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ON A NOTE OF CONVERGENCE THEOREMS FOR ZEROS OF GENERALIZED LIPSCHITZ Φ-QUASI-ACCRETIVE OPERATORS

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Abstract. In this paper, the convergence of Mann iterative process with errors for generalized Lipschitz Φ-quasi-accretive operators is proved in uniformly smooth Banach spaces. Our results improve the corresponding results of Chidume et al.[2].

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1. INTRODUCTION

Let E be a real Banach space and E^* be its dual space. The normalized duality mapping $J : E \to 2^{E^*}$ is defined by

$$J(x) = \{f \in E^* :< x, f >= \|x\|^2 = \|f\|^2\},$$

for all $x \in E$, where $< \cdot, \cdot >$ denotes the generalized duality pairing. It is well known that

(1) If E is a smooth Banach space, then the mapping J is single-valued.
(2) $J(\alpha x) = \alpha J(x)$ for all $x \in E$ and $\alpha \in \Re$.
(3) If E is a uniformly smooth Banach space, then the mapping J is uniformly continuous on any bounded subset of E(see [1] and [3]).

In the sequel, we denote the single-valued normalized duality mapping by j.

Definition 1 ([2]). Let $T : E \to E$ be an operator. T is said to be strongly accretive if there is a positive constant $k \in (0,1)$ such that for every $x, y \in E$, there exists $j(x-y) \in J(x-y)$ such that

$$< Tx - Ty, j(x-y) >\geq k\|x-y\|^2. \tag{1.1}$$

Let τ denote the class of all strictly increasing continuous function $f : [0,+\infty) \to [0,+\infty)$ with $f(0) = 0$. Given $\phi \in \tau$, we say that T is ϕ-strongly accretive if for each $x, y \in E$, there exists $j(x-y) \in J(x-y)$ such that

$$< Tx - Ty, j(x-y) >\geq \phi(\|x-y\|)\|x-y\|. \tag{1.2}$$

Further, let $\Phi \in \tau$, T is called Φ-accretive if there exists $j(x-y) \in J(x-y)$, such that the inequality

$$< Tx - Ty, j(x-y) > \geq \Phi(\|x-y\|) \tag{1.3}$$

holds for all $x, y \in E$.

It is shown that the class of Φ-accretive operators not only properly includes the class of ϕ-strongly accretive operators, but also that of strongly accretive operators. Let $N(T) = \{x \in E : Tx = 0\} \neq \varnothing$. If the above inequalities (1.1), (1.2) and (1.3) hold for any $x \in E$ and $y \in N(T)$, then the corresponding operator T is called strongly quasi-accretive, ϕ-strongly quasi-accretive and Φ-quasi-accretive, respectively. Closely related to the class of accretive operators is that of pseudocontractive types.

Let $F(T) = \{x \in E : Tx = x\} \neq \varnothing$. A mapping $T : E \to E$ is called strongly hemi-contractive, ϕ-strongly hemi-contractive and Φ-hemi-contractive if and only if $I - T$ is strongly quasi-accretive, ϕ-strongly quasi-accretive and Φ-quasi-accretive, respectively. Here I denotes the identity mapping of E.

Definition 2 (see [7])**.** For arbitrary given $x_0 \in E$, Mann iterative process with errors $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1 - a_n - c_n)x_n + a_n T x_n + c_n u_n, n \geq 0, \tag{1.4}$$

where $\{u_n\}$ is any bounded sequence in E; $\{a_n\}$ and $\{c_n\}$ are two real sequences in $[0,1]$ satisfying $a_n + c_n \leq 1$ for any $n \geq 0$.

Definition 3 (see [8,9])**.** A mapping $T : E \to E$ is called generalized Lipschitz if there exists a constant $L > 0$ such that

$$\|Tx - Ty\| \leq L(1 + \|x - y\|)$$

for all $x, y \in E$.

Recently, C. E. Chidume and C. O. Chidume [2] established an approximation theorem for the zeros of generalized Lipschitz Φ-quasi-accretive operators. Their result is as follows.

