

The sundial of Augustus and its survey: unresolved issues and possible solutions

V. Baiocchi¹  · M. Barbarella² · M. T. D'Alessio³ ·
K. Lelo⁴ · S. Troisi⁵

Received: 7 May 2015 / Accepted: 18 August 2015 / Published online: 27 August 2015
© Akadémiai Kiadó 2015

Abstract A lively debate has developed regarding the characteristics of the so-called *Horologium Augusti*, at first known only through a notice in Pliny and subsequently discovered (at least partly) during the course of excavations begun in 1997 (Leonhardt, in: The Horologium of Augustus: debate and context, 2014). The gnomon of the “Horologium” was composed by the obelisk that presently is nearby in “Piazza Montecitorio” in Rome (Fig. 1). A large part of the debate has centred on the very function of the *Horologium*, in particular whether it was a true functioning solar clock or simply a sundial. The scope of the present essay concentrates rather on the metrical accuracy that such a sundial could have had; in particular, we will hazard a hypothesis as to the accuracy with which the direction of the sundial was laid out and the possibility of measuring the azimuth

✉ V. Baiocchi
valerio.baiocchi@uniroma1.it

M. Barbarella
maurizio.barbarella@unibo.it

M. T. D'Alessio
mt.dalessio@uniroma1.it

K. Lelo
keti.lelo@uniroma3.it

S. Troisi
salvatore.troisi@uniparthenope.it

¹ Dipartimento Ingegneria Civile, Edile ed Ambientale, Sapienza University of Rome, Via Eudossiana, 18, 00184 Rome, Italy

² Dipartimento di Ingegneria Civile, Chimica, Ambientale e dei Materiali, Alma mater Studiorum, Università di Bologna, Viale Risorgimento 2, 40126 Bologna, Italy

³ Dipartimento Scienze dell'Antichità, Sapienza University of Rome, via dei Volsci 122, 00100 Rome, Italy

⁴ CROMA, “Roma3” University, Via Ostiense, 139, 00154 Rome, Italy

⁵ Dipartimento di Scienze e Tecnologie, “Parthenope” University, Centro Direzionale, isola C4, 80143 Naples, Italy

in its present placement. Such a detailed geodetic-topographic survey of the portion thus far excavated, could provide useful information for the eventual pursuit of excavations yielding, at the same time, further avenues of research; as an example it would also allow for the deduction of two pieces of information still not entirely established: the exact height of the gnomon, and the exact position of the original placement of the axis of the obelisk.

Keywords Augustus · Sundial · Meridian · Ancient Rome · Solar clock

1 Introduction: “Horologium Augusti”

Several of the most representative monuments of Augustus’ political strategy in Rome occur in the context of a vast urban development operation concerned with the monumental reorganization of the centre of the Campo Marzio. In this area—set between the Tiber and the via *Lata* (which corresponds to the present-day via del Corso)—which was previously outside the city and therefore partially undeveloped, we find for example the Mausoleum destined for the *princeps* and his family, the *Ara Pacis* that celebrated the prosperity and peace restored at the end of the civil wars, the Pantheon, and the *Saepta*, where civic elections took place.

In this area also rose an extraordinary work of engineering that was treated as a sundial and that is known as the *Horologium Augusti*. This monument was inaugurated in 9 BCE and dedicated to the Sun (*CIL*, VI.702), a symbol of the new era initiated by Augustus. Pliny (*Storia naturale*, 36.71–73) records the function of the sundial, which was to measure “the length of the days and nights” and also indicate the passage of the days during the course of the year.

The gnomon was composed of an obelisk in granite 22 m high inscribed with hieroglyphs, brought to Rome by sea in 10 BCE from the Egyptian city of Heliopolis (Strabon, 7.1.27; Pliny, *Storia naturale*, 36.71), where it had been erected by the Pharaoh Psammetichus II at the beginning of the sixth century BCE; this is the obelisk still recognizable in piazza Montecitorio in front of the eponymous palace. Atop the obelisk, a sphere in gilded

