Orientation with a Viking sun-compass, a shadow-stick, and two calcite sunstones under various weather conditions

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It is widely accepted that Vikings used sun-compasses to derive true directions from the cast shadow of a gnomon. It has been hypothesized that when a cast shadow was not formed, Viking navigators relied on crude skylight polarimetry with the aid of dichroic or birefringent crystals, called "sunstones." We demonstrate here that a simple tool, that we call "shadow-stick," could have allowed orientation by a sun-compass with satisfying accuracy when shadows were not formed, but the sun position could have reliably been estimated. In field tests, we performed orientation trials with a set composed of a sun-compass, two calcite sunstones, and a shadow-stick. We show here that such a set could have been an effective orientation tool for Vikings only when clear, blue patches of the sky were visible.

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1. Introduction

From the 8th century Vikings dominated the North Atlantic Ocean and even founded settlements in Greenland and Newfoundland without knowing about the magnetic compass. In 1948 in Uunartoq (Greenland) a fragment of a wooden disk—with a diameter of 70 mm carved in with a compass rose, the mark of North, and two gnomonic lines—was found [1]. Based on this artifact, it has been hypothesized that Vikings used primitive solar positioning instruments, or primitive sun-compasses to derive true directions from the cast shadow of a gnomon at least from the 10th century [2–5].

A sun-compass is an inverted sun-dial: instead of following the movement of the shadow tip along a hyperbolic gnomonic line on a fixed dial, the compass-dial is rotated until fitting a previously drawn hyperbolic gnomonic line right under the shadow tip. In this position, the major axis of the hyperbola points toward the true North. Although gnomonic lines are valid only on given days of the year at given latitudes, deviations originating from using an

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inappropriate curve in the morning and in the afternoon compensate each other [4]. Sun-compasses with dimensions identical with that of the Uunartoq artifact were tested by modern sailors on short-haul trips (maximum about 50 nautical miles) and were found to be remarkably reliable in clear weather [4].

Sun-compasses are sensitive to the changing lighting conditions. Thin cloud layers, dust, smoke, and atmospheric haze scatter or absorb direct sunlight, shadows are poorly contrasted when the intensity of direct sunlight is comparable to or lower than that of scattered skylight. Under such circumstances a Viking navigator could easily underestimate the shadow length and derive compass directions with significant errors. When the sun is occluded, cast shadows do not appear at all, thus sun-compasses cannot be used. Vikings are hypothesized to overcome this problem by crude sky-polarimetry using mysterious sunstones that are mentioned in the saga of King Olaf the Holy [6].

Navigation by means of the polarization pattern of the sky is a natural ability of several animal species (e.g., insects and birds), but it was also applied on board trans-arctic flights of the 1960s [6–8]. Skylight origins mainly from the Rayleigh scattering of sunlight on atmospheric particles and is predominantly partially linearly polarized. Depending on the meteorological conditions and the solar elevation angle, its direction of polarization is more or less perpendicular to the plane of scattering determined by the sun, the observed celestial point, and the observer [9], and forms a celestial polarization pattern, whose axis of mirror symmetry is the solar–antisolar meridian [7,10]. This pattern exists even under thick clouds, although the degree of polarization of skylight is very low in such situations [10–13]. By measuring the direction of polarization of skylight in two or more celestial points, one can mark out celestial great circles (perpendicular to the local direction of polarization), the intersection point of which provides a good estimation of the position of the occluded sun [8].

Sunstones are hypothesized to be dichroic crystals (e.g., tourmaline or cordierite) or birefringent crystals (e.g., calcite), that can be used to identify the direction of polarization of skylight [6,14–16]. Theoretically, Vikings could use such a primitive skylight polarimetry to locate the occluded sun and use this information for navigation [6,8,15,16]. In modern astrophysics, comparison of the irradiances of ordinary and extraordinary beams in birefringent calcite is used to detect extremely weak polarized light produced in the atmospheres of exoplanets [17]. Theoretically, the contrast sensitivity of the human eye should allow human observers to use calcite sunstones to identify the direction of polarization of light with an accuracy of 1°, even if its degree of polarization is very low [15]. Thus, sky-polariemtric navigation under an overcast sky is theoretically possible. Roslund and Beckman [18] criticized this theory, suggesting that the sun can be located in most situations without skylight polarimetry, but the psychophysical survey by Barta *et al.* [19] proved that such estimations are unreliable. The actual threshold minimum degree of polarization allowing reliable non-instrumental measurement of the direction of skylight polarization in the field is still not known. The atmospheric prerequisites and the optical elements of such a sky-polarimetric navigational method have been studied earlier [11–13,15], but complete procedures of using sunstones with sun-compasses have not been tested in practice.