Chidume's Theorem. Let E be a uniformly smooth real Banach space and $A : E \to E$ be a mapping with $N(A) \neq \varnothing$. Suppose A is a generalized Lipschitz generalized Φ-quasi-accretive mapping. Let $\{a_n\}, \{b_n\}$ and $\{c_n\}$ be real sequences in $[0,1]$ satisfying the following conditions: (i) $a_n + b_n + c_n = 1$; (ii) $\Sigma_{n=0}^{\infty}(b_n + c_n) = \infty$; (iii) $\Sigma_{n=0}^{\infty} c_n < \infty$; (iv) $\lim_{n\to\infty} b_n = 0$. Let $\{x_n\}$ be generated iteratively from arbitrary $x_0 \in E$ by,

$$x_{n+1} = a_n x_n + b_n S x_n + c_n u_n, n \geq 0,$$

where $S : E \to E$ is defined by $Sx := f + x - Ax, \forall x \in E$ and $\{u_n\}$ is an arbitrary bounded sequence in E. Then, there exists $\gamma_0 \in \Re$ such that if $b_n + c_n \leq \gamma_0, \forall n \geq 0$, the sequence $\{x_n\}$ converges strongly to the unique solution of the equation $Au = 0$.

This result improves a lot of recent contributions in the area. However, there exists a gap in its provided proof. Precisely, $c_n = min\{\frac{\epsilon}{4\beta}, \frac{1}{2\sigma}\Phi(\frac{\epsilon}{2})\alpha_n\}$ does not holds in line 14 of Claim 2 of page 248, i.e., $c_n \leq \frac{1}{2\sigma}\Phi(\frac{\epsilon}{2})\alpha_n$ is a wrong case, and it was applied to the formula of line 3rd of page 249.

Example 1. Setting the iteration parameters: $a_n = 1 - b_n - c_n$, where
$\{b_n\}_{n=0}^{\infty} : b_0 = b_1 = 0, b_n = \frac{1}{n}, n \geq 2.$
$\{c_n\}_{n=0}^{\infty} : 0, \frac{1}{\sqrt{1^2}}, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{\sqrt{4^2}}, \frac{1}{5^2}, \cdots, \frac{1}{8^2}, \frac{1}{\sqrt{9^2}}, \frac{1}{10^2}, \cdots, \frac{1}{15^2}, \frac{1}{\sqrt{16^2}}, \frac{1}{17^2}, \cdots.$
Then $\Sigma_{n=0}^{\infty} c_n < +\infty$, but $c_n \neq o(b_n + c_n)$. Therefore, the proof of Theorem 3.1 of [2] is incorrect.

The aim of this paper is to establish a convergence result relative to the Mann iteration with errors for generalized Lipschitz Φ-quasi-accretive operators in uniformly smooth real Banach spaces. The following auxiliary facts will be needed.

Lemma 1 (see [3])**.** *Let E be a uniformly smooth real Banach space and let $J : E \to 2^{E^*}$ be a normalized duality mapping. Then*

$$\|x + y\|^2 \leq \|x\|^2 + 2 < y, J(x + y) >$$

for all $x, y \in E$.

Lemma 2 (see [6])**.** *Let $\{\rho_n\}_{n=0}^{\infty}$ be a nonnegative real numbers sequence satisfying the condition*

$$\rho_{n+1} \leq (1 - \theta_n)\rho_n + o(\theta_n), n \geq 0,$$

where $\theta_n \in [0, 1]$ with $\Sigma_{n=0}^{\infty}\theta_n = \infty$. Then $\rho_n \to 0$ as $n \to \infty$.