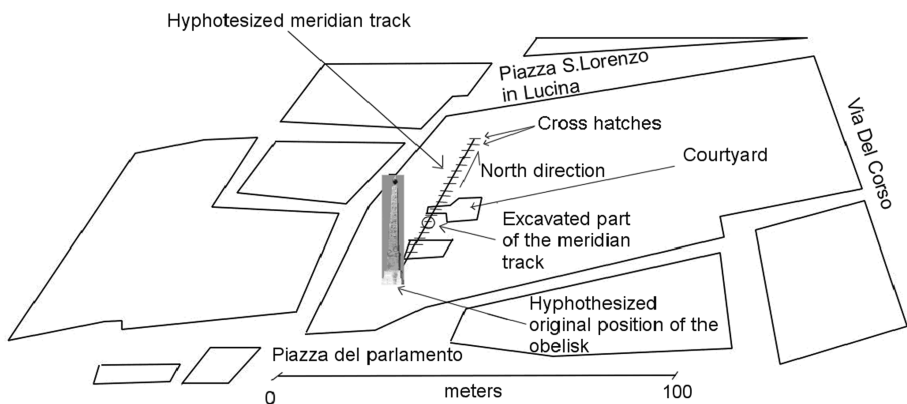


Fig. 1 Approximate position of the “Horologium”: present ground level is approximately 6 m higher than Augustus age ground

bronze surmounted by a needle projected its shadow onto the pavement underneath. Here, a network of lines in bronze and a series of inscriptions on the slabs of the pavement indicated the months, the seasons and the zodiacal signs in accordance with the research of the mathematician *Facundus Novius*. For Augustus, the conception of time and the zodiac held exceptional symbolic value, in some cases fundamental for his implied political messages. In addition, since in his capacity as *pontifex maximus* Augustus was responsible for the public time of Rome as marked by the calendar, one can understand the importance of this monument.

Around the middle of the first century C.E., the monumental clock had already ceased to function correctly: perhaps, as Pliny says (*Storia naturale*, 36.73) for astronomical reasons, perhaps due to an earthquake that had undermined the obelisk, or perhaps as a result of the frequent floods of the Tiber. Because of this inaccuracy, as well as due to an accumulation of sediments in the area following the floods, in the Flavian era the pavement was raised to a new height, as can be seen in the travertine slabs partially conserved in the underground area of the church of San Lorenzo in Lucina (Brandt 2012) and in several cellars in via Campo Marzio which also bear inscriptions in Greek relating to various zodiacal signs: Leo and Virgo on one hand, Aries and Taurus on another. They also indicate the start of the summer and of the Etesian winds (Fig. 6) (nowadays Meltemi) that blow from the north onto the Aegean Sea in the summer months.

There is a good deal of discussion regarding the form that the comprehensive design of the sundial traced on the ground and its full extension. Buchner (1984), who edited the results of the excavation, and was the first scholar to concern himself with the matter, imagined a large piazza entirely paved, with lines extending in the curves typical of sundials, while today the most probable reconstruction seems to reckon on a simple meridian line along a north–south axis (Heslin 2007; Albers 2008). To correctly answer these questions, it would be useful to be able to amplify the archaeological investigations, admittedly complicated in the monumental heart of a multi-layered city such as Rome. However, the use of new methods of documentation and non-destructive investigation—such as, for example, a new survey of the known structures and measurements recalculated with centimetric accuracy, as described below, could contribute to resolving some of the open questions regarding the monument.

1.1 The discovery of the obelisk and its representation on the eighteenth century Nolli map

Giovanni Battista Nolli's *La Nuova Pianta di Roma* from 1748 is the first map of Rome based on a topographical survey carried out according to scientific criteria, and it can be considered an exhaustive visual documentation of the city's aspect after the great urban development of the Renaissance and Baroque periods (Baiocchi and Lelo 2014). Its graphic detail and its indication of "antiquities" and buildings "worthy of note" offer a complete vision of the eighteenth-century city. Nolli's meticulous engraving also provides a vehicle for the representation of the cultural climate in the pontifical capital in the mid-eighteenth century, where the activities of excavation and systemisation of the archaeological remains had become a common practice, in part due to the interest in antiquity on the part of European artists and intellectuals who had made Rome one of the most visited sites on the *Grand Tour*.

Indicated on Nolli's map as "the recumbent Solar Obelisk" with the number 344, the archaeological find was discovered under the cellar of a building on via di Campo Marzio precisely in 1748, the year of publication for the *Nuova Pianta*. Its presence in the area of

the Obelisk of *Psammetichus II* had been known for some time, and was the object of numerous citations on Roman topography, as documented in the collection published by Romano (1944). In the first decades of the sixteenth century, Albertini (1510) recorded the obelisk in his *Opusculum de Mirabilis urbis Romae*, which notes it as “half uncovered.” In 1587, Pope Sixtus V (1585–90) tried to raise it up, assembling some of the pieces previously discovered. The task, which proved too arduous, was quickly abandoned, and the pieces reburied.