In this work, we suggest that a simple tool, called "shadow-stick" could substitute the gnomon shadow at given solar elevation angles and could extend the usability of sun-compasses (Fig. 1). This shadow-stick should be small and elongated in general, but can take many forms, like a carved fang or metal pendant, easy to be carried on the body of the navigator. It must be provided with a series of sockets corresponding to discreet solar elevation angles. Positions of the sockets could be appointed either empirically or by calculation. Theoretically, a set composed of a sun-compass, a shadow-stick, and two sunstones [Figs. 1C, 1D, and 2] could form an ideally compact all-weather solar navigation toolkit with the potential of replacing a magnetic compass.

To reveal the accuracy and reliability of such a navigation toolkit under clear, partially cloudy, or totally overcast skies, we performed an extensive series of field tests. We demonstrate here that the solar elevation angle can be assessed with satisfying accuracy even without dedicated instruments, and the shadow-stick functions perfectly in situations when the sun position can be estimated by the naked eye. However, locating the sun in overcast skies with the aid of sunstones was found to be too inaccurate for navigation purposes.

2. Materials and Methods

A series of orientation trials was performed with a navigational toolkit consisting of a pendulum-levelled sun-compass, two calcite sunstones, and a shadow-stick (Figs. 1 and 2) in the field in Northern Hungary (47° 28' N, 19° 3' E). The toolkit was operated by three male members of the Environmental Optics Laboratory (Eötvös University, Budapest, Hungary), aged between 26 and 37 years. All of them were highly experienced in measuring and analyzing skylight polarization patterns, thus they were eligible to impersonate experienced Viking navigators.

The accuracy of orienting the sun-compass was measured in the field under weather conditions of four categories, classified on the basis of available information on the sun position. *Category 1*: Under illumination dominated by direct sunlight the cast shadow of the gnomon is continuously clearly visible. Then, navigation with the sun-compass was possible by fitting the tip of the gnomon shadow to the gnomonic line. *Category 2*: The gnomon shadow is not formed, but one can see either the faint sun disk behind the clouds, or the sunbursts formed by the atmospheric Tyndall effect unambiguously mark

