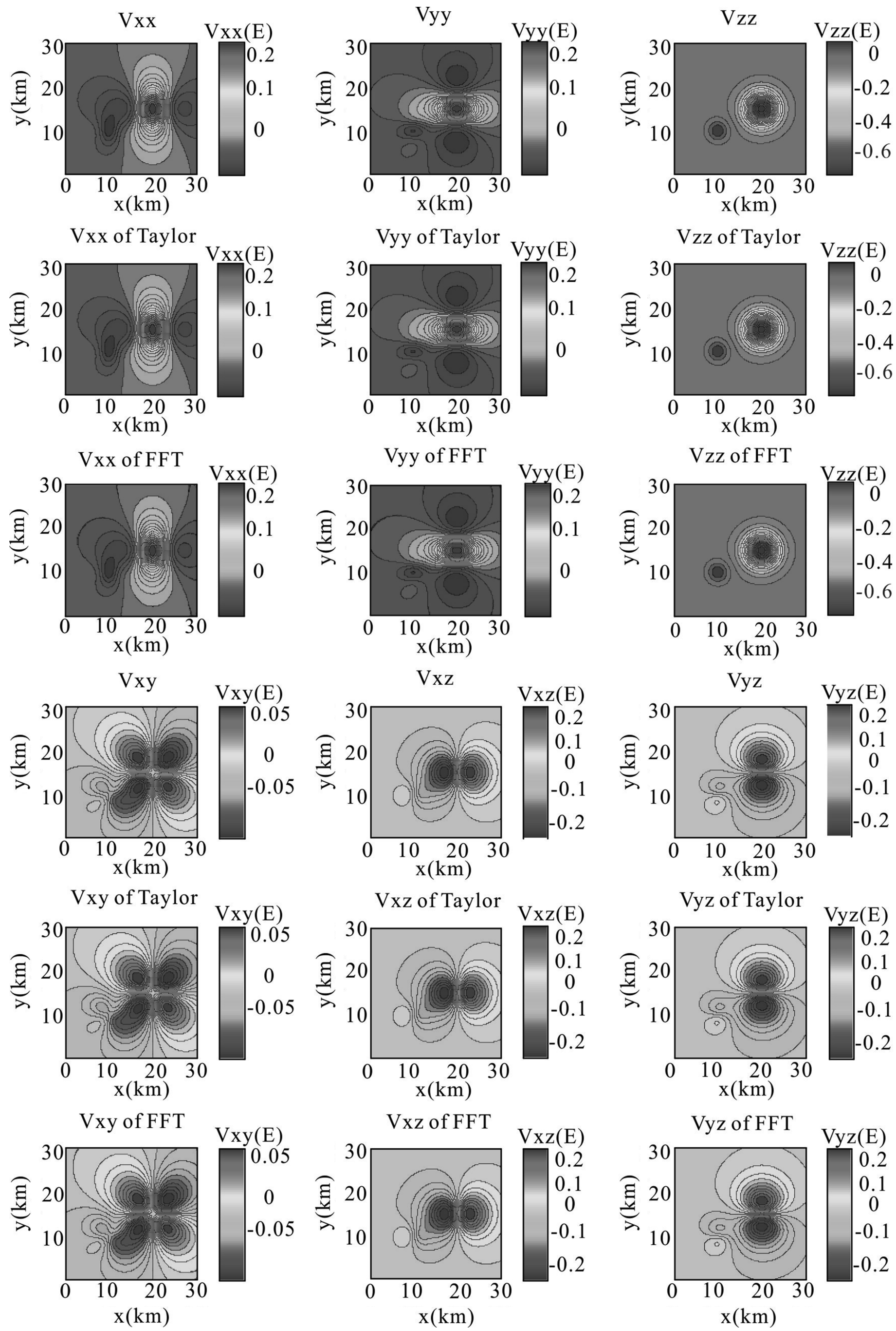

Fig. 2 Contour map of the theoretical gradient tensors, gradient tensors of downward continue to 200 m using Taylor iteration downward continuation method and FFT