In March of 1748, during the demolition of a house corresponding to the present-day n° 3 in piazza del Parlamento, the remains of the obelisk were rediscovered for the third time (LP. 138.64.52v, Archivio di Stato di Roma). Pope Benedict XIV (1740–1758) decided at this point to excavate the site and gave the task of extract the five large pieces of the monolith to Nicola Zabaglia, the chief of the Sampietrini, or street-pavers, of Rome. The complex lifting operations are documented in an engraving by Vasi of the same year (Fig. 2). There is an inscribed stone marker recording the achievement on the façade of the building at n° 3 in piazza del Parlamento bearing this long inscription. “Benedict XIV, at great cost and with great ability, extracted this obelisk elegantly inscribed with Egyptian hieroglyphs, carried to Rome by the emperor Caesar Augustus when Egypt was under Roman dominion, dedicated to the sun, and erected in the Campo Marzio to indicate the light and shadow of the sun and the passing of the days and nights on a stone pavement with inlaid bronze plaques. Broken and overturned by the inclemency of time and of the barbarians, buried underneath the ground and palaces, Benedict transported it to a nearby site for the enjoyment of the public; the support of letters and the order of memory ensured that the ancient role of the obelisk would not be forgotten.”

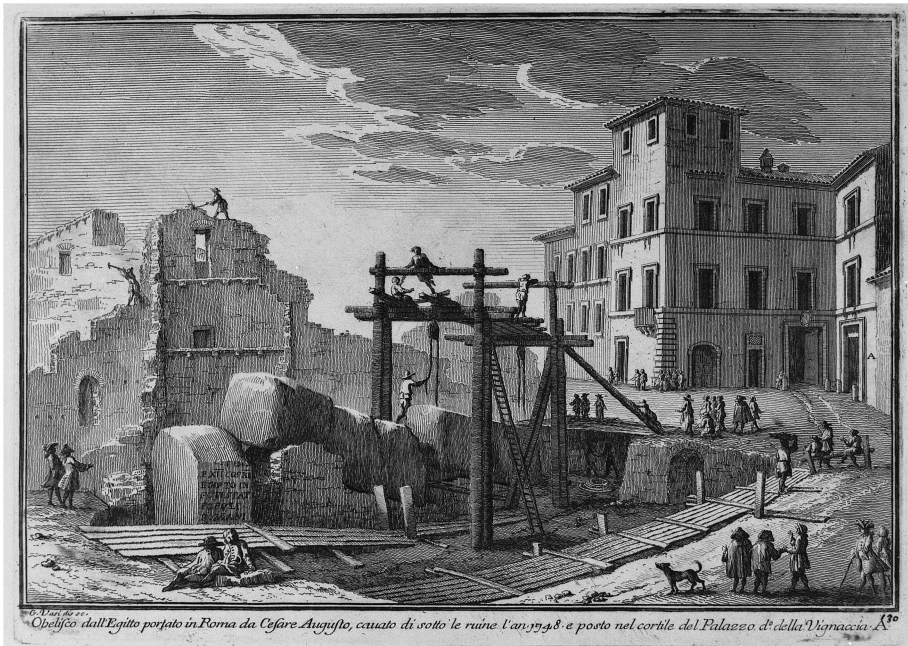


Fig. 2 G. Vasi (1710–1782) The obelisk from Egypt brought to Rome by Caesar Augustus, unearthed from beneath the ruins in the year 1748 and placed in the courtyard of the Palazzo della Vignaccia

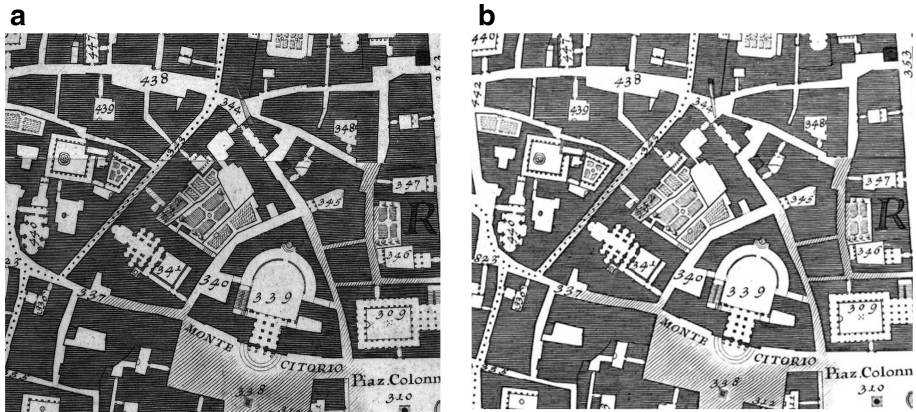


Fig. 3 G. B. Nolli map, 1748 (N° 344 Palazzo Conti with the recumbent solar obelisk): **a** first edition, 1748; **b** subsequent editions