2. Results

Theorem 1. *Let E be an arbitrary uniformly smooth real Banach space and $T : E \to E$ be a generalized Lipschitz Φ-quasi-accretive operator with $N(T) \neq \varnothing$. Let $\{a_n\}, \{c_n\}$ be two real numbers sequences in $[0, 1]$ and satisfy the conditions (i) $a_n + c_n \leq 1$; (ii) $a_n, c_n \to 0$ as $n \to \infty$ and $c_n = o(a_n)$; (iii) $\Sigma_{n=0}^{\infty} a_n = \infty$. For some $x_0 \in E$, let $\{u_n\}$ be any bounded sequence and $\{x_n\}$ be Mann iterative sequence with errors defined by*

$$x_{n+1} = (1 - a_n - c_n)x_n + a_n S x_n + c_n u_n, n \geq 0, \tag{2.1}$$

where $S : E \to E$ is defined by $Sx = x - Tx$ for any $x \in E$. Then sequence $\{x_n\}$ converges strongly to the unique solution of the equation $Tx = 0$.

Proof. Let $q \in N(T)$, that is $q \in F(S)$. Since $T : E \to E$ is a generalized Lipschitz Φ-quasi-accretive operator, then S is a generalized Lipschitz Φ-hemi-contractive, i.e., there exists $\Phi \in \tau$ such that

$$< Sx - Sq, J(x - q) > \leq \|x - q\|^2 - \Phi(\|x - q\|),$$

and

$$\|Sx - Sy\| \leq L(1 + \|x - y\|)$$

for any $x, y \in E$.

Step 1: There exists $x_0 \in D$ and $x_0 \neq Sx_0$ such that

$$r_0 = \|x_0 - Sx_0\| \cdot \|x_0 - q\| \in R(\Phi).$$

Indeed, if $\Phi(r) \to +\infty$ as $r \to +\infty$, then $r_0 \in R(\Phi)$; if $\sup\{\Phi(r) : r \in [0, +\infty)\} = r_1 < +\infty$, then for $q \in E$, there exists a sequence $\{w_n\}$ in E such that $w_n \to q$ as $n \to \infty$ with $w_n \neq q$. Furthermore, we obtain that $\{w_n - Sw_n\}$ is bounded. Hence there exists a natural number n_0 such that $\|w_n - Sw_n\| \cdot \|w_n - q\| < \frac{r_1}{2}$ for $n \geq n_0$, then we renew define $x_0 = w_{n_0}$ and $\|x_0 - Sx_0\| \cdot \|x_0 - q\| \in R(\Phi)$.

Step 2: For any $n \geq 0$, $\{x_n\}$ is bounded.

Setting $R = \Phi^{-1}(r_0)$, then from Definition 2, we obtain that $\|x_0 - q\| \leq R$. Denote

$$B_1 = \{x \in D : \|x - q\| \leq R\}, B_2 = \{x \in D : \|x - q\| \leq 2R\}.$$

Since S is generalized Lipschitz, so S is bounded. Let

$$M = \sup_{x \in B_2} \{\|Sx - q\| + 1\} + \sup_n \{\|u_n - q\|\}.$$

Next, we want to prove that $x_n \in B_1$. If $n = 0$, then $x_0 \in B_1$. Now assume that it holds for some n, i.e., $x_n \in B_1$. We prove that $x_{n+1} \in B_1$. Suppose it is not the case, then $\|x_{n+1} - q\| > R$. Since J is uniformly continuous on bounded subset of E, then for $\epsilon_0 = \frac{\Phi(\frac{R}{2})}{4[L+(1+L)R]}$, there exists $\delta > 0$ such that $\|Jx - Jy\| < \epsilon$ when $\|x - y\| < \delta, \forall x, y \in B_2$. Now denote

$$\tau_0 = \min\{\frac{R}{2M}, \frac{\Phi(\frac{R}{2})}{8R(M+2R)}, \frac{\delta}{2(M+2R)}, \frac{R + L(1+R)}{2(M+R)}\}.$$