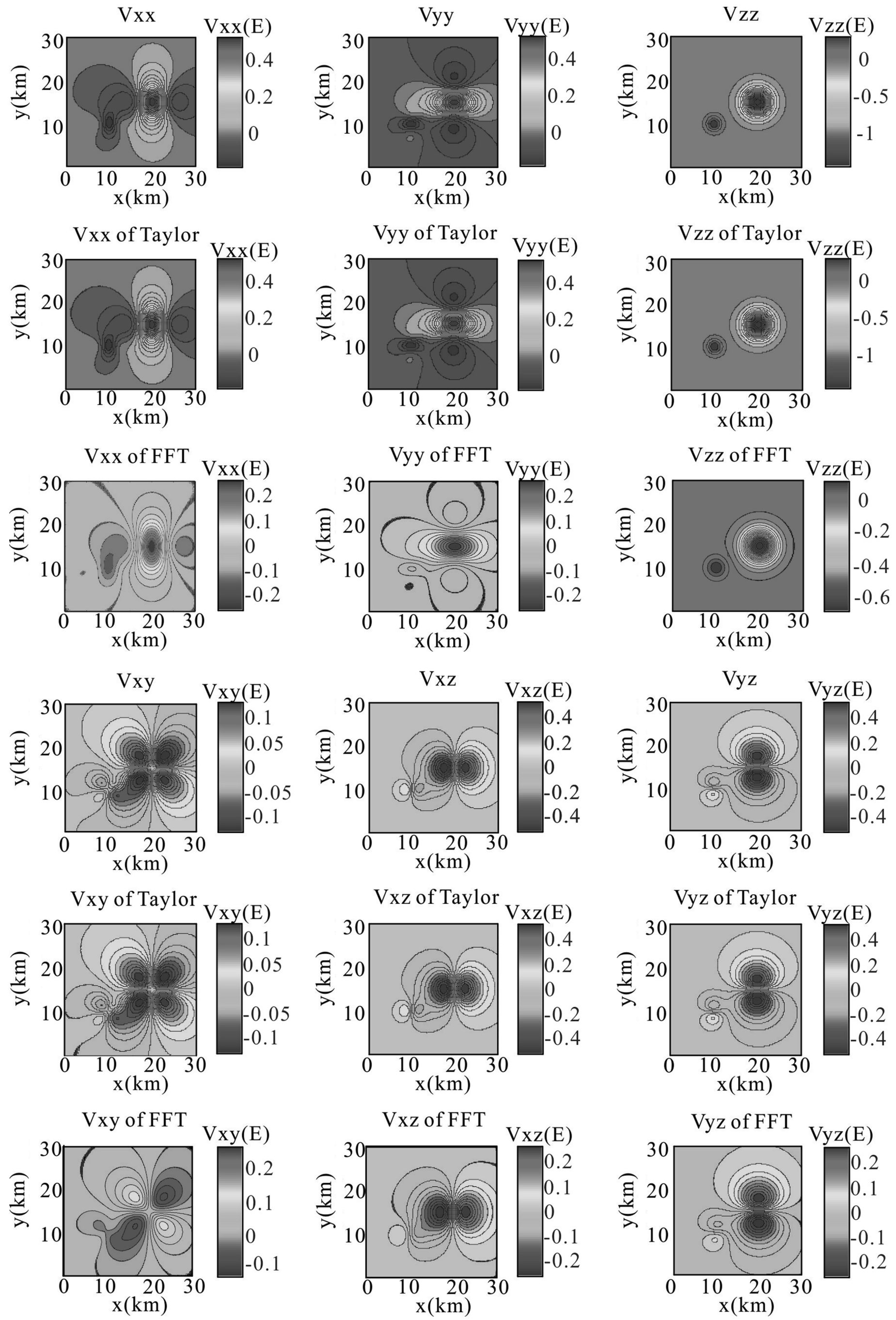

Fig. 3 Contour map of the theoretical gradient tensors at 1100 m, gradient tensors of downward continue to 1100 m using Taylor iteration downward continuation method and gradient tensors of downward continue to 250 m using FFT

4 Model experiment

We selected a model with two spheres that have different depth and residual density. Their radiuses are 0.4 and 0.9 km, and the residual densities are 0.2 and -0.3 g/cm^3 respectively. The coordinates of the sphere center are (10, 10, 4) and (20, 15, 6) with the unit in a km. The grid height is 100 m. We analyzed the gravity tensors that were calculated by Taylor iteration downward continuation and FFT to 200, 1100 and 2000 m (Figs. 2, 3, 4). Error statistics are shown in Table 1.