The representation of the obelisk in the first edition of Nolli's *Nuova Pianta* in 1748 is peculiar: it is depicted with the base inside palazzo Conti (n° 344), and the body lying along largo dell'Impresa, with the point inside the building opposite the aforementioned palazzo, close to the place where it was effectively found (Fig. 3a). The buildings are all represented in their entirety. This particularity suggests that the obelisk was still underground at the date of the publication of the *Nuova Pianta*. The track is in fact identical to the one we find in the preparatory drawing for the *Nuova Pianta*, which is dated 1738. It is known that every significant intervention in the meantime was carried over directly to the matrices of the “large map” with a technique that implied a careful and meticulous job of beating and subsequently re-engraving the copper plates (Bevilacqua 1998).

Nolli was certainly aware of the existence of the archaeological remains and their placement. The excavations conducted by Zabaglia in 1748 clearly evidenced the different position of the monolith with respect to that reported on the map. Nolli was therefore constrained to correct the copper plates in order to give a correct representation on the map of the remains in the subsequent re-editions of the *Nuova Pianta* (Fig. 3b).

Regarding the 1748 excavations, the notices from the time, cited by Romano (1944) and Severino (1997) describe a disposition of the remains corresponding to the second version of the *Nuova Pianta*: “When the excavation commenced in the courtyard of the house, the top of the pedestal was discovered still standing up in its original placement, and the lower part of the obelisk still remained supported atop its terminal, fallen toward the noonday aspect. This lay broken in five pieces, with the lower part more elevated, and resting at the beginning above the pedestal.”

The story of this relic of ancient Rome concluded in 1789 with the erection of the restored obelisk in piazza Montecitorio.

1.2 Accuracy of the meridian orientation with the methods of ancient Rome

If the heavenly body (sun or moon) is capable of projecting a shadow, one need only plant a vertical style in the ground and periodically mark the point to which the tip of the shadow projects: the instant that the heavenly body crosses the meridian will correspond to the mark nearest the base of the style (and therefore the shortest shadow).

Table 1 Markings of the sun's shadow every 2 min

Time	Sun azimuth (Az)	Sun elevation (h)	Shadow length (m)
T1	175°18'35"	29°41'46"	52.605
T2 = T1 + 2 min	175°51'19"	29°43'41"	52.530
T3 = T1 + 4 min	176°24'04"	29°45'03"	52.485
T4 = T1 + 6 min	176°56'51"	29°46'22"	52.440
T5 = T1 + 8 min	177°29'39"	29°47'28"	52.395
T6 = T1 + 10 min	178°02'27"	29°48'21"	52.365
T7 = T1 + 12 min	178°35'16"	29°49'02"	52.345
T8 = T1 + 14 min	179°08'06"	29°49'30"	52.331
T9 = T1 + 16 min	179°40'56"	29°49'45"	52.321
T10 = T1 + 18 min	180°13'46"	29°49'48"	52.318
T11 = T1 + 20 min	180°46'36"	29°49'37"	52.326

Because the elevation angle speed of the heavenly bodies is minimal at the instant they cross the meridian, the observer will determine this instant with a degree of uncertainty that depends on the care taken in marking the tip of the shadow, its length, and the frequency of the markings (in the sense that the smaller the interval between two successive marking, the smaller the uncertainty as to the instant of the meridian crossing).

On the other hand the azimuth speed of the heavenly bodies reaches its maximum value in the same instant, so that the direction of the meridian will have maximum uncertainty precisely because the instant of crossing is indeterminate (Loreti 2014).

Example (Table 1):

Date January 30, 9 BCE.

Latitude in Rome 41°53'4 N.

L is the length of a vertical style [30 m, according to Auber (2014)].

Ls is the length of the shadow h is the sun elevation Ls is the $L/\tan(h)$.

It is evident that an uncertainty in the measurement of the length of the shadow of even a millimetre produces an indeterminacy of 24 s in the identification of the instant of the meridian crossing (which occurs at a moment near T10) and an uncertainty of almost 8' in the direction of the meridian.

Another method available to draw the direction of the meridian could be the following: the endpoints of the shadows on the pavement form a hyperbola, and the north direction is the axis of the hyperbola. This can be determined using a fixed length measuring rod or chain as a bow, to intersect two points on the hyperbola, that have equal distance from the pole, and using these to intersections as endpoints of line segment's perpendicular bisector.

