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Generalized A-statistical convergence and a Korovkin type approximation theorem for double sequences

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GENERALIZED A-STATISTICAL CONVERGENCE AND A KOROVKIN TYPE APPROXIMATION THEOREM FOR DOUBLE SEQUENCES

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Abstract. In this paper, we use the notion of ideal to obtain two more general concepts and call them as I-A-statistical convergence and ideal A-summability. We also use the the concept of I-A-statistical convergence to prove a Korovkin type approximation theorem for double sequences of positive linear operators and present an interesting example to show that our Korovkin theorem is stronger than those proved earlier.

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Keywords: ideal convergence, statistical convergence, A-summability, Korovkin theorem, double sequence

1. INTRODUCTION

In [12], Gadjiev and Orhan has given a more general form of the classical Korovkin theorem (see [14]) by means of statistical convergence. Later, using different summability methods, some Korovkin type theorems have been obtained by many authors in several ways. For instance, see [4], [6–9], [10]. In recent years, the studies on double sequences has a rapid growth. Some concepts related to the single sequences have been extended to double sequences. In this work, we first generalize the concepts of A-statistical convergence and statistical A-summability for double sequences via ideals, and we call them as $\mathcal{I}$-A-statistical convergence and ideal A-summability. The another main purpose of this paper is to obtain a Korovkin type approximation theorem through the concept of $\mathcal{I}$-A-statistical convergence.
We refer the readers to [16] and [3] for more details on statistical convergence and ideal convergence of double sequences.

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2. $S_A^2(\mathcal{I})$-CONVERGENCE AND $\mathcal{I}^2(A)$-SUMMABILITY

Let $A=\left(a_{jk}^{mn}\right), m,n,j,k\in\mathbb{N}$, be a four dimensional matrix and $x=\left(x_{jk}\right)$ be a double sequence. Then the double (transformed) sequence, $Ax:=(y_{mn})$, is denoted by

$$y_{mn}:=\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}a_{jk}^{mn}x_{jk} \tag{2.1}$$

where it is assumed that the summation exists as a well-known Pringsheim convergence (or P-limit) for each $(m,n)\in\mathbb{N}\times\mathbb{N}$. Now, let $A=\left(a_{jk}^{mn}\right)$ be a nonnegative RH-regular matrix (see, [13],[17]), $x=\left(x_{jk}\right)$ be a double sequence and (y_{mn}) be defined as in (2.1). If (y_{mn}) is statistically convergent to L, then x is said to be statistically A-summable to L (see [1, 5]). Note that the concept of statistical A-summability for single sequences is introduced by Edely and Mursaleen in [11].
In this section, we first generalize the concepts of A-statistical convergence and statistical A-summability to $\mathcal{I}$-A-statistical convergence and ideal A-summability for double sequences, when $A=\left(a_{jk}^{mn}\right)$ is a nonnegative RH-regular matrix and $\mathcal{I}$ is an admissible ideal in $\mathbb{N}\times\mathbb{N}$. We further establish the relation between these more general forms of summability methods.

Definition 1. A double sequence $x=\left(x_{jk}\right)$ is said to be $\mathcal{I}$-A-statistically convergent (or briefly $S_A^2(\mathcal{I})$-convergent) to a number L if for each $\varepsilon>0$ and $\delta>0$

$$\left\{(m,n)\in\mathbb{N}\times\mathbb{N}:\sum_{(j,k):\left|x_{jk}-L\right|\geq\varepsilon}a_{jk}^{mn}\geq\delta\right\}\in\mathcal{I}.$$

In this case we write $S_A^2(\mathcal{I})-\lim x=L$.

Note that $\mathcal{I}$-A-statistically convergence for single sequences has been recently introduced by Savaş et al. (see [18]).

Definition 2. A double sequence $x=\left(x_{jk}\right)$ is said to be ideal A-summable (or briefly $\mathcal{I}^2(A)$-summable) to a number L if for each $\delta>0$

$$\{(m,n)\in\mathbb{N}\times\mathbb{N}:|y_{mn}-L|\geq\delta\}\in\mathcal{I}.$$

In this case we write $\mathcal{I}^2(A)-\lim x=L$.