Since $a_n, c_n \to 0$ as $n \to \infty$, without loss of generality, we assume that $0 \leq a_n, c_n \leq \tau_0$ for any $n \geq 0$. Since $c_n = o(a_n)$, denote $c_n < a_n \tau_0$. So we have

$$\|u_n - x_n\| \leq \|x_n - q\| + \|u_n - q\| \leq R + M, \tag{2.2}$$

$$\|x_n - Sx_n\| \leq L + (1+L)\|x_n - q\| \leq L + (1+L)R, \tag{2.3}$$

$$\|x_n - q\| \geq \|x_{n+1} - q\| - a_n\|Sx_n - q\| - c_n\|u_n - q\| > \frac{R}{2}, \tag{2.4}$$

$$\begin{aligned} \|x_{n+1} - q\| &\leq (1 - a_n - c_n)\|x_n - q\| + a_n\|Sx_n - q\| + c_n\|u_n - q\| \\ &\leq R + \tau_0 M \leq 2R, \end{aligned} \tag{2.5}$$

and

$$\begin{aligned}\|(x_{n+1}-q)-(x_n-q)\| &\le a_n\|Sx_n-x_n\|+c_n\|u_n-x_n\| \\ &\le a_n(\|Sx_n-q\|+\|x_n-q\|) \\ &+c_n(\|u_n-q\|+\|x_n-q\|) \\ &\le \tau_0(M+2R) \\ &< \delta.\end{aligned} \tag{2.6}$$

So

$$\|J(x_{n+1}-q)-J(x_n-q)\| < \epsilon_0.$$

Using Lemma 1 and above formulas, we obtain

$$\begin{aligned}\|x_{n+1}-q\|^2 &\le \|x_n-q\|^2+2a_n < Sx_n-x_n, J(x_{n+1}-q) > \\ &+2c_n < u_n-x_n, J(x_{n+1}-q) > \\ &\le \|x_n-q\|^2-2a_n\Phi(\|x_n-q\|)+2a_n\|x_n-Sx_n\| \\ &\cdot\|J(x_{n+1}-q)-J(x_n-q)\| \\ &+2c_n\|u_n-x_n\|\cdot\|x_{n+1}-q\| \\ &\le R^2-2a_n\Phi(\frac{R}{2})+2a_n(L+(1+L)R)\epsilon_0 \\ &+4a_n\tau_0(M+R)R \\ &\le R^2,\end{aligned} \tag{2.7}$$

which is a contradiction. Hence $x_{n+1}\in B_1$, i.e., $\{x_n\}$ is a bounded sequence.

Step 3: We want to prove that $\|x_n-q\|\to 0$ as $n\to\infty$.

Setting

$$M_1=\sup_n\|x_n-q\|+\sup_n\|u_n-q\|.$$

Since $a_n, c_n\to 0$ as $n\to\infty$, then

$$\|(x_{n+1}-q)-(x_n-q)\|\to 0.$$

Hence

$$\|J(x_{n+1}-q)-J(x_n-q)\|\to 0$$

as $n \to \infty$. From (2.7), we have

$$\begin{aligned}
\|x_{n+1}-q\|^2 \le\ & \|x_n-q\|^2 - 2a_n\Phi(\|x_n-q\|) \\
& + 2a_n\|x_n - Sx_n\| \cdot \|J(x_{n+1}-q) - J(x_n-q)\| \\
& + 2c_n\|u_n - x_n\| \cdot \|x_{n+1}-q\| \\
\le\ & \|x_n-q\|^2 - 2a_n\Phi(\|x_n-q\|) \\
& + 2a_n[(1+L)\|x_n-q\| + L]A_n \\
& + 2c_n\|u_n - x_n\| \cdot \|x_{n+1}-q\| \\
\le\ & \|x_n-q\|^2 - 2a_n\Phi(\|x_n-q\|) \\
& + a_nA_n(1+L)\|x_n-q\|^2 + a_nA_n(1+L) \\
& + 2a_nLA_n + 4c_nM_1^2 \\
\le\ & \|x_n-q\|^2 - 2a_n\Phi(\|x_n-q\|) \\
& + a_nA_n(1+L))M_1^2 + a_nA_n(1+3L) + 4c_nM_1^2 \\
=\ & \|x_n-q\|^2 - 2a_n\Phi(\|x_n-q\|) + C_n \\
=\ & \|x_n-q\|^2 + 2a_n[B_n - \Phi(\|x_n-q\|)],
\end{aligned} \tag{2.8}$$