As a matter of fact the curve described from the endpoint of the shadow is not a perfect hyperbolae because the declination of the sun changes constantly; only in the days of the solstice the variation of the declination reaches the minimum value and therefore the described method is more reliable.

1.3 Use of the sundial as clock

The length of the interval between two consecutive passages of the "true" sun at the meridian by a fixed observer (the true day) is not constant; in fact, the earth, in addition to rotating on its axis at a speed which we can consider constant, also revolves around the sun

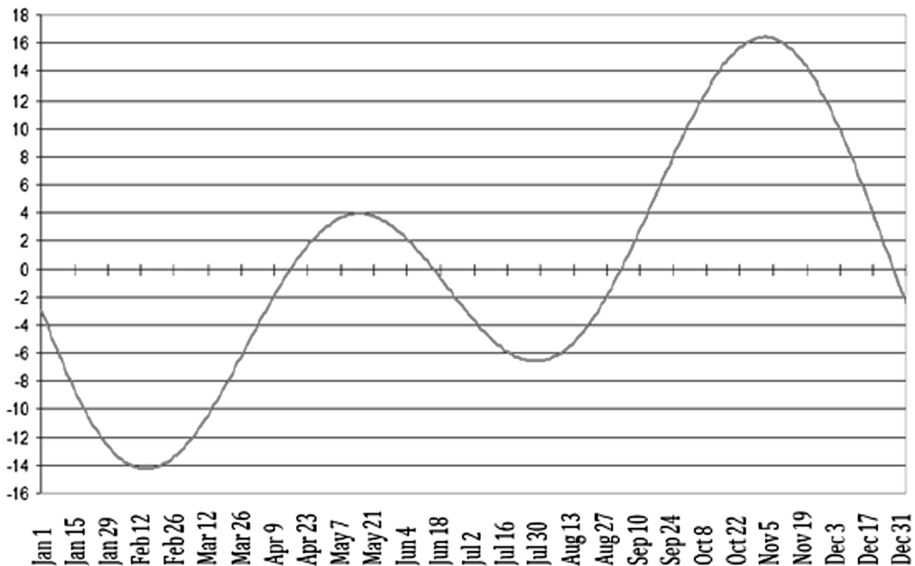


Fig. 4 The equation of time as expressed in minutes

with a non-constant speed, accordingly to the laws of Kepler. This means that the sun crosses the meridian of the same observer in a longer time that which the earth takes to complete a rotation on its axis, and this difference is a function of the position of the earth in its orbit around the sun.

If we imagine that it is the sun that has an elliptical orbit around the earth, the equation of time expresses how much the true sun anticipates or delays with respect to an imaginary sun that has a circular orbit (and hence at a constant speed) around the earth.

Our life is regulated by time measured with an atomic or mechanical clock (standard clock time) that has no direct link with the movement of the sun: therefore, if one uses the shadow of the sun as the indicator of time one needs to provide a correction to the hours indicated by the direction of the shadow to obtain the mean solar time: such a correction is in fact provided by the equation of time (Fig. 4) that on many sundials is given in the form of tables.

1.4 Use of the sundial as calendar

The length of the shadow of the sun on the meridian line depends on the day of year, reaching the maximum length in the winter solstice and the minimum in the summer solstice. If we mark on the meridian line each day the point reached by the shadow we have a sundial calendar. However, how reliable is such a calendar after more than 2000 years? Alas very little! Indeed the obliquity of the ecliptic, the inclination of the Earth's equator with respect to the ecliptic, changes slowly but it has reached almost three tenths of a degree in 2000 years (Laskar 1986a, b).

Therefore, if we used the sundial of Augustus nowadays, assuming, as in Table 1, a height of 30 m for obelisk as, for example, hypothesized by Auber (2014), in 30 January of 2015 the length of the shadow at the instant of meridian crossing would amount to 51,065 m.

1.5 Possible methods for determining the actual azimuth of the sundial and its position

The surveys conducted so far have been based on the use of electronic theodolites or total stations (TS), whose setup reference points have unfortunately been lost to us (Leonhardt 2014), thus rendering it impossible to determine the correct three dimensional referencing, so for this reason these measurements need to be repeated because they have only relative value.

The biggest problem in the survey lies in the fact that the *meridian line of the sundial* on which the *cross-hatches* and inscriptions are found is located in an underground cellar at a lower level, and planimetrically displaced with respect to the level of the open-air courtyard which could establish a reference allowing a correspondence with the surrounding buildings and to an absolute reference system (Datum).