Remark 1. (i) Let $\mathcal{I}_0:=\{A\subset\mathbb{N}\times\mathbb{N}:(\exists\, m(A)\in\mathbb{N})\ \ (j,k\geq m(A)\Rightarrow(j,k)\notin A)\}$.
If we take $\mathcal{I}=\mathcal{I}_0$ in Definition 1, then $S_A^2(\mathcal{I})$-convergence is reduced to A-statistical convergence. Also for $A=C(1,1)$, $S_A^2(\mathcal{I})$-convergence becomes $S_2(\mathcal{I})$ -convergence introduced by Belen and Yildirim in [2].
(ii) Let $\mathcal{I}_{\delta_2}=\{K\subset\mathbb{N}\times\mathbb{N}:\delta_2(K)=0\}$, where $\delta_2(K)$ is the double A-density of K.

If we take $\mathcal{I} = \mathcal{I}_{\delta_2}$ and $A = C(1,1)$, which is the double Cesàro matrix, in Definition 2, then $\mathcal{I}^2(A)$-summability coincides with statistical $C(1,1)$-summability introduced by Móricz in [15]. But the choice of $\mathcal{I} = \mathcal{I}_{\delta_2}$ in Definition 2 gives us the concept of statistical A-summability (see [1] and [5]).

Recall that a nontrivial ideal of $\mathbb{N}\times\mathbb{N}$ is called strongly admissible if $\{i\}\times\mathbb{N}$ and $\mathbb{N}\times\{i\}$ belong to $\mathcal{I}$ for each $i \in \mathbb{N}$.

Theorem 1. *Let $A = \left(a_{jk}^{mn}\right)$ be a nonnegative RH-regular matrix and $\mathcal{I}$ be a strongly admissible ideal in $\mathbb{N}\times\mathbb{N}$. If a double sequence is bounded and $S_A^2(\mathcal{I})$-convergent to L, then it is $\mathcal{I}^2(A)$-summable to L but not conversely.*

Proof. Let $x = (x_{jk})$ be bounded and $S_A^2(\mathcal{I})$-convergent to L. Write $M := \sup_{j,k}\left|x_{jk}-L\right|$ and $K(\varepsilon) := \left\{(j,k), j \le m, k \le n : \left|x_{jk}-L\right| \ge \varepsilon\right\}$ for any $\varepsilon > 0$. Then, we have

$$
\begin{aligned}
&|y_{mn}-L| = \\
&= \left|\sum_{j,k=1,1}^{\infty,\infty} a_{jk}^{mn}\left(x_{ij}-L\right) + L\left(\sum_{j,k=1,1}^{\infty,\infty} a_{jk}^{mn}-1\right)\right| \\
&\le \left|\sum_{j,k=1,1}^{\infty,\infty} a_{jk}^{mn}\left(x_{jk}-L\right)\right| + |L|\left|\sum_{j,k=1,1}^{\infty,\infty} a_{jk}^{mn}-1\right| \\
&\le \left|\sum_{(j,k)\in K(\varepsilon)} a_{jk}^{mn}\left(x_{jk}-L\right)\right| + \left|\sum_{(j,k)\notin K(\varepsilon)} a_{jk}^{mn}\left(x_{jk}-L\right)\right| + |L|\left|\sum_{j,k=1,1}^{\infty,\infty} a_{jk}^{mn}-1\right| \\
&\le M\sum_{(j,k)\in K(\varepsilon)} a_{jk}^{mn} + \varepsilon\sum_{(j,k)\notin K(\varepsilon)} a_{jk}^{mn} + |L|\left|\sum_{j,k=1,1}^{\infty,\infty} a_{jk}^{mn}-1\right| \\
&\le B\left\{\sum_{(j,k)\in K(\varepsilon)} a_{jk}^{mn} + \left|\sum_{j,k=1,1}^{\infty,\infty} a_{jk}^{mn}-1\right|\right\} + \varepsilon\|A\|
\end{aligned}
$$