where

$$A_n = \|J(x_{n+1}-q) - J(x_n-q)\|, \;\; B_n = \frac{C_n}{2a_n},$$

$$C_n = a_nA_n(1+L))M_1^2 + a_nA_n(1+3L) + 4c_nM_1^2.$$

Letting $\inf_{n\ge 0} \frac{\Phi(\|x_n-q\|)}{1+\|x_{n+1}-q\|^2} = \lambda$, then $\lambda = 0$. If it is not the case, we assume that $\lambda > 0$. Let $0 < \gamma < \min\{1, \lambda\}$, then $\frac{\Phi(\|x_n-q\|)}{1+\|x_{n+1}-q\|^2} \ge \gamma$, i.e.,

$$\Phi(\|x_n-q\|) \ge \gamma + \gamma\|x_{n+1}-q\|^2 \ge \gamma\|x_{n+1}-q\|^2.$$

Thus, from (2.8) that

$$\|x_{n+1}-q\|^2 \le \|x_n-q\|^2 + 2a_n(B_n - \gamma\|x_{n+1}-q\|^2), \tag{2.9}$$

which implies that

$$\begin{aligned}
\|x_{n+1}-q\|^2 & \le \tfrac{1}{1+2a_n\gamma}\|x_n-q\|^2 + \tfrac{2a_nB_n}{1+2a_n\gamma} \\
& = (1 - \tfrac{2a_n\gamma}{1+2a_n\gamma})\|x_n-q\|^2 + \tfrac{2a_nB_n}{1+2a_n\gamma}.
\end{aligned} \tag{2.10}$$

Let $\rho_n = \|x_n-q\|^2, \lambda_n = \frac{2a_n\gamma}{1+2a_n\gamma}, \sigma_n = \frac{2a_nB_n}{1+2a_n\gamma}$. Then we get that

$$\rho_{n+1} \le (1-\lambda_n)\rho_n + \sigma_n.$$

Applying Lemma 2, then $\rho_n \to 0$ as $n \to \infty$. Thus $\lambda = 0$, it is a contradiction. Therefore, there exists an infinite subsequence such that $\frac{\Phi(\|x_{n_i}-q\|)}{1+\|x_{n_i+1}-q\|^2} \to 0$ as $i \to$

∞. Since $0 \leq \frac{\Phi(\|x_{n_i}-q\|)}{1+M_1^2} \leq \frac{\Phi(\|x_{n_i}-q\|)}{1+\|x_{n_i+1}-q\|^2}$, then $\Phi(\|x_{n_i}-q\|) \to 0$ as $i \to \infty$. By the strictly increasing and continuity of Φ, we get $\|x_{n_i}-q\| \to 0$ as $i \to \infty$. Next we prove $\|x_n - q\| \to 0$ as $n \to \infty$. Let $\forall \varepsilon \in (0,1)$, there exists n_{i_0} such that $\|x_{n_i}-q\| < \epsilon, a_n, c_n < \frac{\epsilon}{8M_1}, B_n < \frac{1}{2}\Phi(\frac{\epsilon}{2})$, for any $n_i, n \geq n_{i_0}$. First, we prove $\|x_{n_i+1}-q\| < \epsilon$. Suppose that it is not this case, then $\|x_{n_i+1}-q\| \geq \epsilon$. By Definition 2, we estimate the following formula:

$$\begin{aligned}
\|x_{n_i}-q\| &\geq \|x_{n_i+1}-q\| - a_{n_i}\|Sx_{n_i}-x_{n_i}\| - c_{n_i}\|u_{n_i}-x_{n_i}\| \\
&> \epsilon - a_{n_i}[\|Sx_{n_i}-q\| + \|x_{n_i}-q\|] - c_{n_i}[\|u_{n_i}-q\| + \|x_{n_i}-q\|] \\
&\geq \epsilon - (b_{n_i}+c_{n_i})2M_1 \\
&> \frac{\epsilon}{2}.
\end{aligned} \tag{2.11}$$