As is shown in Fig. 2, the six gravity tensors downward continued to 200 m by both the FFT and Taylor iteration method are very closed to the theory values, and the contour shape is consistent. But Table 1 shows that all relative square errors of Taylor iteration downward continuation method are all smaller than FFT, and the minimum value is 1.4×10^{-5} E. All average relative errors of Taylor iteration downward continuation method are also smaller than FFT, which are listed on the 3rd and 7th column of Table 1. In other words, although the six gravity gradient tensors produced by Taylor iteration

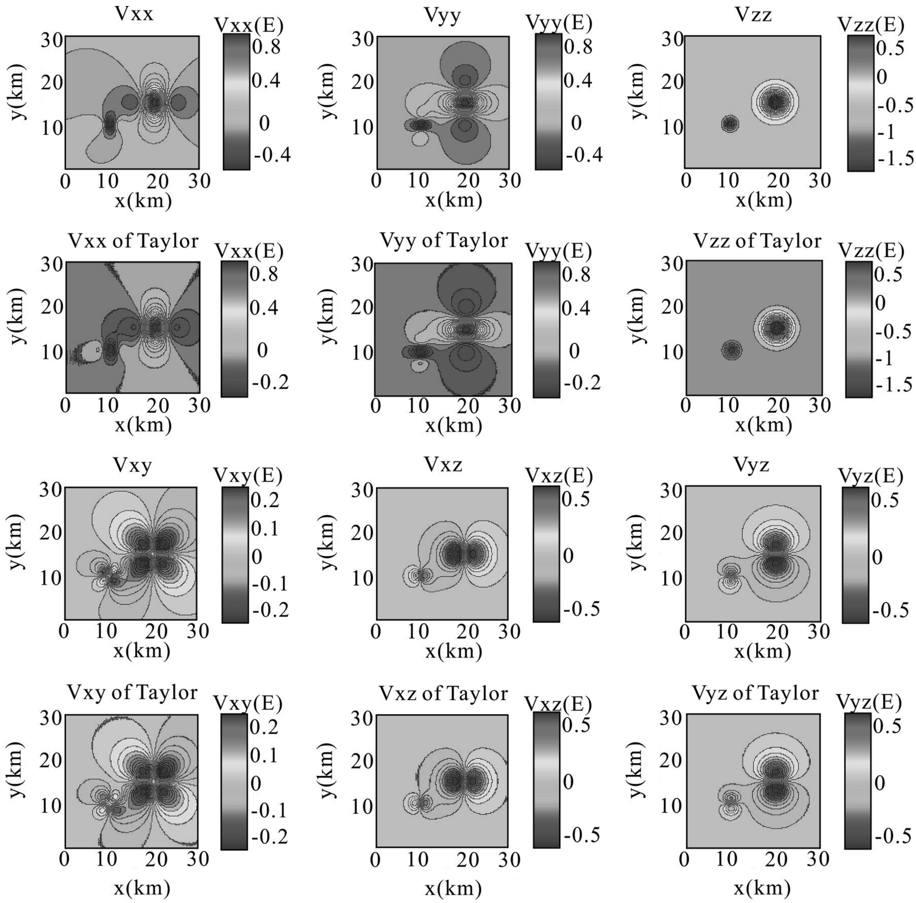

Fig. 4 Contour map of the theoretical gradient tensors, gradient tensors of downward continue to 2000 m using Taylor iteration downward continuation method

Table 1 Error analysis of different depth gradient tensors of Taylor iteration method and FFT

	Taylor downward continuation			FFT		
	200 (m)	1100 (m)	2000 (m)	200 (m)	250 (m)	350 (m)
V_{xx}						
e (E)	1.6×10^{-5}	2.46×10^{-4}	5.7×10^{-3}	2.8×10^{-4}	0.0024	0.24
ε (%)	0.0029	0.31	4.65	0.50	4.22	97.05
V_{yy}						
e (E)	1.7×10^{-5}	2.83×10^{-4}	4.8×10^{-3}	2.2×10^{-4}	0.0021	0.22
ε (%)	0.0031	0.36	3.92	0.41	3.74	96.57
V_{zz}						
e (E)	2.2×10^{-5}	1.90×10^{-4}	8.2×10^{-3}	1.6×10^{-4}	2.7012e-5	0.05
ε (%)	0.0024	0.15	4.14	0.18	1.08	42.37
V_{xy}						
e (E)	1.4×10^{-5}	1.81×10^{-4}	2.7×10^{-3}	1.5×10^{-4}	0.0016	0.18
ε (%)	0.0044	0.40	3.86	0.49	4.95	98.21
V_{xz}						
e (E)	3.2×10^{-5}	3.19×10^{-4}	3.6×10^{-3}	1.4×10^{-4}	0.0012	0.11
ε (%)	0.0048	0.34	2.50	0.22	1.74	84.67
V_{yz}						
e (E)	2.2×10^{-5}	1.82×10^{-4}	3.2×10^{-3}	9.3×10^{-4}	6.6161e-4	0.05
ε (%)	0.0034	0.20	2.29	0.14	0.99	54.74