It helps to distinguish three levels of elevation: the cellar containing the markings, the catwalk above it, and the courtyard on which the reference points can be established (Figs. 5, 6). The passage from one to the other is not direct, but rather composed of landings and narrow staircases in varying directions (Fig. 5).

Another problem lies in the obvious impossibility of realize precise countermarks on the original inscribed pavement, to preserve the pavement itself, as once any characteristic points of interest (for example, the centre of the strips that identify the cross-hatches) are selected, the positioning on them of surveying instruments or reflector targets cannot be precise, and this leads to centring errors which are particularly influential due to the short distance between the survey positions. In addition, and by no means least problematic, the original pavement is covered with a layer of around ten centimetres of water (Fig. 6) which effect has to be properly modelled (Menna et al. 2013; Troisi et al. 2015).

Without going into details with regards to the survey schema that could be carried out, one can delineate the principal elements: in the survey operations it is useful to separate those direct to determining the azimuth of the axis of the meridian line from those relating to the three dimensional positioning of the sundial itself with respect to an external reference system that can be used to provide a framework for this and other monuments.

1.6 Determining the position of the meridian line: survey of the centres of the cross-hatches

The determination of the intersection of the line of the meridian with the centre of the transversal lines (cross hatches) is carried out with a TS and reflector targets with high-precision prisms; on the cellar pavement, at a distance from the find, signals could be positioned that permit a repositioning of the TS with good accuracy (Fig. 7). The two countermarks materialized (R1, R2) permit the institution of a reference system with respect to which one can calculate the position of the points of interest. The measurement of angles and distances by means of reflector targets placed on poles with bipods from both of the reference points allows for a redundant number of observations and therefore a control of the same measurements and of the calculation of the positions of the points by means of a least squares adjustment.

To proceed, one needs to realize with markers (at least) two points on the catwalk and determine the position relative to the R1–R2 reference; a possible choice of these points could allow for the connection between the measurements of angles and distances, albeit



Fig. 5 The complex of stairs from the level of the courtyard (gate in “a”) to the entrance at the catwalk (door on the *right* in “d”)

with extremely inclined visuals (for this it is necessary to have a Total Station with bi-axial compensators).

An alternative solution for the pavement in the cellar would be to locate at least two points on the vertical of the edge of the catwalk that is around 3 m above it and on which one could take positions with a zenithal level that allows for positioning the vertical of the two points at the level of the catwalk.

From the points on the catwalk one can develop a three dimensional open traverse survey that allows for their linkage to the courtyard outside the cellar.



Fig. 6 Views of the track of the meridian and the cross hatches from the catwalk; note reflections of tube neon lamps in the water

On the courtyard itself one could set up three points in a stable way that establish an external reference system of which could itself be linked to all of the structures of interest.

The three-dimensional network indicated suffers from a rather bad geometry that could result in notable errors; once one or more survey schemas and their approximate geometries have been settled, it would be opportune to carry out a simulation with the method of

other measurements. It helps to find two points that identify the line from which one seeks the direction, as far as possible from each other and from which it is possible to establish a station with the theodolite and a reflector target; one must consider that once the vertical line for the centre of the instrument and the reflector target is established on the points, a small but still significant “centring error” is present: the two errors on the position of the instrument and the position of the sight-line, if not correlated, result in a misalignment of the two effective positioning points with respect to the line individuated by the centre of the marks. In the worst case scenario of two directions opposed to each other by a centring error r between the two points of distance D , such a misalignment yields $\varepsilon = \arctan(2r/D)$; to quantify the value of ε , assuming that $r = 2$ mm and $D = 10$ m, this results in $\varepsilon = 82''$.

A more articulated model to determinate the effect of the centring errors could be the following. Considering a reference system with its origin in the centre of the mark that identifies the first point A and its axis x , passing through the second point B at a distance D , the centring error on the station produces a position for the station point (X_A, Y_A) different from $(0,0)$ and for the vertical of the reflector target (X_B, Y_B) different from $(D,0)$. One assumes that the shift in x would be completely uncorrelated to that of y in both points, and σ_{X_A} , σ_{Y_A} , σ_{X_B} , σ_{Y_B} indicate the standard deviation of the coordinates of the points A and B .

