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CONVERGENCE FIELDS OF REGULAR MATRIX TRANSFORMATIONS OF SEQUENCES OF ELEMENTS OF BANACH SPACES

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Abstract. In [4, 7] some properties of convergence fields of regular matrix transformations of bounded sequences of real numbers are presented. We shall prove a generalization of Steinhaus' theorem for sequences of a Banach space and show that results of [4] cen be generalize for a space of sequences of elements of a Banach space $(X, \|\cdot\|)$.

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1. Introduction

Let $(X, \|\cdot\|)$ be a Banach space. The sequence $\alpha = (\alpha_k)$ converges to c if $\forall \varepsilon > 0$ $\exists k_0 \in \mathbb{N}$ $\forall k > k_0$: $\|\alpha_k - c\| < \varepsilon$. We write $\lim_{k\to\infty} \alpha_k = c$.

We define the following sets:

(a) $b = \{\alpha = (\alpha_k) : \alpha_k \in X, k = 1, 2, \ldots\}$;
(b) $B_\infty = \{\alpha = (\alpha_k) : \alpha_k \in X, k = 1, 2, \ldots : \exists K_\alpha > 0\ \forall k = 1, 2, \ldots \|\alpha_k\| \le K_\alpha\}$;
(c) $\Omega = \{\alpha = (\alpha_k) : \alpha_k \in X, k = 1, 2, \ldots : \forall k = 1, 2, \ldots \|\alpha_k\| = 0\ \vee\ \|\alpha_k\| = 1\}$.

The set b contains all sequences of elements of X, the set B_∞ is the set of all bounded sequences of X and Ω is the set of all sequences of elements of X which have norm null or one. In what follows, for the set B, the following inclusions hold:

$$\Omega \subset B_\infty \subset B \subset b. \tag{1}$$

The notion of porosity has been introduced in [10]. It is a suitable tool to describe small sets in a metric space.

Let (Y, d) be a metric space, $Z \subset Y$. Let $y \in Y$, $\delta > 0$, and let $B(y, \delta)$ denote the set $\{x \in Y : d(x, y) < \delta\}$. We put

$$P(y, Z, \delta) = \sup\{t > 0 : \exists z \in B(y, \delta) \text{ such that } B(z, t) \subset B(y, \delta) \text{ and } B(z, t) \cap Z = \emptyset\}.$$

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If such $t > 0$ does not exist, we put $P(y, Z, \delta) = 0$. The numbers

$$\underline{p}(y, Z) = \liminf_{\delta \to 0^+} \frac{P(y, Z, \delta)}{\delta}$$

and

$$\overline{p}(y, Z) = \limsup_{\delta \to 0^+} \frac{P(y, Z, \delta)}{\delta}$$

are called the lower and upper porosity of the set Z at $y \in Y$, respectively.

A set $Z \subset Y$ for which $\overline{p}(y, Z) > 0$ for every $y \in Y$ is said to be porous in Y. Obviously every set porous in Y is nowhere dense in Y. If $\underline{p}(y, Z) = \overline{p}(y, Z) = p(y, Z)$, then the number $p(y, Z)$ is called porosity of Z at $y \in Y$. If $p(y, Z) = 1$, then Z is said to be strongly porous at y. If for all $y \in Y$ we have $p(y, Z) = 1$ for $y \notin Z$ and $p(y, Z) = \frac{1}{2}$ for $y \in Z$, then Z is said to be strongly porous at Y. The set W is said to be σ-porous (σ-strongly porous) at Y if $W = \bigcup_{n=1}^{\infty} Z_n$ and each Z_n is porous (strongly porous) at Y.

Further on, we recall the definition of a matrix transformation. Let $\mathscr{A} = (a_{nk})$ be an infinite matrix of real numbers. A sequence $\alpha = (\alpha_k)$, $\alpha_k \in X$, is said to be $\mathscr{A}$-limitable (limitable by the method $\mathscr{A}$) to the element $c \in X$ if, for $\beta = (\beta_n(\alpha))$, $\beta_n = \beta_n(\alpha) = \sum_{k=1}^{\infty} a_{nk}\alpha_k$, we have $\lim_{n\to\infty} \beta_n = c$.