where $\|A\| = \sup_{m,n}\sum_j\sum_k\left|a_{jk}^{mn}\right| < \infty$ and $B = \max\left(M, |L|\right)$. Now choose a $\delta > 0$ such that $\delta - \varepsilon\|A\| > 0$. Hence from the last inequality we obtain

$$
\begin{gathered}
\{(m,n) \in \mathbb{N}\times\mathbb{N} : |y_{mn}-L| \ge \delta\} \\
\subset \left\{n \in \mathbb{N} : \sum_{(j,k)\in K(\varepsilon)} a_{jk}^{mn} \ge \frac{\delta-\varepsilon\|A\|}{2B}\right\} \cup \left\{n \in \mathbb{N} : \left|\sum_{j,k=1,1}^{\infty,\infty} a_{jk}^{mn}-1\right| \ge \frac{\delta-\varepsilon\|A\|}{2B}\right\} \\
=: K_1 \cup K_2.
\end{gathered}
$$

By using definition of $S_A^2(\mathcal{I})$-convergence we get $K_1 \in \mathcal{I}$. Also since $\mathcal{I}$ is strongly admissible and A is RH-regular, we obtain that $\mathcal{I}-\lim_{m,n}\left|\sum_{j,k} a_{jk}^{mn}-1\right| = 0$, so $K_2 \in \mathcal{I}$. Hence, by definition of an ideal we conclude that $\mathcal{I}^2(A)-\lim x = L$.
To show that the converse is not true in general, let $\mathcal{I}$ be a strongly admissible ideal, $A = C(1,1)$ and $x = (x_{jk})$ be defined as

$$x_{jk} = (-1)^j \text{, for all } k.$$

Since

$$P-\lim_{m,n}\frac{1}{mn}\sum_{j=1}^{m}\sum_{k=1}^{n} x_{jk} = 0,$$

x is $C(1,1)$-summable to zero (hence $\mathcal{I}^2(C(1,1))$-summable to zero), but obviously x is not $C(1,1)^{\mathcal{I}}$-statistical convergent. This completes the proof of theorem. □

3. $S_A^2(\mathcal{I})$-APPROXIMATION

In this section we prove a Korovkin type approximation theorem via the concept of $S_A^2(\mathcal{I})$-convergence for a double sequence of positive linear operators defined on $C(K)$, where $C(K)$ is the space of all continuous real valued functions on any compact subset of the real two dimensional space. Note that $C(K)$ is a Banach space with the norm $\|.\|_{C(K)}$ defined by

$$\|f\|_{C(K)} := \sup_{(x,y)\in K} |f(x,y)|,\ (f \in C(K)).$$

Let L be a linear operator from $C(K)$ into $C(K)$. Then as usual, we say that L is positive linear operator provided that $f \geq 0$ implies $Lf \geq 0$. Also, we denote the value of Lf at a point (x,y) by $L(f;x,y)$. Before proceeding further, we quote here the classical and statistical forms of Korovkin-type theorems introduced in [20] and [6], respectively.

Theorem 2. *Let $\{L_{jk}\}$ be a double sequence of positive linear operators acting from $C(K)$ into itself. Then for all $f \in C(K)$,*

$$P-\lim_{j,k}\left\|L_{jk}(f)-f\right\|_{C(K)} = 0$$

if and only if

$$P-\lim_{j,k}\left\|L_{jk}(f_i)-f_i\right\|_{C(K)} = 0 \quad for \quad i = 0,1,2,3,$$

where $f_0(x,y) = 1,\ f_1(x,y) = x,\ f_2(x,y) = y,\ f_3(x,y) = x^2+y^2.$

Theorem 3. *Let $A = \left(a_{jk}^{mn}\right)$ be a nonnegative RH-regular matrix. Let $\{L_{jk}\}$ be a double sequence of positive linear operators acting from $C(K)$ into itself. Then for all $f \in C(K)$,*

$$st_A^2 - \lim_{j,k} \|L_{jk}(f) - f\|_{C(K)} = 0$$

if and only if

$$st_A^2 - \lim_{j,k} \|L_{jk}(f_i) - f_i\|_{C(K)} = 0 \quad for \quad i = 0,1,2,3,$$

where $f_0(x,y) = 1,\ f_1(x,y) = x,\ f_2(x,y) = y,\ f_3(x,y) = x^2 + y^2$.

