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Corrigendum to Multiplicity of positive weak solutions to subcritical singular elliptic Dirichlet problems

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Abstract. This paper serves as a corrigendum to the paper "Multiplicity of positive weak solutions to subcritical singular elliptic Dirichlet problems", published in *Electron J. Qual. Theory Differ. Equ.* **2017**, No. 100, 1–30. We modify one of the assumptions of that paper and we present a correct proof of the Lemma 2.11 of that paper.

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1 Introduction

Lemma 2.11 in [1], under the assumptions stated there, is false. In order to correct this situation, the assumption *H2)* of [1], Theorem 1.1 (assumed, jointly with *H1)* and *H3)–H5)*, in the quoted lemma and throughout the whole article [1]) must be replaced (throughout the whole article [1]) by the (slightly stronger) following new version of it:

H2) $a \in L^{\infty}(\Omega)$, $a \geq 0$ *a.e.* in Ω, *and there exists* $\delta > 0$ *such that* $\inf_{A_\delta} a > 0$.

Here and below, for $\rho > 0$,

$$A_\rho := \{x \in \Omega : d_\Omega(x) \leq \rho\},$$

where $d_\Omega := \operatorname{dist}(\cdot, \partial\Omega)$; and, for a measurable subset E of Ω, $\inf_E$ means the essential infimum on E. In the next section we give (assuming the stated new version of *H2)*) a correct proof of [1, Lemma 2.11]. With these changes, all the results contained in [1] hold.

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2 Correct proof of [1, Lemma 2.11]

Below, "problem (2.4)" refers to the problem labeled (2.4) in [1]; i.e., refers to the problem

$$\begin{cases} -\Delta u = \chi_{\{u>0\}} a(x) u^{-\alpha} + \zeta & \text{in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \\ u \geq 0 \text{ in } \Omega, \ u > 0 \ a.e. \text{ in } \{a > 0\}, \end{cases}$$

where $\zeta \in L^{\infty}(\Omega)$. Recall that the new version of *H2)* is assumed in the following lemma.

Lemma 2.1 ([1, Lemma 2.11]). *Assume* $1 < \alpha < 3$, *and let* $\zeta \in L^{\infty}(\Omega)$ *be such that* $\zeta \geq 0$. *Let* u *be the solution to problem* (2.4) *given by [1, Lemma 2.5] (in the sense stated there). Then there exists a positive constant* c, *independent of* ζ, *such that* $u \geq c d_{\Omega}^{\frac{2}{1+\alpha}}$ *in* Ω.

Proof. From [1, Lemma 2.5], there exists a positive constant c', independent of ζ, such that $u \geq c' d_{\Omega}$ *a.e.* in Ω. Then (since $\inf_{\Omega \setminus A_{\frac{\delta}{4}}} d_{\Omega} > 0$), there exists a positive constant c'' (that depends on δ, but not on ζ) such that

$$u \geq c'' d_{\Omega}^{\frac{2}{1+\alpha}} \quad a.e. \text{ in } \Omega \setminus A_{\frac{\delta}{4}}. \tag{2.1}$$

Let U be a $C^{1,1}$ domain such that $A_{\frac{3\delta}{4}} \subset U \subset A_{\delta}$. Note that $\partial U \setminus \partial\Omega \subset \Omega \setminus A_{\frac{\delta}{2}}$. Indeed, let $z \in \partial U \setminus \partial\Omega$. Since $\overline{U} \subset A_{\delta} \cup \partial\Omega$, we have $z \in \Omega$. If $z \in A_{\frac{\delta}{2}}$, then, for some open set V_z such that $z \in V_z \subset \Omega$, we would have $d_{\Omega} \leq \frac{3}{4}\delta$ on V_z, and so $V_z \subset A_{\delta} \subset U$, which contradicts that $z \in \partial U$. Then $\partial U \setminus \partial\Omega \subset \Omega \setminus A_{\frac{\delta}{2}}$.