If $\alpha = (\alpha_k)$ is $\mathscr{A}$-limitable to the element c, we write $\mathscr{A}\text{-}\lim_{k\to\infty} \alpha_k = c$. The method $\mathscr{A}$ defined by the matrix $\mathscr{A}$ is said to be regular if $\lim_{k\to\infty} \alpha_k = c$ implies that $\mathscr{A}\text{-}\lim_{k\to\infty} \alpha_k = c$.

Let $\mathscr{A}$ be a regular method. The symbol $F(\mathscr{A})$ denotes the set of all $\mathscr{A}$-limitable sequences of X. We put

$$F(\mathscr{A}) = \Big\{ \alpha = (\alpha_k) \mid \alpha_k \in X, k = 1, 2, \dots : \text{ there exists } \lim_{n\to\infty} \beta_n, \\ \text{where } \beta_n = \sum_{k=1}^{\infty} a_{nk}\alpha_k \Big\}.$$

The set $F(\mathscr{A})$ is called the convergence field of the matrix transformation $\mathscr{A}$.

It is proved in [4] that $F(\mathscr{A})$ is a set of first Baire category in S, where S is a linear space of sequences of real numbers and $F(\mathscr{A})$ is strongly porous in l_∞ (l_∞ is the set of all bounded sequences of real numbers).

In this paper we show that these results can be generalized for the sequences of elements of Banach space $(X, \|\cdot\|)$.

2. Main results

Monograph [6] gives the Toeplitz theorem which provides necessary and sufficient condition for matrix $\mathscr{A}$ be a regular, i. e., when $\lim_{k\to\infty} x_k = t$ implies $\mathscr{A}\text{-}\lim_{k\to\infty} x_k = t$ for real sequences (x_k).

In [9], it is proved that Toeplitz theorem holds also for sequences of elements of a Banach space.

Theorem A. *Let $(X, \|\cdot\|)$ be a Banach space. Let $\mathscr{A} = (a_{nk})$ be an infinite matrix of real numbers. The necessary and sufficient condition for a sequence $\beta_n(\alpha) = \sum_{k=1}^{\infty} a_{nk}\alpha_k$, to converge to $c \in X$ as $n \to \infty$ where $\lim_{k\to\infty} \alpha_k = c$, $\alpha_k \in X$ is that matrix $\mathscr{A}$ satisfies the following three conditions:*

(a) $\exists M > 0,\ \forall n = 1, 2, \ldots\ \sum_{k=1}^{\infty} |a_{nk}| \leq M$*;*
(b) $\forall k = 1, 2, \ldots\ \lim_{n\to\infty} a_{nk} = 0$*;*
(c) $\lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk} = 1$.

Further we will use the following theorem concerning the functions of Baire class one (see [8, p. 185]).

Theorem B. *Let A and B be metric spaces. Let $\delta_n : A \to B$, $n = 1, 2, \ldots$, be a sequence of continuous operators, which converges pointwise to an operator δ, i. e.,*

$$\forall a \in A : \lim_{n\to\infty} \delta_n(a) = \delta(a).$$

Then the set of all discontinuity points of the operator $\delta : A \to B$ is a set of the first Baire category.

In [2], the Steinhaus theorem is proved under the condition that there does not exist a regular matrix which limits all sequences of 0's and 1's. Now we will show an analogue of the Steinhaus theorem for sequences of elements of X. First we prove the following:

Lemma 1. The metric space (Ω, d), where $d(\alpha,\beta) = \sum_{k=1}^{\infty} 2^{-k} \|\alpha_k - \beta_k\|$, $\alpha = (\alpha_k) \in \Omega$, $\beta = (\beta_k) \in \Omega$, is a complete metric space.

Proof. The function $d : \Omega \times \Omega \to \langle 0, \infty)$ is a metric.