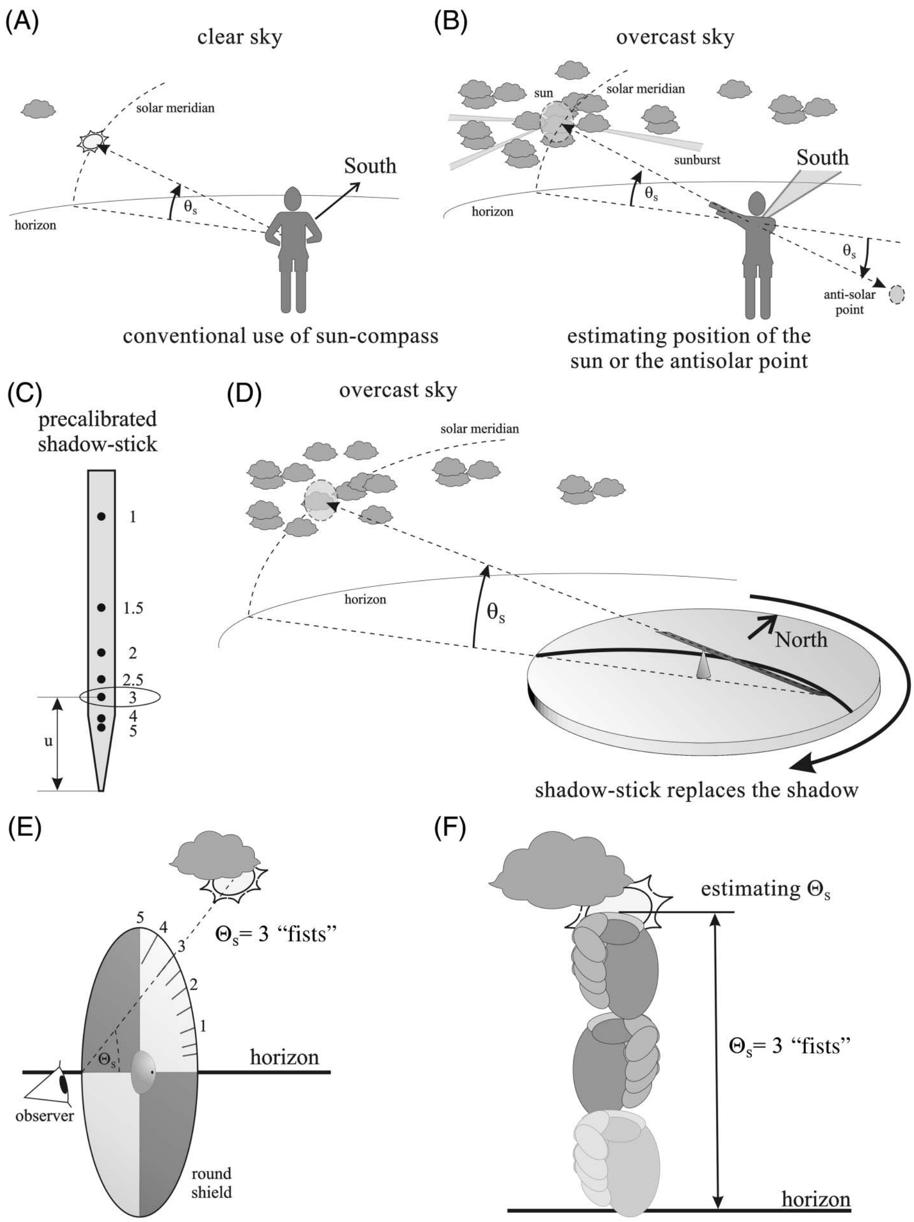

Fig. 1. Deriving true compass directions with a Viking sun-compass, a shadow-stick and sunstones. A Under clear skies the gnomon casts a clear sharp shadow on the horizontal dial of a levelled sun-compass. Navigation is possible without auxiliary tools. B A cast shadow cannot be seen when the sun is occluded. The Viking navigator must estimate the elevation and azimuth angles of the sun. These data can be gained also by estimating the position of the antisolar point. C A shadow-stick is a small item provided with a series of sockets representing various solar elevation angles. Since the sockets must not overlap, the smallest resolution of elevation angles is determined by the dimensions of the shadow-stick and the diameter of the sockets. D To derive true compass directions with a shadow-stick and a sun-compass, the socket on the shadow-stick corresponding to a given solar elevation is applied on the gnomon tip, and then the end of the stick is turned to point toward the solar meridian. The shadow-stick now replaces the missing or poorly visible cast shadow. To find true compass directions, the navigator must rotate the sun-compass while keeping the shadow-stick still until the gnomonic line fits to the tip of the shadow-stick. E A marked replica Viking round-shield with a diameter of 80 cm was used as a crude sextant with satisfying precision to provide a secondary estimation of the solar elevation as suggested by Captain Jensen [4]. F The estimation of elevation angles of celestial bodies or celestial points with fists and extended arms is a practice frequently used by amateur astronomers. The observer counts the numbers of fists and fingers needed to subtend the arc in question.

the sun position. Then, the sun position was estimated by the naked eye, and a shadow-stick was used to replace the cast gnomon shadow. To measure the accuracy of orienting the sun-compass with the aid of sunstones, the position of the antisolar point was estimated to ensure that the observers are not influenced by the sight of the sun disk. *Category 3*: The exact position of the sun disk cannot be estimated by the naked eye, but the intensity pattern of the sky unambiguously marks the solar hemisphere. Then, the sun position or the antisolar point was estimated with two calcite sunstones, and a shadow-stick was used to replace the cast shadow. *Category 4*: When the sky is totally overcast, neither the exact sun position, nor the solar hemisphere can be identified by the naked eye. Then, the sun position or the antisolar point was estimated with two calcite sunstones, and a shadow-stick was used to replace the cast shadow.

Measurements in weather situations 1 and 2 were carried out at a single location on an undisturbed platform of a steel-frame building of the Eötvös University in Budapest (47° 28.43' N, 19° 3.69' E). The local magnetic compass deviation of +6.8° was calculated using the method of Indian circles and was taken into consideration during the evaluation. All measurements in weather categories 3 and 4 were carried out at different locations in four suburban areas of Budapest (47° 24.5' N, 19° 5.5' E; 47° 25.5' N, 19° 0.6' E; 47° 35' N, 19° 6.1' E; 47° 23.8' N, 18° 53.5' E) characterized by irregular street networks. The +2.8° average magnetic compass deviation in Northern Hungary [20] was considered at these localities. Between measurements, the observers were blindfolded and transported by a car that followed a zig–zag path. After all observers signaled that they had lost their sense of direction, the driver chose a new locality, from which at least about 80% of the sky was visible. All observers made an independent educated guess on the direction of North; the adequate directional angles were recorded by the driver. Then, they independently estimated the sun position using two calcite sunstones and oriented the sun-compass using the shadow-stick. The real solar elevation angle was obtained from the data service of the USNO Naval Oceanography Portal [21].

Two polished rhombohedra of Icelandic spar (that is a transparent variety of calcite) sized 5 cm × 5 cm × 2.5 cm was used as sunstones (Figs. 2–4). These crystals were purchased in set from a specialized mineral bourse trader (Kristálycentrum Kft., Budapest, Hungary; www.kristalycentrum.hu), thus their exact geographical origin was untraceable. A calcite crystal with similar size was found between navigational instruments in a 16th century shipwreck at Alderney [16]. All faces of our rhombohedra were covered by a black adhesive carton paper only leaving clear a 3 mm wide incoming slit and a 6 mm wide exit slit, both being perpendicular to the crystallographic c-axis of the calcite (Figs. 2 and 4). The greatest faces of the rhombohedra were used as incoming and exit faces. Partially linearly polarized light entering the birefringent calcite through the incoming slit is separated into totally linearly polarized ordinary and extraordinary rays with a walk-off distance of 3 mm, and these rays form two parallel images in the entrance slit on the exit face (Figs. 2 and 4). The irradiances of the two slit images are proportional to the square of sine or cosine of the angle enclosed by the slit axis and the direction of polarization of light entering the crystal. The slit images are equally bright when the axis of the entrance slit encloses 45° with the direction of polarization of incoming light. This direction was marked on the exit face [Fig. 4B]. Maximal and minimal irradiances of