We claim that

$$d_U = d_{\Omega} \quad \text{in } A_{\frac{\delta}{8}}, \tag{2.2}$$

where $d_U := \operatorname{dist}(\cdot, \partial U)$. Indeed, let $x \in A_{\frac{\delta}{8}}$, let $y_x \in \partial\Omega$ be such that $d_{\Omega}(x) = |x - y_x|$, and let $w \in \partial U \setminus \partial\Omega$. Since $\partial U \setminus \partial\Omega \subset \Omega \setminus A_{\frac{\delta}{2}}$, we have $|w - y_x| \geq d_{\Omega}(z) > \frac{\delta}{2}$. Also, $|x - y_x| = d_{\Omega}(x) \leq \frac{\delta}{8}$. Therefore, by the triangle inequality, $|w - x| \geq |w - y_x| - |x - y_x| > \frac{\delta}{2} - \frac{\delta}{8} = \frac{3\delta}{8}$. Then $\operatorname{dist}(x, \partial U \setminus \partial\Omega) \geq \frac{3\delta}{8}$ for any $x \in A_{\frac{\delta}{8}}$, and so (since $d_{\Omega}(x) \leq \frac{\delta}{8}$), $d_U(x) = \min\{\operatorname{dist}(x, \partial U \setminus \partial\Omega), d_{\Omega}(x)\} = d_{\Omega}(x)$ for all $x \in A_{\frac{\delta}{8}}$.

Since $U \subset A_{\delta}$ we have that $\underline{a} := \inf_U a > 0$. Let σ_1 be the principal eigenvalue for $-\Delta$ in U with homogeneous Dirichlet boundary condition and weight function a, and let ψ_1 be the corresponding positive principal eigenfunction, normalized by $\|\psi_1\|_{\infty} = 1$. Observe that $\psi_1^{\frac{2}{1+\alpha}} \in H_0^1(U) \cap L^{\infty}(U)$ (because $1 < \alpha < 3$), and that a computation gives

$$\begin{aligned} -\Delta\left(\psi_1^{\frac{2}{1+\alpha}}\right) &= \frac{2}{1+\alpha}\sigma_1 a \psi_1^{\frac{2}{1+\alpha}} + \frac{2}{1+\alpha}\frac{\alpha-1}{1+\alpha}\left(\psi_1^{\frac{2}{1+\alpha}}\right)^{-\alpha}|\nabla\psi_1|^2 \\ &\leq \beta a \left(\psi_1^{\frac{2}{1+\alpha}}\right)^{-\alpha} \qquad a.e. \text{ in } U, \end{aligned}$$

where $\beta := \frac{2}{1+\alpha}\sigma_1 + \frac{2}{1+\alpha}\frac{\alpha-1}{1+\alpha}\frac{1}{\underline{a}}\|\nabla\psi_1\|_{\infty}^2$. Then

$$-\Delta\left(\beta^{-\frac{1}{1+\alpha}}\psi_1^{\frac{2}{1+\alpha}}\right) \leq a\left(\beta^{-\frac{1}{1+\alpha}}\psi_1^{\frac{2}{1+\alpha}}\right)^{-\alpha} \qquad \text{in } U$$

in the weak sense of [1, Lemma 2.5] (i.e., with test functions in $H_0^1(U) \cap L^\infty(U)$). Moreover, again in the weak sense of [1, Lemma 2.5], $-\Delta u \geq a u^{-\alpha}$ in U. Also $u \geq \beta^{-\frac{1}{1+\alpha}} \psi_1^{\frac{2}{1+\alpha}}$ in ∂U. Then, by the weak maximum principle in [2, Theorem 8.1], $u \geq \beta^{-\frac{1}{1+\alpha}} \psi_1^{\frac{2}{1+\alpha}}$ *a.e.* in U; therefore, for some positive constant c''' independent of ζ, $u \geq c''' d_U^{\frac{2}{1+\alpha}}$ *a.e.* in U. In particular,

$$u \geq c''' d_U^{\frac{2}{1+\alpha}} \quad \textit{a.e. } \text{in } A_{\frac{\delta}{8}}. \tag{2.3}$$

From (2.1), (2.3), and (2.2), we get $u \geq c d_\Omega^{\frac{2}{1+\alpha}}$ *a.e.* in Ω, with $c := \min\{c'', c'''\}$ and the lemma follows. □

References

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