Let $\alpha^{(n)} = (\alpha_k^{(n)}), n = 1, 2, \ldots$ be a Cauchy sequence of elements of Ω. Thus,

$$\forall \varepsilon > 0\ \exists n_0 \in \mathbb{N}\quad \forall m, n > n_0 :\ d(\alpha^{(n)}, \alpha^{(m)}) < \varepsilon.$$

Choose $j \in \mathbb{N}$. Then

$$\varepsilon > d(\alpha^{(n)}, \alpha^{(m)}) = \sum_{k=1}^{\infty} \frac{\|\alpha_k^{(n)} - \alpha_k^{(m)}\|}{2^k} \geq \frac{1}{2^j} \|\alpha_j^{(n)} - \alpha_j^{(m)}\|.$$

The sequences $(\alpha_j^{(n)})_{n=1}^{\infty}$ of elements of X are Cauchy sequences. X is a complete metric space, therefore $(\alpha_j^{(n)})_{n=1}^{\infty}$ is convergent.

Let $\lim_{n\to\infty} \alpha_j^{(n)} = \alpha_j$. If $\alpha = (\alpha_j)_{j=1}^{\infty}$, then one can easily verify that that $\lim_{n\to\infty} \alpha^{(n)} = \alpha$ and, for each $j = 1, 2, \ldots$, $\|\alpha_j\| = 0$ or $\|\alpha_j\| = 1$. □

Theorem C. *For any regular matrix $\mathscr{A} = (a_{nk})$ there exists a sequence in the set Ω, which is not limitable by the method $\mathscr{A}$.*

Proof. Let $\mathscr{A} = (a_{nk})$ be a regular matrix. Then there exists an $M > 0$ such that, for all $n = 1, 2, \ldots$,

$$\sum_{k=1}^{\infty} |a_{nk}| \leq M. \tag{2}$$

We prove that the operator $\delta_n : \Omega \to X$ is continuous at $\alpha = (\alpha_k) \in \Omega$ for each $n = 1, 2, \ldots$, where $\delta_n(\alpha) = \sum_{k=1}^{\infty} a_{nk}\alpha_k,\ \alpha = (\alpha_k) \in \Omega$. Let $\varepsilon > 0$ and $\alpha = (\alpha_k) \in \Omega$, $\beta = (\beta_k) \in \Omega$. Then

$$\|\delta_n(\alpha) - \delta_n(\beta)\| = \|\sum_{k=1}^{\infty} a_{nk}\alpha_k - \sum_{k=1}^{\infty} a_{nk}\beta_k\| \leq \sum_{k=1}^{\infty} |a_{nk}|\|\alpha_k - \beta_k\|.$$

We choose $n_0 \in \mathbb{N}$ so that $\sum_{k>n_0} |a_{nk}| < \frac{\varepsilon}{4}$. Then

$$\sum_{k>n_0} |a_{nk}|\|\alpha_k - \beta_k\| \leq \sum_{k>n_0} |a_{nk}|(\|\alpha_k\| + \|\beta_k\|) \leq 2 \cdot \frac{\varepsilon}{4} = \frac{\varepsilon}{2}. \tag{3}$$

If $d(\alpha, \beta) < \frac{\varepsilon}{2M2^{n_0}}$ for α and β, then we have

$$\frac{\|\alpha_k - \beta_k\|}{2^k} \leq d(\alpha, \beta) < \frac{\varepsilon}{2M2^{n_0}}$$

for all $k = 1, 2, \ldots n_0$. Therefore, $\|\alpha_k - \beta_k\| < \frac{\varepsilon}{2M}$ and

$$\sum_{k=1}^{n_0} |a_{nk}|\|\alpha_k - \beta_k\| < \frac{\varepsilon}{2M} \sum_{k=1}^{n_0} |a_{nk}| \leq \frac{\varepsilon}{2}. \tag{4}$$

According to (2)–(4), the inequality $d(\alpha, \beta) < \frac{\varepsilon}{2M2^{n_0}}$ implies that $\|\delta_n(\alpha) - \delta_n(\beta)\| < \varepsilon$. The operator δ_n, $n = 1, 2, \ldots$, is continuous at $\alpha = (\alpha_k)$.