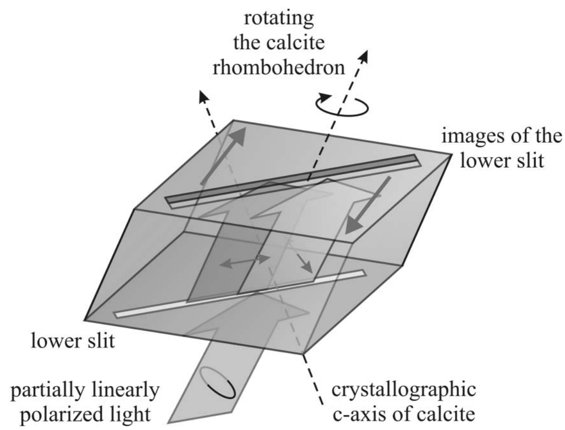

Fig. 2. Calcite rhombohedron used to measure the direction of polarization of transmitted skylight. All faces of the crystal are covered by a black adhesive carton paper, and only two narrow slits perpendicular to the crystallographic c-axis of the calcite remain open. Partially linearly polarized skylight entering the lower slit is separated into totally linearly polarized ordinary and extraordinary rays and form two images of the lower slit in the upper slit in the exit face.

slit images are transposed when the rhombohedron is rotated by 90° (Fig. 4). Observers measured the direction of polarization of skylight by rotating the crystals—further on "sunstones"—until reaching equally bright slit images.

In weather situations 2, 3, and 4, the position of either the sun or the antisolar point was estimated by performing skylight polarimetry using the two sunstones (Figs. 4). The sun position can be estimated by appointing the celestial great circles perpendicular to the local direction of polarization of skylight in sky patches characterized with high degree of polarization. The intersection of such two great circles or the error triangle defined by three great circles appoints the sun position. The accuracy of such an estimation is influenced chiefly by the angular distance between the sun and the clear sky patches, the intersecting angle κ of the appointed great circles and the accuracy of determining the direction of polarization.

Totally overcast skies are characterized by low and fluctuating degrees of polarization, and the observers must systematically look up points with relatively high degrees of polarization (Fig. 3). According to the single-scattering Rayleigh model, the degree of polarization of skylight is the highest in a zone d, at 90° from the sun. To locate zone d, a horizontal zone b of the sky about 20°–30° above the horizon is scanned with a sunstone held with its slits enclosing about 45° with the local meridian a until finding the greatest contrast of the two slit images [Fig. 3A]. This method provides a fair contrast of the slit images in most situations when the sun is not close to the horizon or the zenith. Held toward the point of b characterized by the highest contrast of slit images, the sunstone is rotated to reach equal intensities of the slit images to read the direction of polarization of skylight [Fig. 3B]. Then, the sun is located along