method and FFT are closed to the theoretical values when the downward continued height is small, the relative square errors of Taylor iteration method is smaller than that of FFT. It demonstrates that Taylor iteration method is better than FFT.

Figure 3 shows that the contour of calculated values downward continued to 1100 m by Taylor iteration downward continuation method agrees well with the theoretical values, but all contour shapes of calculated values that are downward continued to 250 m by FFT without filter suppress factor present oscillation. Table 1 shows the relative square errors and average relative errors that are downward continued to 1100 m by Taylor iteration downward continuation method, both of them have high accuracy, see the 4th column in Table 1. When the tensors are downward continued to 350 m by FFT, the average relative errors are generally above 50 %, see the last column in Table 1. Figure 4 shows that the contour shape of calculated values that are downward continued to 2000 m by Taylor iteration downward continuation method are consistent with theoretical values. Although V_{xx}, V_{yy} and V_{zz} present oscillation, the results are trustworthy. It indicates that Taylor iteration downward continuation method is much better than FFT when the downward continuation height is large. From Figs. 3 and 4, we found that V_{zz}, V_{zx} and V_{zy} are always consistent with theoretical characteristics when the downward continuation height is large. However, when the continuation height is 1100 m, V_{xy} is a little worse than the other tensors. This is because V_{xy} mainly reflects the change rate of V_{xx} in N–S direction. There is a Gibbs effect on the boundary of model, which may be related to the selected model. The radius and residual density of the sphere at (10, 10, 4) are both small, so the gravity anomaly is small too. In addition, it is also related to the extension range of the extension function. When the downward continuation height is 2000 m, V_{xx}, V_{yy} and V_{yy} are a little

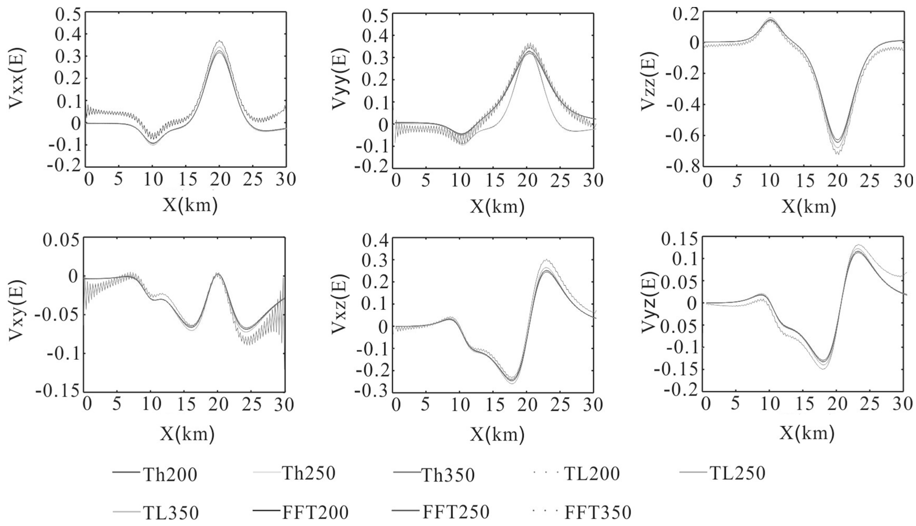

Fig. 5 Gravity gradient tensors profile map at different depth using Taylor iteration downward continuation method and FFT. Th200, Th250 and Th350 are the theoretical values at 200, 250 and 350 m respectively. TL200, TL250 and TL350 are the calculated values of downward continued to 200, 250 and 350 m respectively using Taylor iteration method. FF200, FF250 and FF350 are the calculated values of downward continued to 200, 250 and 350 m respectively using FFT

worse than the other three tensors, while the other tensors keep well with the theoretical tensors. From V_{zz}, we can simply find the boundary of these two spheres.