Suppose that all sequences in Ω are limitable by the matrix $\mathscr{A} = (a_{nk})$. Hence, there exists $\lim_{n\to\infty} \delta_n(\alpha) = \delta(\alpha)$ for each $\alpha = (\alpha_k) \in \Omega$. Then, by Theorem B, the set of discontinuity points of the operator $\delta : \Omega \to X$ is a set of the first Baire category in Ω. On the other hand if $\alpha = (\alpha_k) \in \Omega$ and $\eta > 0$, then, in the open ball $S(\alpha, \eta)$, it is possible to find elements $\beta, \gamma \in \Omega$ such that

$$\|\delta(\beta) - \delta(\gamma)\| > \frac{1}{2}.$$

We choose $k_1 \in \mathbb{N}$ so that

$$\sum_{k>k_1} \frac{\|\alpha_k\|}{2^k} \leq \sum_{k>k_1} \frac{1}{2^k} < \frac{\eta}{4},$$

where, $\alpha = (\alpha_k) \in \Omega$. Put $\beta = (\beta_k)$, $\gamma = (\gamma_k)$, where $\beta_k = \gamma_k = \alpha_k$ for $k = 1, 2, \ldots k_1$ and $\beta_k = \Theta$, $\gamma_k = \xi$ for $k > k_1$. The element Θ is the null-element of X and $\xi \in X$ is an element with the property $\|\xi\| = 1$. Then

$$d(\alpha, \beta) = \sum_{k>k_1} \frac{\|\alpha_k - \Theta\|}{2^k} < \frac{\eta}{4}$$

and

$$d(\alpha, \gamma) = \sum_{k>k_1} \frac{\|\alpha_k - \xi\|}{2^k} < \frac{\eta}{2}.$$

Therefore, β and γ belong to $S(\alpha, \eta)$.

Due to the regularity the matrix $\mathscr{A}$ and Theorem A, it follows that

$$\delta(\beta) = \lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk}\beta_k = \lim_{n\to\infty} \sum_{k=1}^{k_1} a_{nk}\alpha_k$$

and

$$\delta(\gamma) = \lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk}\gamma_k = \lim_{n\to\infty} \sum_{k=1}^{k_1} a_{nk}\alpha_k + \lim_{n\to\infty} \sum_{k>k_1} a_{nk}\xi.$$

Since $\lim_{n\to\infty} \sum_{k>k_1} a_{nk} > \frac{1}{2}$, we have $\left\|\sum_{k>k_1} a_{nk}\xi\right\| = \left|\sum_{k>k_1} a_{nk}\right| \|\xi\| > \frac{1}{2}$ for every $n \in \mathbb{N}$ sufficiently large. Thus,

$$\|\delta(\gamma) - \delta(\beta)\| = \left\|\lim_{n\to\infty} \sum_{k>k_1} a_{nk}\xi\right\| > \frac{1}{2}.$$

Therefore the operator δ is discontinuous at each element α of Ω. Since Ω is a complete metric space, the set of all discontinuity points of δ is the set of the second Baire category in Ω — a contradiction. □

We introduce the notion of FK-space (see [1]).

Definition 1. A complete metric linear space (B, d) of sequences, i. e., $B \subset b$ (see (1)), is said to be an FK-space provided that the linear operators $\delta_k : B \to X$,

$$\delta_k(\alpha) = \alpha_k, \quad \alpha = (\alpha_k) \in B, \quad k = 1, 2 \ldots,$$

are continuous on B.