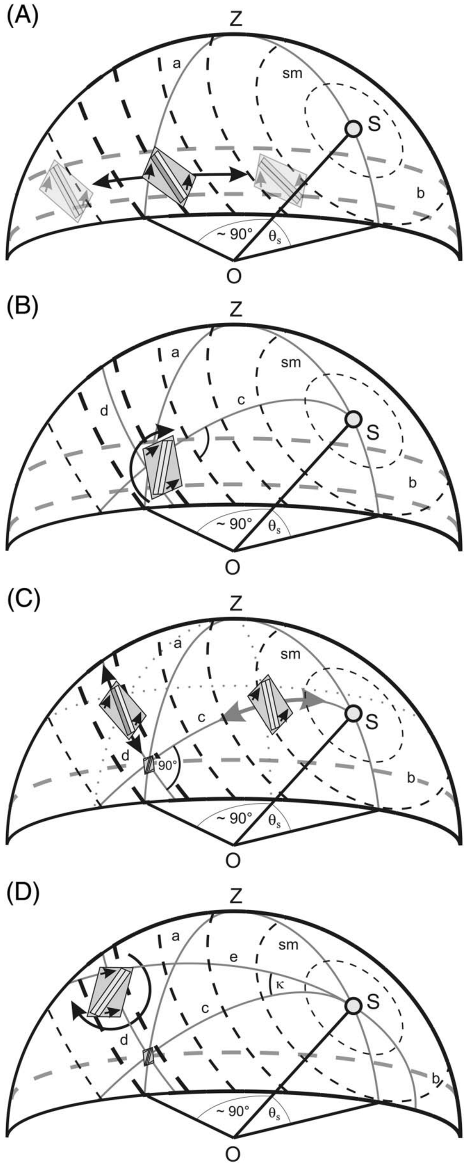

Fig. 3. Estimating the sun position with two birefringent calcite sunstones under totally overcast skies. The direction of polarization of skylight is symbolized by a dashed line, the degree of linear polarization of skylight is symbolized by the thickness of the dashed line. A Zone d of high degrees of polarization of skylight is located by scanning along zone b about 20°–30° above the horizon with a sunstone held with its slits enclosing about 45° with the local meridian a. This method provides a fair contrast of the slit images in the sunstone when the sun (S) is not close to the horizon or the zenith (Z). B The sunstone is rotated to reach equal intensities of the slit images to read the direction of polarization of skylight. The sun is located along the celestial great circle c that lies in the plane of scattering and is perpendicular to d. C The zone of highly polarized skylight d is verified. The sunstone is moved along d and then along c with its slit parallel to d. Along d the contrast of slit images is expected to remain perceivable. Along c the contrast of slit images is expected to quickly decrease. D A second patch appropriate for sky-polarimetry is chosen along d. The second sunstone is used to identify the direction of polarization of skylight here to mark out the great circle e. The intersections of c and e mark the positions of the sun and the antisolar point. Both celestial positions can be used to estimate the solar elevation angle Θ_S and the direction of the solar meridian sm. The estimation is more reliable, if the intersecting angle κ of c and e is close to 90°.

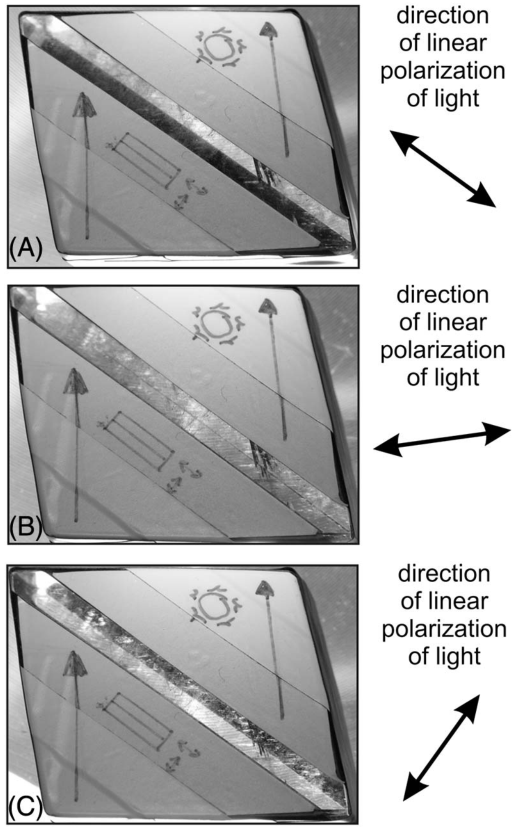

Fig. 4. Photographs of a calcite crystal prepared as a sunstone and transilluminated by totally linearly polarized light. A If the direction of polarization of the transilluminating light is parallel to the axis of the slits, only one of the slit images can be seen. B When the direction of polarization is rotated by 45°, the slit images will have equal light intensities. When analyzing the polarization properties of skylight, this orientation is appropriate to appoint the direction of the sun. C When the direction of polarization is rotated by further 45°, the other slit image will reach maximal intensity, while the first one darkens.

the celestial great circle c perpendicular to zone d, lying in the plane of scattering. The zone d of highly polarized skylight should be verified [Fig. 3C]. The sunstone is turned until its slit is parallel to the assumed zone d, then it is moved along d and also along c. Along d the contrast of slit images is expected to fluctuate due to the changing degree of skylight polarization, but to remain perceivable. Along c the contrast of slit images is expected to decrease while the sunstone deviates from d. After verifying zone d, a second patch appropriate for sky-polarimetry is chosen along d [Fig. 3D]. The second sunstone is used to identify the direction of polarization in this point to mark out the great circle e. The observer sweeps c and e along with the sunstones until pointing toward their intersections, which is the estimated position of the sun or the antisolar point. Either point can be used to estimate the solar elevation angle Θ_S and the position of the solar meridian [Fig. 1B]. The estimation is more reliable if the intersecting angle κ of c and e is close to 90°.

In weather situations 2, 3, and 4 the elevation angle of the sun or the antisolar point was estimated using two alternative methods: (i) The observers performed an unaided estimation by counting with their arm extended the numbers of quarters of a fist (digits) needed to subtend the arc between the sun disk and the horizon [Fig. 1F]. (ii) In weather situation 2 a more accurate second estimation was made using a replica Viking round shield with a diameter of 80 cm as a primitive sextant [Fig. 1E], as it was suggested by Captain Jensen [4]. Markings on the perimeter of the shield matched with angular ranges subtended by one or more fists of the observer and allowed recording solar elevation angle in "fists." In weather situations 3 and 4 the round shield was not used, because it would have required "dropping" the presumed position of the sun.