In order to compare the difference among Taylor iteration downward continuation method, FFT, and the downward continuation height of Taylor iteration downward continuation method, we plotted the gravity gradient tensors profile maps at different downward continuation heights (Figs. 5, 6). The profiles (dot curve in Figs. 2 and 5) show that the values calculated by both Taylor iteration downward continuation method and FFT are consistent with the theoretical values, when the downward continuation heights are 200 and 250 m. But the curve oscillation of FFT is bigger than that of Taylor iteration downward continuation method when the downward continuation height is 350 m. When the downward continuation height is 1100 m, the calculated values by Taylor iteration downward continuation method are consistent with the theoretical values (Fig. 6). The results are trustworthy when the downward continuation height is 2000 m, although the extreme points calculated by Taylor iteration downward continuation method are less than the theoretical values.

When using Taylor iteration downward continuation method to downward continue large point spacing, both the extension range of the extension function and the iteration numbers will impact the final results. In order to achieve the best result in practice, we should choose the appropriate truncation parameter and downward continue height. In general, the continue height should be less than 20 times spot spacing. For the truncation parameter, we can select $n = 1$or $n = 2$ in Eq. (4).

In conclusion, Taylor iteration method is generally better than FFT in all the downward continue heights, and the advantage of Taylor iteration method is especially distinct when the downward continue height is relatively large.

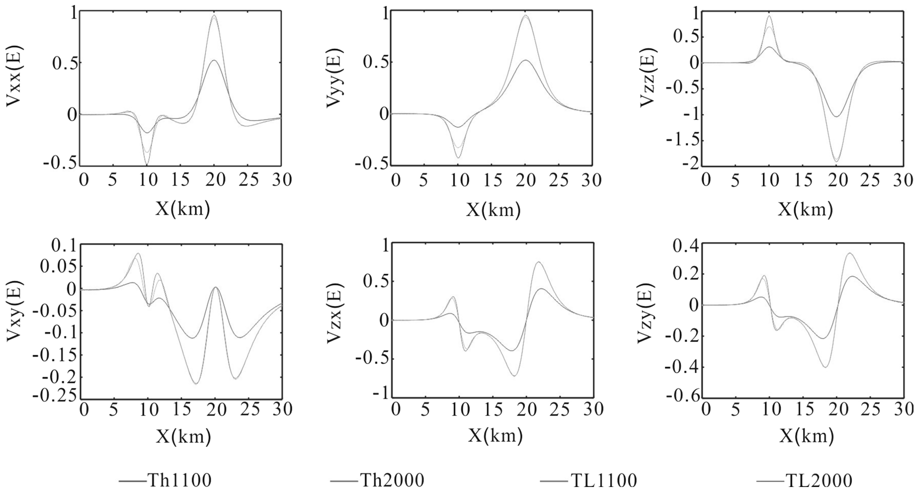

Fig. 6 Gravity gradient tensors profile map of downward continued to 1100 and 2000 m using Taylor iteration downward continuation method. Th1100 and Th2000 are the theoretical values at 1100 and 2000 m respectively. TL1100 and TL2000 are the calculated values of downward continued to 1100 and 2000 m respectively using Taylor iteration method

5 Application to real data

To test the practicality of this method, we downward continued the actual tensors in Vinton, LA using Taylor Iteration downward continuation method. The actual data were measured in 2008 by Bell Geospace company in Vinton Dom, where is in the southwest Louisiana and near to the border of Texas. In order to highlight the feature details of Taylor iteration downward continuation, we only selected a small area near Vinton Dom for analysis. The data are in the WGS84 reference frame, and the coordinate range is x (441, 445), y (3333, 3336) with the unit in km. The distance between measuring points is 4 m, and the distance between measuring line height is 24 m. The real data were downward continued to 8 m which is two times the measuring point distance using both FFT and Taylor iteration downward continuation methods (Fig. 7). We downward continued the six tensors to 44 m which was 11 times the measuring point distance using Taylor iteration downward continuation (Fig. 8).

Figure 7 shows the results of original value and calculated values of downward continued to 8 m using FFT and Taylor iteration downward continued methods. The shallow anomaly information was enhanced after downward continuation. The results of Taylor iteration downward continuation method and FFT are consistent with each other, but the high frequency noise of FFT was amplified without filter suppress factor. The Taylor iteration downward continuation method is better in suppressing the high frequency noise.