Proposition 1. *Let $(X, \|\cdot\|_1)$ be a Banach space. Let (B, d) be a complete metric linear space of sequences of elements of X. Then (B, d) is an FK-space if and only if the convergence in the sense of the metric d implies the pointwise convergence in the sense of the norm $\|\cdot\|_1$ for each $k = 1, 2, \ldots$.*

Proof. Let (B, d) be an FK-space. The linear operator $\delta_k : B \to X$, $\delta_k(\alpha) = \alpha_k$, is continuous for each $k = 1, 2, \ldots$. According to the Heine definition of continuity, the operator $\delta_k : B \to X$ is continuous if for each sequence $(\alpha^{(r)})_{r=1}^{\infty}$, $\alpha^{(r)} = (\alpha_s^{(r)})_{r=1}^{\infty}$, of elements of B such that $\alpha^{(r)} \to \alpha = (\alpha_s)$ as $r \to \infty$ by the metric d, we have $\delta_k(\alpha^{(r)}) \to \delta_k(\alpha)$ by the norm $\|\cdot\|_1$ as $r \to \infty$. In our case, $\delta_k(\alpha^{(r)}) = \alpha_k^{(r)}$ and $\delta_k(\alpha) = \alpha_k$ for each $r = 1, 2, \ldots$. Consequently, $\alpha_k^{(r)} \to \alpha_k$ as $r \to \infty$. □

Now we denote by $F(\mathscr{A}) = \{\alpha = (\alpha_k) : \alpha_k \in B, k = 1, 2, \dots : \lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk}\alpha_k$ exists and equals c for $\alpha_k \to c\}$.

In the proof of Theorem 1 we shall use the following form of Banach's Subgroup Theorem (see [3]): *If G is any proper subgroup of a linear topological space E, then either G is of the first category in E, or G fails to satisfy the condition of Baire.*

Theorem 1. *Let (B, d) be an FK-space, $B_\infty \subset B \subset b$. Let $\mathscr{A} = (a_{nk})$ be a regular matrix of real numbers. Then the set $F(\mathscr{A})$ is of the first Baire category in B for every regular matrix $\mathscr{A}$.*

Proof. We set $\delta_{ns}(\alpha) = \sum_{k=1}^{s} a_{nk}\alpha_k,\ \alpha = (\alpha_k) \in B,\ n, s = 1, 2, \dots$. From the above assumption it follows that each of the operators $\delta_{ns} : B \to X$ is continuous on B. Thus, $\delta_n(\alpha) = \lim_{s\to\infty} \delta_{ns}(\alpha)$ is a linear operator of the first Baire class defined on the set $D_n = \{\alpha \in B :\ \lim_{s\to\infty} \delta_{ns}(\alpha) \text{ exists}\}$.

Obviously,

$$D_n = \left\{\alpha \in B : \forall p = 1, 2, \dots\ \exists s_0 \in \mathbb{N}\ \forall s, q \geq s_0 :\ \|\delta_{ns}(\alpha) - \delta_{nq}(\alpha)\|_1 \leq \frac{1}{p}\right\}$$
$$= \bigcap_{p=1}^{\infty} \bigcup_{s_0=1}^{\infty} \bigcap_{s\geq s_0} \bigcap_{q\geq s_0} \left\{\alpha \in B : \|\delta_{ns}(\alpha) - \delta_{nq}(\alpha)\|_1 \leq \frac{1}{p}\right\}.$$

Let $s > q$. Then every operator $\|\delta_{ns}(\alpha) - \delta_{nq}(\alpha)\|_1$ is continuous, every set D_n is an $F_{\sigma\delta}$ set. The common domain of all the operators δ_n, the set $\bigcap_{n=1}^{\infty} D_n$, is also an $F_{\sigma\delta}$ set.

Put $S(\mathscr{A}) = \bigcap_{n=1}^{\infty} D_n$. Then $F(\mathscr{A}) \subset S(\mathscr{A})$ and

$$F(\mathscr{A}) = \left\{\alpha \in S(\mathscr{A}) :\ \lim_{n\to\infty} \delta_n(\alpha) \text{ exists}\right\}$$
$$= \left\{\alpha \in\ S(\mathscr{A}) :\ \forall p = 1, 2, \dots\ \exists n_0 \in \mathbb{N}\ \forall m, n \geq n_0 :\ \|\delta_n(\alpha) - \delta_m(\alpha)\|_1 \leq \frac{1}{p}\right\}$$
$$= \bigcap_{p=1}^{\infty} \bigcup_{n_0=1}^{\infty} \bigcap_{n\geq n_0} \bigcap_{m\geq n_0} \left\{\alpha \in S(\mathscr{A}) :\ \|\delta_n(\alpha) - \delta_m(\alpha)\|_1 \leq \frac{1}{p}\right\}.$$