In the lack of cast shadow of the gnomon we used a previously calibrated shadow-stick wearing sockets corresponding to solar elevation angles measurable with arcs subtended by multiples of quarters of an extended fist of the observer (Table 1). The shadow-stick was produced by engraving sockets on a sharpened metallic plate (5 mm × 70 mm) at distances u from the tip of the plate [Fig. 1C]. Averages of subtended arcs were measured in the laboratory. The observers were set the task of reading the lengths of a series of sections subtended by their extended fists on a vertical gauge fixed in 2 m distance with its zero point set to the eye-level of the observer. Corresponding elevation angles were calculated from the averaged values of ten readings of all observers (Table 1). Elevation angles with a difference greater than 0.5° were considered to be reliably distinguishable by the estimation based on the arc that the sun disk subtends in the sky (0.52°). Distances u between the gnomon tip and the tip of the gnomon shadow were calculated from the gnomon height and the given solar elevation angles (Table 1).

The orientation procedure was repeated at least 30 times under each conditions of all four weather categories using a pendulum-leveled dial-plate (12.5 cm × 24.5 cm) supported by ball-and-socket joint of a portable tripod. The dial-plate supported a water-proof sun-compass dial card with dimensions identical with those of the artifact dial found at Uunartoq (Greenland) in 1948 and a central gnomon.

Table 1. Discrete Solar Elevation Angle Θ_S Measurable with Fist and Fingers, and Corresponding Lengths of the Shadow and Shadow-Stick for a 9.8 mm High Gnomon

Counted Fists	Elevation Angle Θ_S [°]	Shadow Length [mm]	Stick Length u [mm]
0.75	5.7	98.2	98.7
1.0	9.0	61.9	62.6
1.5	13.5	40.8	42.0
2.0	18.0	30.2	31.7
2.5	22.5	23.7	25.6
3.0	27.0	19.2	21.6
4.0	36.0	13.5	16.7
5.0	45.0	9.8	13.9

A magnetic compass (Freiberger Präzisionsmechanik, Sport 4) with an opaque cover fixed on the dial-plate served to measure the orientation error. A broad conical central gnomon with a height of 9.8 mm and a diameter of 17 mm was applied. Unique dial cards wearing gnomonic lines calculated for the actual date and latitude and well-visible straight lines pointing to true North and the magnetic North were printed for all measurements.

The sun-compass was always oriented with covered magnetic compass as follows: (i) Sun position was estimated either by the naked eye, or using two sunstones. (ii) The solar elevation angle was estimated either with bare hands, or with the round shield [Figs. 1E and 1F]. (iii) The sun-compass was rotated until the gnomonic line fitted to the shadow tip, or to the tip of the shadow-stick, which pointed toward the presumed antisolar meridian [Figs. 1C and 1D]. After orientation, an assistant removed the cover of the magnetic compass without turning the dial-plate and took a photograph with a digital camera (Nikon CoolPix 8700) looking at the dial-plate in normal angle from 60 cm above it. Observers were not allowed to see the magnetic compass. The directional angles considered to be true North were calculated by computer from the deviation of the line considered to point to the magnetic North from the magnetic needle measured to 0.1 degree.

The distributions of directional angles taken as true North under the four weather conditions were analyzed separately using circular statistics [22]. The direction a and the length r of the mean vector of the directional angles considered as North were calculated. The uniformity of the directional angles considered as North was tested by Rayleigh test on the mean vector separately in all four weather categories. The uniformity distribution of the guessed North in weather situations 3 and 4 was tested likewise. The 95% confidence limits of the mean angles were calculated. Correlations between directions of North guessed or derived at a given locality by a given observer using sunstones, a shadow-stick and a sun-compass in weather situations 3 and 4 were tested using a circular rank correlation test.

3. Results

The results of our orientation trials are summarized in Figs. 5 and 6. The solar elevation angle was estimated with small errors with a clear tendency of overestimation (Fig. 6). Since estimating the solar elevation by bare hands or with the round shield always provided identical values, the data gained with the round shield are not presented here. In weather situation 1, the orientation of the sun-compass was highly precise despite of the small size of the compass. The mean vector of the directions considered to be North deviated less than 1° from true North [$a = 359.1° \pm 5.9°$, $r = 0.997$, $p < 0.001$, $N = 90$, Fig. 5A]. The deviation of the mean vector from true North origins from the different numbers