As shown in Fig. 7, the high anomaly in V_{zz} is Vinton Dom. The anomaly after downward continuation agrees with the initial anomaly at high values, and the anomaly shape is directly related to the source shape. The anomaly after downward continuation reflects the boundary information of anomaly, and the sign of anomaly directly represents the residual density of abnormal body. In contrary to V_{zz}, V_{xx} and V_{yy} mainly reflect the change of gravity anomaly in north–south and east–west direction respectively, which show negative in the anomaly area where V_{zz} is high value, see V_{xx} of Taylor and V_{yy} of

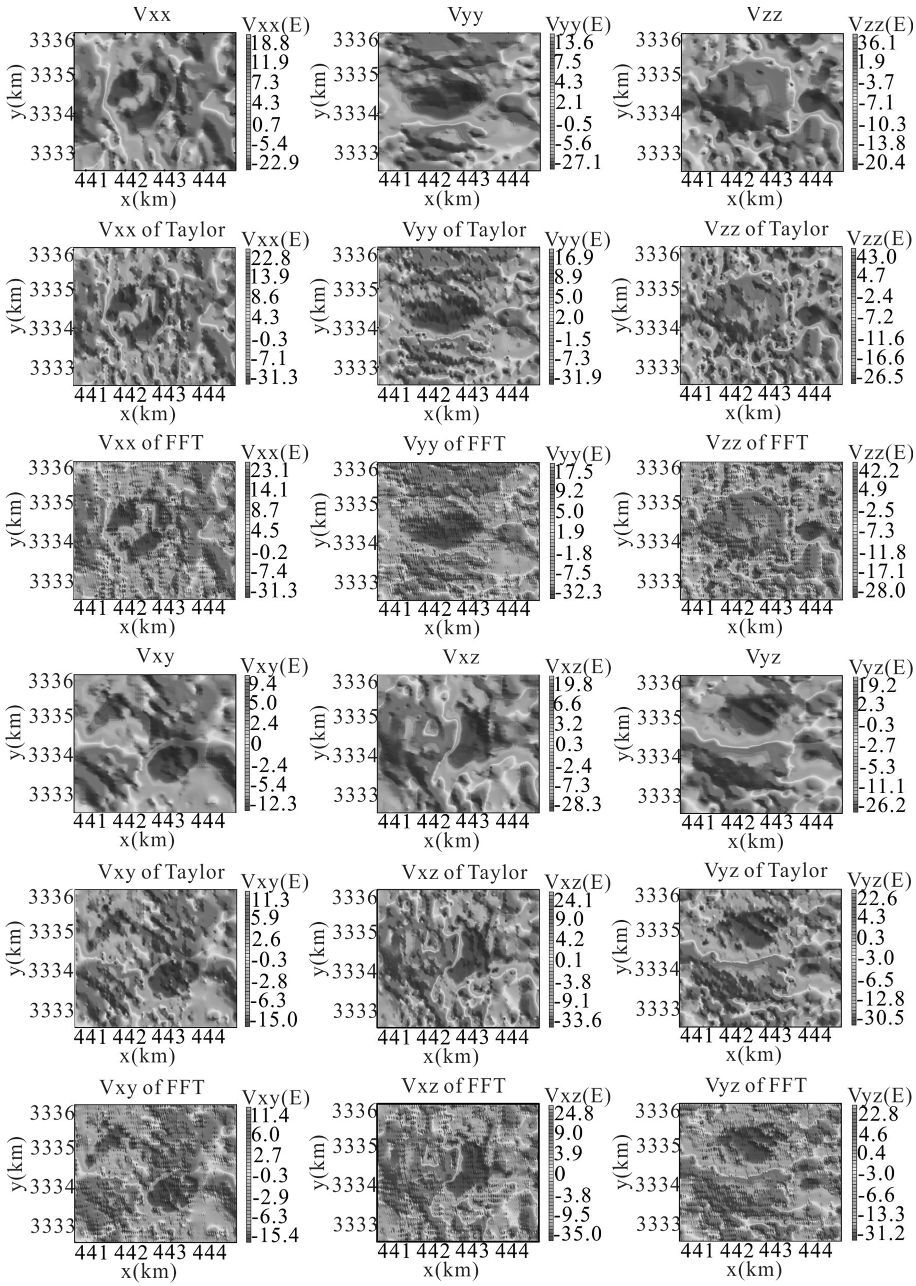

Fig. 7 Contour map of Vinton, LA gradient tensors and tensors after downward continuation 8 m using Taylor iteration downward continuation method and FFT