Each of the operators $\|\delta_n(\alpha) - \delta_m(\alpha)\|_1$ is a Baire one operator, hence the set $F(\mathscr{A})$ is a $G_{\delta\sigma\delta}$ set, therefore $F(\mathscr{A})$ satisfies the condition of Baire. Since $F(\mathscr{A})$ is a proper subgroup of B, Theorem 1 is a consequence of Banach's Subgroup Theorem. □

The following Lemma is proved in [5].

Lemma 2. Let Z be a convex nowhere dense set in a Banach space X. Then Z is strongly porous in X.

We shall show that in the set B_∞ endowed with the norm $\|\alpha\|_2 = \sup\{\|\alpha_k\|_1\}$, $\alpha = (\alpha_k) \in B_\infty$, the set $F(\mathscr{A})$ is strongly porous.

Theorem 2. *In the Banach space $(B_\infty, \|\cdot\|_2)$ the set $F(\mathscr{A})$ is strongly porous in B_∞ for any regular matrix $\mathscr{A}$.*

Proof. Each of the operators $\delta_n : B_\infty \to X$, $\delta_n(\alpha) = \sum_{k=1}^\infty a_{nk}\alpha_k$, $n = 1, 2, \ldots$, fulfils the Lipschitz condition with the same constant M.

Due to Theorem A, for $\alpha = (\alpha_k) \in B_\infty$, $\beta = (\beta_k) \in B_\infty$, we have

$$\|\delta_n(\alpha) - \delta_n(\beta)\|_1 = \left\| \sum_{k=1}^\infty a_{nk}(\alpha_k - \beta_k) \right\|_1 \le \|\alpha - \beta\|_2 \sum_{k=1}^\infty |a_{nk}| \le M\,\|\alpha - \beta\|_2\,. \tag{5}$$

The set $F(\mathscr{A})$ is a subspace of B_∞ and it is convex. We show that $B_\infty \setminus F(\mathscr{A})$ is open in B_∞. Let $\alpha = (\alpha_k) \in B_\infty \setminus F(\mathscr{A})$. Then the sequence $(\delta_n(\alpha))$ does not satisfy Cauchy's conditions, i. e.,

$$\exists \varepsilon > 0\ \forall n_0 \in \mathbb{N}\ \exists m, n > n_0 :\ \|\delta_n(\alpha) - \delta_m(\alpha)\|_1 \ge \varepsilon.$$

Let $\beta = (\beta_k) \in B_\infty$ such that

$$\|\alpha - \beta\|_2 < \frac{\varepsilon}{4M}. \tag{6}$$

Then, by (5), we have

$$\begin{aligned}\varepsilon &\le \|\delta_n(\alpha) - \delta_m(\alpha)\|_1 \\ &\le \|\delta_n(\alpha) - \delta_n(\beta)\|_1 + \|\delta_n(\beta) - \delta_m(\beta)\|_1 + \|\delta_m(\beta) - \delta_m(\alpha)\|_1 \\ &< 2M\|\alpha - \beta\|_2 + \|\delta_n(\beta) - \delta_m(\beta)\|_1.\end{aligned}$$

By (6), we have

$$\frac{\varepsilon}{2} < \|\delta_n(\beta) - \delta_m(\beta)\|_1.$$

Thus, $\beta = (\beta_k) \notin F(\mathscr{A})$ and

$$S\left(\alpha, \frac{\varepsilon}{4M}\right) \subset B_\infty \setminus F(\mathscr{A}),$$

which means that $B_\infty \setminus F(\mathscr{A})$ is open in B_∞.

Therefore, $F(\mathscr{A})$ is a closed nowhere dense subset of B_∞, and Theorem 2 follows from Lemma 2. □

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