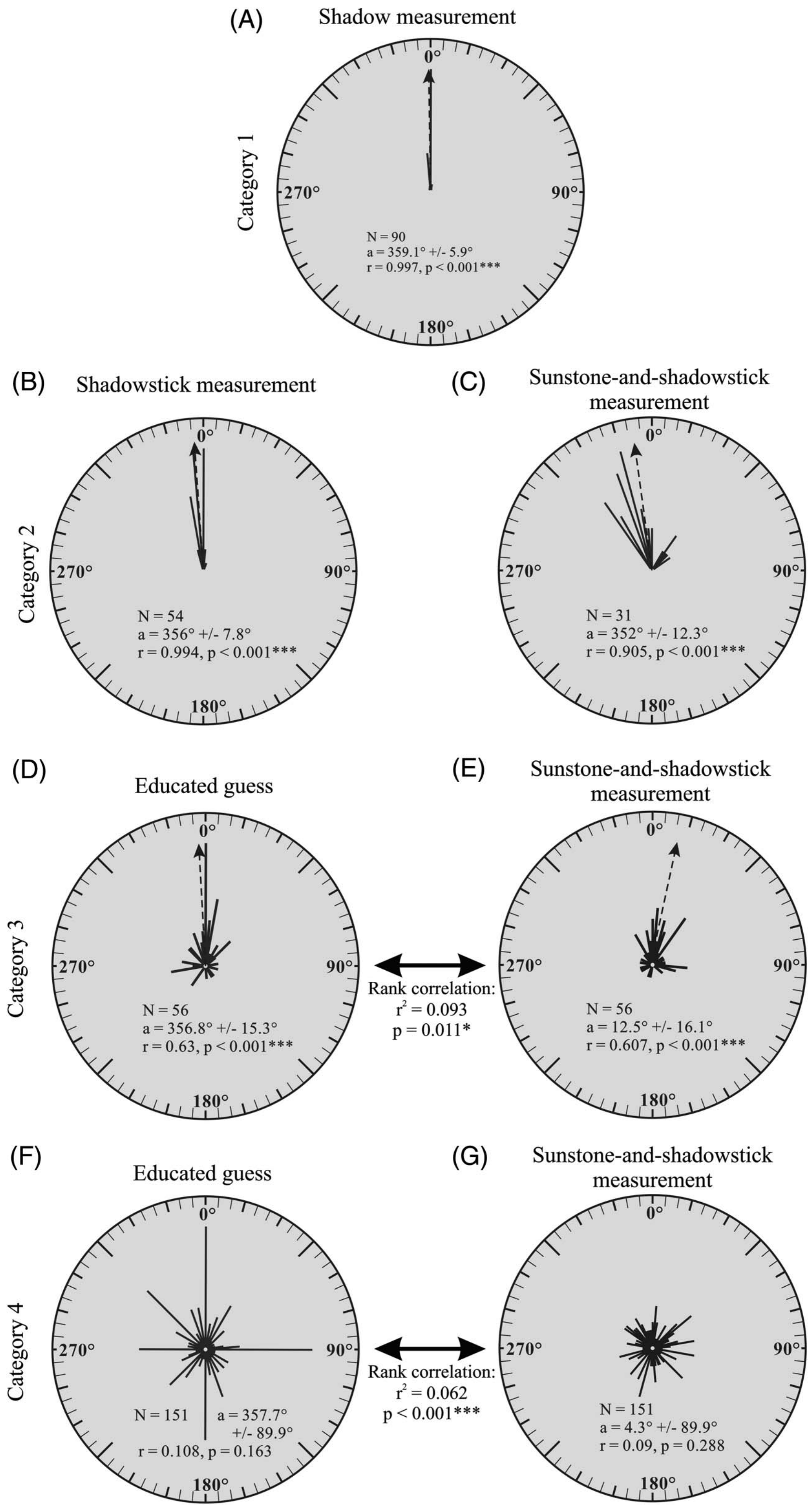

Fig. 5. Histograms of directional angles considered to be true North (0°) under weather situations A 1; B, C 2; D, E 3; F, G 4. Dashed arrows mark the directions of the mean vectors. The number of individual orientations N, the direction of the mean vector a with 95% confidence limits, the length of the mean vector r and the significance level p of the Rayleigh test for uniformity of distribution of the measured directions are given in the lower half of the dials. A–E The distributions of direction considered as North are significantly directional under weather situations 1, 2, and 3; F and G while they are uniform under weather situation 4. E and G the test values r^2 and significance levels p of circular rank correlation tests mark significant correlations between directional angles identified as true North by the observer using the sunstone and shadow-stick, and D and F between their unaided guesses.

of orientations performed in forenoons and afternoons.

In weather situation 2, the accuracy of orientation with the shadow-stick was slightly inferior to that of orienting the sun-compass with cast shadow [$a = 356° \pm 7.8°$, $r = 0.994$, $p < 0.001$, $N = 54$, Fig. 5B]. Estimating the sun position with sunstones under weather condition 2, the dispersion of directional angles considered as North increased significantly [$a = 352° \pm 12.3°$, $r = 0.905$, $p < 0.001$, $N = 31$, Fig. 5C]. In weather situation 3, the sense of direction of the observers was distracted, but they still could perform significantly directional guessing on the direction of North [$a = 356.8° \pm 15.3°$, $r = 0.63$, $p < 0.001$, $N = 56$, Fig. 5D]. Again, using the sunstones to estimate the sun position, the dispersion of the directional angles considered as North increased [$a = 12.5° \pm 16.1°$, $r = 0.607$, $p < 0.001$, $N = 56$, Fig. 5E]. The significant correlation between the guessed North and the North derived with sunstones ($r^2 = 0.093$, $p = 0.011$) marks that the sense of direction of the observers influenced the skylight polarimetric measurements. In weather situation 4, the sense of direction of the observers was lost. The distribution of guessed North was quadrimodal ($N = 151$, $r = 0.201$, $p = 0.001$), rather than unimodal [$a = 357.7° \pm 89.9°$, $r = 0.108$, $p = 0.163$, $N = 151$, Fig. 5F]. Using sunstones to estimate the sun position under such weather conditions resulted in directions correlated with the North guessed by the observers ($r^2 = 0.062$, $p < 0.001$), but were uniformly distributed [$a = 4.3° \pm 89.9°$, $r = 0.288$, $p = 0.09$, $N = 151$, Figs. 5F and 5G].