The angle ε , composed by the direction of the station points referred to the axis x (the “true” direction), is $\varepsilon = \arctan((Y_B - Y_A)/(X_B - X_A))$; so applying the propagation of variance (Mikhail 1982), for ε results a standard variation equal to:

$$\sigma_\varepsilon^2 = [(\Delta Y)^2 \sigma_{X_A}^2 + (\Delta X)^2 \sigma_{Y_A}^2 + (\Delta Y)^2 \sigma_{X_B}^2 + (\Delta X)^2 \sigma_{Y_B}^2] / D^4$$

If one considers the standard deviation of the coordinates x and y to be equal for each point

$$\sigma_{X_A} = \sigma_{X_B} = \sigma_X \text{ and } \sigma_{Y_A} = \sigma_{Y_B} = \sigma_Y, \text{ one arrives at } \sigma_\varepsilon^2 = [\sigma_X^2 + \sigma_Y^2] / D^2$$

and finally, if one assumes the standard deviation for the two points $\sigma_x = \sigma_y = \sigma$ to be equal, the result is:

$$\sigma_\varepsilon^2 = (2\sigma^2) / D^2$$

For a distance of 10 m (7 m is apparently the maximum presently visible portion of meridian track) and a resulting random centring error of 2 mm, the resulting random misalignment of the azimuth has a standard deviation of $58''$ ($83''$).

One should keep in mind that all of this is true if the centring error of the targets is equal to the two millimetres hypothesized, so that if in the actual conditions of the site the centring error turns out to be, for example, four millimetres, the estimated indeterminacies would obviously double. A final consideration can be that also a gyro theodolite a little less accurate than DMT-Gyromat can properly measure the azimuth considering all the cited limitations of the site.

2 Final remarks and unresolved issues

From the above one can deduce that the meridian line of the sundial currently visible should indicate the direction of the emergence of the axis of terrestrial rotation at the age of the sundial, with an accuracy of a few minutes. Even the most accurate measurement currently possible (given the impossibility of astronomical measurement in an underground

environment) would have an accuracy of a few dozen arc seconds. A possible cause of eventual change of the original azimuth of the track is a differential settlement due to the alluvial nature of the ground (Bozzano et al. 2000; Camponeschi and Nolasco 1982). Geodynamic movements since antiquity of the sundial's meridian line presumably do not significantly influence the azimuth itself, while an accurate evaluation of geodynamic rotations would require a further study, both for the choice of the correct model and reference De Mets (2013), as well as in function of the specific movements of the Italian peninsula (Devoti et al. 2014).

The position and height transformation of the measurements already carried out in the 1990s or the effectuation of new measurements with the techniques described above could permit the localization of the exact positions of the remaining, unexcavated parts, making it possible to excavate smaller areas and thereby limiting the interference with the many buildings currently present. A high-precision altimetric survey would also produce further data on the level to which one can refer the excavated meridian line, given that a series of rises in the level of the Campo Marzio are documented and commonly accepted which are probably behind the makeover with the so-called “baseboards” or supports that were observed around the base of the sundial itself at the time of its discovery (Haselberger 2011); it has also to be considered that surely the position and height reference system (Datum) is significantly different from the one used when the base was surveyed (Baiocchi and Lelo 2010; Barbarella 2014).

An accurate measurement of the distance between the various cross hatches observable along the principal track of the meridian line would also allow for the deduction of two pieces of information still not entirely established: the exact height from ground level of the sphere placed on the top of the obelisk, and the exact position of the centre of the original base, which represents the original placement of the axis of the obelisk. The sphere was in fact the exact point of reference from which the shadow was individuated while the axis of the obelisk was in practice the “origin” from which all the distances of the various grooves were calculated. The geometry of the “sight lines” is not perfect because the angles are very narrow but with redundant observations a least squares fit can be properly applied, improving the estimations made till now (Schütz 2011, 2014).

Acknowledgments Authors want to thanks Dott. G. Pietrantonio Ph.D. of INGV (Italian National Institute for Geophysics and Vulcanology) for her precious bibliographic hints, Dott. F. Vatore Ph.D. for her support with the illustrations and Dott. T. Aebischer for the organization of two inspections of the site.