Since Φ is strictly increasing, (2.11) leads to

$$\Phi(\|x_{n_i}-q\|) \geq \Phi(\frac{\epsilon}{2}).$$

From (2.8), we have

$$\begin{aligned}
\|x_{n_i+1}-q\|^2 &\leq \|x_{n_i}-q\|^2 + 2a_{n_i}[B_{n_i} - \Phi(\|x_{n_i}-q\|)] \\
&< \epsilon^2 + 2a_{n_i}[\frac{1}{2}\Phi(\frac{\epsilon}{2}) - \Phi(\frac{\epsilon}{2})] \\
&\leq \epsilon^2,
\end{aligned} \tag{2.12}$$

which is a contradiction. Thus $\|x_{n_i+1}-q\| < \epsilon$. Suppose that $\|x_{n_i+m}-q\| < \epsilon$ holds. Repeating the above course, we can easily show that $\|x_{n_i+m+1}-q\| < \epsilon$ holds. Therefore, we obtain that $\|x_{n_i+m}-q\| < \epsilon$ for any positive integer m, which means $\|x_n - q\| \to 0$ as $n \to \infty$. This completes the proof. □

Remark 1. In above theorem we assume a condition to have $N(T) \neq \varnothing$ for generalized Lipschitz Φ-quasi-accretive operator. However before presenting the main results, we note that this assumption is not necessary in [4] and [5]. The reason is that for Lipschitz or continuous Φ-accretive operators we have $N(T) \neq \varnothing$, but for a generalized Lipschitz mapping it can not be assumed that it must be Lipschitz or continuous, which means that $N(T)$ may be empty. For this, we add a sufficient condition. Therefore our Theorem 1 includes the past results of [4] and [5] which are known as the existence theorems for Lipschitz or continuous Φ-accretive operators and which are special cases of our Theorem 1.

Theorem 2. *Let D be a nonempty closed convex subset of uniformly smooth real Banach space E, and $T : D \to D$ a generalized Lipschitz Φ-hemi-contractive mapping with $q \in F(T) \neq \varnothing$. Let $\{a_n\}, \{c_n\}$ be real sequences in $[0,1]$ and satisfy the conditions (i) $a_n + c_n \leq 1$; (ii) $a_n \to 0$ as $n \to \infty$; (iii) $\Sigma_{n=0}^{\infty} a_n = \infty$; (iv)$c_n = o(a_n)$.*

Let $\{u_n\}$ be any bounded sequence in D. For some $x_0 \in D$, let $\{x_n\}$ be Mann iterative scheme with errors defined by (1.4). Then $\{x_n\}$ converges strongly to the unique fixed point q of T.

Proof. Since $T : D \to D$ is a generalized Lipschitz Φ-hemi-contractive mapping, then

$$Tx - Tq, J(x-q) >\leq \|x-q\|^2 - \Phi(\|x-q\|)$$

and

$$\|Tx - Ty\| \leq L(1 + \|x-y\|)$$

hold for any $x, y \in E, q \in F(T)$. The rest follows as in Theorem 1. □

Remark 2. In Theorem 1 and Theorem 2, the condition $\Sigma_{n=0}^{\infty} c_n < \infty$ of the iteration parameter $\{c_n\}$ is replaced by $c_n = o(a_n)$, and these two conditions are not included each other (See above Counterexample). Up to now, it is unknown whether the results of Chidume et al. [2] hold for the condition $\Sigma_{n=0}^{\infty} c_n < \infty$. Hence our Theorem 1 and Theorem 2 improve Theorem 3.1 and Theorem 3.2 of [2], respectively.

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