Now we give the main result of this section.

Theorem 4. *Let $\{L_{jk}\}$ be a double sequence of positive linear operators acting from $C(K)$ into itself. Then for all $f \in C(K)$,*

$$S_A^2(\mathcal{I}) - \lim_{j,k} \|L_{jk}(f) - f\|_{C(K)} = 0 \tag{3.1}$$

if and only if

$$S_A^2(\mathcal{I}) - \lim_{j,k} \|L_{jk}(f_i) - f_i\|_{C(K)} = 0 \quad for\ i = 0,1,2,3, \tag{3.2}$$

where $f_0(x,y) = 1,\ f_1(x,y) = x,\ f_2(x,y) = y,\ f_3(x,y) = x^2 + y^2$.

Proof. Condition (3.2) follows immediately from condition (3.1) since each $f_i \in C(K)$, $(i = 0,1,2,3)$. Let us prove the converse. By the continuity of f on compact set K, we can write $|f(x,y)| \le M$, where $M = \|f\|_{C(K)}$. Also since $f \in C(K)$, for every $\varepsilon > 0$, there is a number $\delta > 0$ such that $|f(u,v) - f(x,y)| < \varepsilon$ for all $(u,v) \in K$ satisfying $|u - x| < \delta$ and $|v - y| < \delta$. Hence we get

$$|f(u,v) - f(x,y)| < \varepsilon + \frac{2M}{\delta^2}\left\{(u-x)^2 + (v-y)^2\right\}. \tag{3.3}$$

Since $\{L_{jk}\}$ is linear and positive, we obtain from (3.3) that

$$\begin{aligned}
&\left|L_{jk}(f;x,y) - f(x,y)\right| \\
&\le L_{jk}(|f(u,v) - f(x,y)|;x,y) + |f(x,y)|\left|L_{jk}(f_0;x,y) - f_0(x,y)\right| \\
&\le L_{jk}\left(\varepsilon + \frac{2M}{\delta^2}\left((u-x)^2 + (v-y)^2\right);x,y\right) \\
&\quad + |f(x,y)|\left|L_{jk}(f_0;x,y) - f_0(x,y)\right| \\
&\le \varepsilon + (\varepsilon + M)\left|L_{jk}(f_0;x,y) - f_0\right| + \frac{2M}{\delta^2}\left|L_{jk}(f_3;x,y) - f_3(x,y)\right| \\
&\quad + \frac{4M}{\delta^2}|x|\left|L_{jk}(f_1;x,y) - f_1(x,y)\right| + \frac{4M}{\delta^2}|y|\left|L_{jk}(f_2;x,y) - f_2(x,y)\right| \\
&\quad + \frac{2M}{\delta^2}\left(x^2 + y^2\right)\left|L_{jk}(f_0;x,y) - f_0(x,y)\right|
\end{aligned}$$

$$\leq \varepsilon + \left(\varepsilon + M + \frac{2M}{\delta^2}\left(C^2 + D^2\right)\right)\left|L_{jk}\left(f_0; x, y\right) - f_0\left(x, y\right)\right|$$
$$+ \frac{2M}{\delta^2}\left|L_{jk}\left(f_3; x, y\right) - f_3\left(x, y\right)\right| + \frac{4MC}{\delta^2}\left|L_{jk}\left(f_1; x, y\right) - f_1\left(x, y\right)\right|$$
$$+ \frac{4MD}{\delta^2}\left|L_{jk}\left(f_2; x, y\right) - f_2\left(x, y\right)\right|$$

where $C := \max|x|$, $D := \max|y|$. Taking supremum over $(x, y) \in K$ we get

$$\left\|L_{jk}(f) - f\right\|_{C(K)} \leq B \sum_{i=0}^{3} \left\|L_{jk}\left(f_i(u, v); x, y\right) - f_i(x, y)\right\|_{C(K)}$$

where

$$B := \max\left\{\varepsilon + M + \frac{2M}{\delta^2}\left(C^2 + D^2\right), \frac{2M}{\delta^2}, \frac{4MC}{\delta^2}, \frac{4MD}{\delta^2}\right\}.$$