References

- Albèri Auber P (2014) Reconstructing Augustus' Montecitorio obelisk: a gnomonist's point of view. In: Haselberger L (ed), *The Horologium of Augustus: debate and context*, “JRA” supplementary series number 99, Portsmouth
- Albers J (2008) Das Marsfeld. Die Entwicklung der urbanen Struktur aus topographischer, traditioneller und rechtlicher Perspektive. In: Albers J, Graßhoff G, Heinzelmann M, Wäfler M (ed), *Das Marsfeld in Rom*, Beiträge der Berner Tagung, Bern, 23–24 November 2007, pp 13–26
- Albertini F (1510) *Opusculum de Mirabilis urbis Romae*. Mazzocchi, Rome (re-edited in 1515 and 1522)
- Baiocchi V, Lelo K (2010) Accuracy of 1908 high to medium scale cartography of Rome and its surroundings and related georeferencing problems. *Acta Geodaetica et Geophysica Hungarica*. doi:10.1556/AGeod.45.2010.1.14
- Baiocchi V, Lelo K (2014) Assessing the accuracy of historical maps of cities: Methods and problems. *Citta e Storia*. Anno IX, n. 1/gennaio-giugno, 61–89

- Barbarella M (2014) Digital technology and geodetic infrastructures in Italian cartography. *Citta e Storia*. Anno IX, n. 1/gennaio-giugno: 91–110
- Bevilacqua M (1998) Roma nel secolo dei lumi. Architettura erudizione scienza nella Pianta di G.B. Nolli “celebre geometra”. Electa, Napoli
- Bozzano F, Andreucci A, Gaeta M, Salucci R (2000) A geological model of the buried Tiber River valley beneath the historical centre of Rome. *Bull Eng Geol Environ* 59(1):1–21. doi:10.1007/s100640000051
- Brandt O (2012) The early Christian basilica of San Lorenzo in Lucina. In: Brandt O (ed) *San Lorenzo in Lucina. The transformations of a Roman quarter*, “ActaInstRomFin”, series 4°, pp 123–154
- Buchner E (1984) L’orologio solare di Augusto. *Rendiconti della Pontificia Accademia di Archeologia* 53–54(1980–82):331–345
- Camponeschi B, Nolasco F (1982) *Le Risorse Naturali della Regione Lazio*. Roma ei Colli Albani (vol 7), Regione Lazio editions
- De Mets et al. (2013) Plate motion calculator http://ofgs.aori.u-tokyo.ac.jp/~okino/platecalc_new.html. Accessed 25 June 2014
- Devoti R, Pietrantonio G, Riguzzi F (2014) GNSS networks for geodynamics in Italy. *Física de la Tierra*. doi:10.5209/rev_FITE.2014.v26.46968
- Haselberger L (2011) A debate on the Horologium of Augustus: controversy and clarifications. In: Haselberger L (ed), *The Horologium of Augustus: debate and context*, “JRA” supplementary series number 99, Portsmouth, Rhode Island 2014
- Heslin P (2007) Augustus, Domitian and the so-called Horologium Augusti. *J Rom Stud* 97:1–20
- Laskar J (1986a) Secular terms of classical planetary theories using the results of general theory. *Astron Astrophys* 157:59–70
- Laskar J (1986b) Erratum—Secular terms of classical planetary theories using the results of general theory. *Astron Astrophys* 164:437–438
- Leonhardt G (2014) Horologium and Mausoleum Augusti: an overview of the fieldwork (1979–97) and the existing documentation (2013), with Addendum (2014). In: Haselberger L (ed), *The Horologium of Augustus: debate and context*, “JRA” supplementary series number 99, Portsmouth, Rhode Island 2014
- Loreti G (2014) *Disegno automatico di Orologi Solari (Sundials drawing)*. Dissertation, “Parthenope” University of Naples, Italy
- Menna F, Nocerino E, Troisi S, Remondino F (2013) A photogrammetric approach to survey floating and semi-submerged objects. *Proc SPIE Int Soc Opt Eng*. doi:10.1117/12.2020464
- Mikhail EM (1982) *Observations and least squares*. University Press of America, Lanham
- Romano P (1944) *Orologi di Roma*. Autonomia Romana Stampa, Rome
- Schütz M (2011) The Horologium on the Campus Martius reconsidered. In: Haselberger L (ed), *The Horologium of Augustus: debate and context*, “JRA” supplementary series number 99, Portsmouth, Rhode Island 2014
- Schütz M (2014) Ancient and modern gnomonics: concerns and clarifications. In: Haselberger L (ed), *The Horologium of Augustus: debate and context*, “JRA” supplementary series number 99, Portsmouth, Rhode Island 2014
- Severino N (1997) *L’altezza dell’Obelisco in Storia dell’obelisco e dell’orologio solare di Augusto in Campo Marzio*. Prima edizione, Roccasecca
- Troisi S, Del Pizzo S, Gaglione S, Miccio A, Testa RL (2015) 3D models comparison of complex shell in underwater and dry environments. *Int Arch Photogramm Remote Sens Spat Inf Sci*. doi:10.5194/isprsarchives-XL-5-W5-215-2015