Taylor in Fig. 8. After downward continuation, V_{xx} shows more sophisticated northeast-southwest tensile characteristics, which agrees with the characteristics before downward continuation. V_{yy} shows east–west tensile characteristics after downward continuation, and

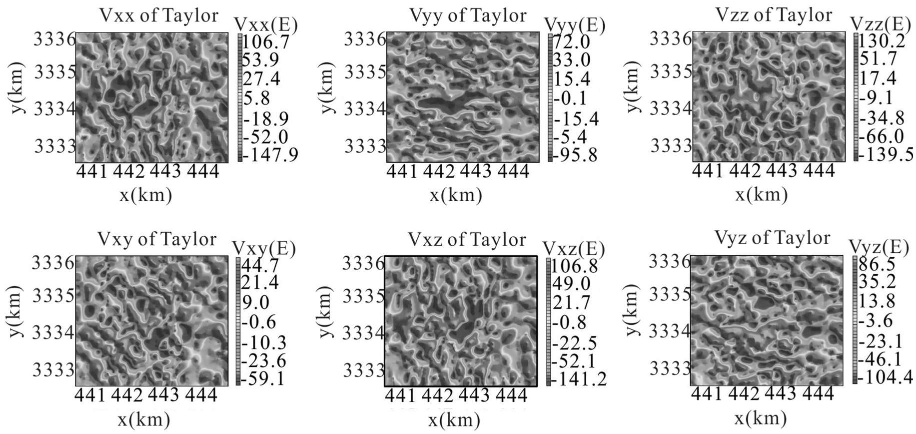

Fig. 8 Contour map after downward continuation 44 m using Taylor iteration downward continuation method

the boundary is more obvious, which agrees with the characteristics before downward continuation too. V_{xz} and V_{yz} represent the change of gravity anomaly V_z in north–south and east–west respectively, which are zero just above the anomaly, while they have higher values in the boundary of the source body, which were increased after downward continuation, see V_{xz} of Taylor and V_{yz} of Taylor in Fig. 8. V_{xy} mainly reflects the change characteristics of V_x in north–south direction, see V_{xy} of Taylor in Fig. 5, the effect is not very prounounced, which is consistent with the conclusion of the model research. In general, the tensor characteristics after downward continuation agree with the characteristics before downward continuation, but the tensors after downward continuation can reflect the sophisticated structure of underground abnormal body, meanwhile the shallow abnormal body information was enhanced.

6 Conclusions

Taylor iteration downward continuation formula in frequency domain was derived theoretically in this paper, and the convergence of this method was proved. We analyzed the relationship of different iteration number N, and also quantified the relative square error and average relative error of Taylor iteration downward continuation.

With the model analysis, we conclude that Taylor iteration shows clear advantages compared with FFT method when the downward continuation height is large point spacing. Although six tensors continued to 200 m by both FFT and Taylor iteration methods are very close to the theoretical values, the relative square errors from Taylor iteration method are always smaller than those from FFT method. Moreover, the tensors that were downward continued to 1100 m by Taylor iteration method still keep high accuracy, while the contour shape of the calculated values downward continued to 250 m by FFT method all presents oscillation, and when the tensors are downward continued to 350 m by FFT method, the average relative errors are generally above 50 %. It has been shown that contour shape of the calculated values downward continued to 2000 m by Taylor iteration method is consistent with that of theoretical values in Fig. 4. In other words, Taylor iteration method largely outperformed the FFT method when the downward continuation height is large point spacing. The gravity gradient tensors profile maps shown in Figs. 5 and 6 also confirm this conclusion.

Using Taylor iteration downward continuation method to actual gravity tensors shows that it has good practical effect. The gravity gradient tensors downward continued can reflect the sophisticated information of the underground geological body, and the shallow abnormal body information can also be enhanced.

We should note that in this manuscript, we are limited to downward continuation problems with a small height. Thus, the problems under study would only be slightly ill-posed. In the case of downward continuation of (satellite) data with a large height, one will have to choose the truncation number for a best solution, as in the truncated SVD case of Xu (1998).

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