For any $\sigma > 0$ define

$$D = \left\{(j,k) \in \mathbb{N} \times \mathbb{N} : \left\|L_{jk}(f; x, y) - f(x, y)\right\|_{C(K)} \geq \sigma\right\}$$
$$D_i = \left\{(j,k) \in \mathbb{N} \times \mathbb{N} : \left\|L_{jk}(f_i; x, y) - f_i(x, y)\right\|_{C(K)} \geq \frac{\sigma}{4B}\right\}, \; i = 0, 1, 2, 3.$$

Then $D \subset \bigcup_{i=0}^{3} D_i$ and hence

$$\left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \sum_{(j,k)\in D} a_{jk}^{mn} \geq \eta\right\} \subset \bigcup_{i=0}^{3} \left\{(m,n) \in \mathbb{N} \times \mathbb{N} : \sum_{(j,k)\in D_i} a_{jk}^{mn} \geq \eta\right\}$$

for any $\eta > 0$. From this inclusion and (3.2), we immediately get (3.1). This completes the proof of theorem. □

We remark that if we take $\mathcal{I} = \mathcal{I}_0$ in Theorem 4, we obtain Theorem 3.

Now, we will show that Theorem 4 is stronger than its classical and statistical forms.

Example 1. Let $\mathcal{I}$ be a strongly admissible ideal and $A \in \mathcal{I}$ be infinite set. Further let $\lambda = (\lambda_m)$ and $\mu = (\mu_n)$ be two non-decreasing sequences of positive numbers tending to ∞ such that

$$\lambda_{m+1} \leq \lambda_m + 1, \lambda_1 = 1 \;\text{ and }\; \mu_{n+1} \leq \mu_n + 1, \mu_1 = 1.$$

For the intervals $J_m := [m - \lambda_m + 1, m]$ and $I_n := [n - \mu_n + 1, n]$, consider the four dimensional matrix $A = \left(a_{jk}^{mn}\right)$ and the double sequence $x = \left(x_{jk}\right)$ given by

$$a_{jk}^{mn} = \begin{cases} \frac{1}{\lambda_m \mu_n} & ;\text{if } j \in J_m \text{ and } k \in I_n \\ 0 & ;\text{otherwise.} \end{cases}$$

and

$$x_{jk}=\begin{cases} jk & ;\text{if } m-\left[\sqrt{\lambda_m}\right]+1\le j\le m,\ n-\left[\sqrt{\mu_n}\right]+1\le k\le n,\ (m,n)\notin A \\ jk & ;\text{if } m-\lambda_m+1\le j\le m,\ n-\mu_n+1\le k\le n,\ (m,n)\in A \\ 0 & ;\text{otherwise} \end{cases} \tag{3.4}$$

respectively. In this case $S_A^2(\mathcal{I})$-convergence coincides with $S_{\lambda,\mu}(\mathcal{I})$-convergence introduced in [2]. Also x is neither P-convergent nor A-statistically convergent but $S_A^2(\mathcal{I})-\lim x=0$ (see again [2]). Now consider the following Bernstein operators (see [19]) given by

$$B_{jk}(f;x,y)=\sum_{p=0}^{j}\sum_{q=0}^{k} f\left(\frac{p}{j},\frac{q}{k}\right)\binom{j}{p}\binom{k}{q})x^p(1-x)^{j-p}y^q(1-y)^{k-q},$$

where $0\le x,y\le 1$ and $f\in C\left(I^2\right)$; $K:=I^2=[0,1]\times[0,1]$. Let $L_{jk}:C\left(I^2\right)\to C\left(I^2\right)$ be defined by

$$L_{jk}(f;x,y)=\left(1+x_{jk}\right)B_{jk}(f;x,y) \tag{3.5}$$

where $\left(x_{jk}\right)$ is defined as in (3.4). Then observe that

$$\begin{aligned} L_{jk}(f_0;x,y)&=\left(1+x_{jk}\right)f_0(x,y)\\ L_{jk}(f_1;x,y)&=\left(1+x_{jk}\right)f_1(x,y)\\ L_{jk}(f_2;x,y)&=\left(1+x_{jk}\right)f_2(x,y)\\ L_{jk}(f_3;x,y)&=\left(1+x_{jk}\right)\left(f_3(x,y)+\frac{x-x^2}{j}+\frac{y-y^2}{k}\right). \end{aligned}$$

Since $\mathcal{I}$ is strongly admissible ideal (in this case P -convergence implies $S_A^2(\mathcal{I})$-convergence), we obtain

$$S_A^2(\mathcal{I})-\lim_{j,k}\left\|L_{jk}(f_i)-f_i\right\|_{C(K)}=0$$

for $i=0,1,2,3$. Thus $\left(L_{jk}\right)$ satisfies the condition (3.2) of Theorem 4. Hence we have

$$S_A^2(\mathcal{I})-\lim_{j,k}\left\|L_{jk}(f)-f\right\|_{C(K)}=0,$$

for all $f\in C(K)$. But, since $\left(x_{jk}\right)$ is not convergent in Pringsheims's sense and A-statistical sense, we see that Theorem 2 and Theorem 3 do not work for our operators defined by (3.5). Hence it is shown that our version is more general to deal with the situation when the operators L_{jk} do not satisfy the conditions of Theorem 2 and Theorem 3.

Finally by using trigonemetric test functions, we give the statement of Theorem 4 for a double sequence of positive linear operators defined on $C^*(\mathbb{R}^2)$, the space of all 2π-periodic and real valued continuous functions on $\mathbb{R}^2$. This space is equipped with the supremum norm

$$\|f\|_{C^*(\mathbb{R}^2)} := \sup_{(x,y)\in\mathbb{R}^2} |f(x,y)|, \ \left(f \in C^*(\mathbb{R}^2)\right).$$

Then using the similar technique as in the proof of Theorem 4, one can also get the following result.

Theorem 5. *Let $\{L_{jk}\}$ be a double sequence of positive linear operator acting from $C^*(\mathbb{R}^2)$ into itself. Then for all $f \in C^*(\mathbb{R}^2)$*

$$S_A^2(\mathcal{I}) - \lim_{j,k} \|L_{jk}(f) - f\|_{C^*(\mathbb{R}^2)} = 0$$

if and only if

$$S_A^2(\mathcal{I}) - \lim_{j,k} \|L_{jk}(f_i) - f_i\|_{C^*(\mathbb{R}^2)} = 0,$$

where $f_0(x,y) = 1,\ f_1(x,y) = \sin x,\ f_2(x,y) = \sin y,\ f_3(x,y) = \cos x,\ f_4(x,y) = \cos y$.

If the ideal $\mathcal{I}$ in Theorem 5 is replaced by $\mathcal{I}_0$, we immediately get the following result proved in [4].

Corollary 1. *Let $\{L_{jk}\}$ be a double sequence of positive linear operator acting from $C^*(\mathbb{R}^2)$ into itself. Then for all $f \in C^*(\mathbb{R}^2)$*

$$st_A^2 - \lim_{j,k} \|L_{jk}(f) - f\|_{C^*(\mathbb{R}^2)} = 0$$

if and only if

$$st_A^2 - \lim_{j,k} \|L_{jk}(f_i) - f_i\|_{C^*(\mathbb{R}^2)} = 0,$$

where $f_0(x,y) = 1,\ f_1(x,y) = \sin x,\ f_2(x,y) = \sin y,\ f_3(x,y) = \cos x,\ f_4(x,y) = \cos y$.

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