4. Discussion

Unlike modern mariners bound to preset routes and guiding vessels by autonomous steering devices, medieval sailors leaned to wind directions that frequently are constant for hours or even days in high seas. Since instantaneous information on the true directions was generally not required, the accuracy of navigation with a sun-compass depended on the frequency of situations favorable for reorientation. The instantaneous accuracy of our proposed all-weather solar navigation kit is determined by the net error of steps of the orientation procedure in a given situation. Wind, extreme temperature, precipitation, and the actual intensity and polarization pattern of the sky worsen the estimation by a navigator, thus accuracy in the field can be significantly worse than that achieved in the laboratory. Since the frequencies of favorable and unfavorable situations change from region to region and season to season, the reliability of sun-compasses in general could not be assessed without detailed information on the climate of the area of use. Two elements of the presumed toolkit were found to be reliable: (i) We confirmed that small sun-compasses, like the artifact dial found in Uunartoq (Greenland), could be oriented very precisely under ideal (clear, cloudless) weather conditions. (ii) Shadow-sticks can be used to replace the missing cast shadow allowing orientation with an error of less than 8°.

The shadow-stick and simple procedures for estimating solar elevation angles could have been developed by past navigators without known artifact precursors. Personal articles, e.g., staffs, weapons, or shields could have been provided with appropriate markings to serve as crude sextants aiding the estimation of solar elevations [Fig. 1E]. The simple method of assessing elevation angles of celestial bodies with the extended fist of the observer is a general practice of amateur astronomers [Fig. 1F]. Such an assessment may seem to be too crude, but can be quickly performed in most situations, and provides replicable values for a given person. Since general anatomical proportions of grown men are similar, the method provides comparable values also for different, independent observers.

Calcite sunstones, forming the third element of our hypothesized toolkit, failed to increase the accuracy of orientation, although theoretical calculations, polarimetric surveys, and laboratory measurements predicted the contrary [12,13,15]. In weather situation 2 (when the sun position can be estimated either by the naked eye or by analyzing skylight polarization patterns) the use of sunstones significantly increased the orientation error [Fig. 5B and 5C]. An important factor deteriorating the efficacy of skylight polarimetry with sunstones are the inhomogeneous intensity and polarization patterns of overcast skies and the nature of the human eye: (i) When the observers rotate the sunstones or scan the sky with them, the irradiances of the observed slit images of the sunstones change either because of the changing degree and direction of skylight polarization, or due to the different skylight intensities in the observed celestial points. Small differences in the contrast of the slit images are hardly perceivable under such circumstances. (ii) The observers must perceive the contrast of the slit images in front of the brighter background of the sky. Since the human retinal photoreceptors are nonlinear intensity detectors, distinguishable differences in light intensity depends on the eye adaptation. Unlike under controlled laboratory lighting conditions, in the field the eyes of the observers continuously adapt to the intensity of the background skylight, and not to the intensity of the slit images seen in the sunstones. Consequently, the contrast levels distinguished by the observers are not constant. The observers rotating the sunstones to reach equal irradiances of the slit images can mark out only an eligible angular range, but not a specific direction, which causes a great error in estimating the direction of skylight polarization.

The observers are greatly influenced also by their sense of direction when the contrast of the slit images is hardly perceivable. The derived directions considered as North are characterized by a greater dispersion than the guessed directions. Skylight polarimetry with sunstones under overcast skies should not be treated as a refinement of guessed directions, but rather as a representation of the sense of directions. Surprisingly, the observers managed to estimate solar elevation angles with a satisfying accuracy under all weather conditions, even when the sun was not visible at all (Fig. 6). This suggests that the observers were experienced enough to estimate solar elevation in the given part of the day in the given season, and this knowledge undeniably influenced how they judged the actual contrast of the slit images in the sunstone. Note, that the guessed North directions in weather situation 4 show a significant quadrimodal manner (Fig. 5). Since the observers were disoriented in this situation and the intensity pattern of the sky did not offer any navigational clue, the most probable reason for quadrimodality of distribution was the street network. Although the observers were instructed to disregard terrestrial cues, they unintentionally aligned themselves to the directions of streets.

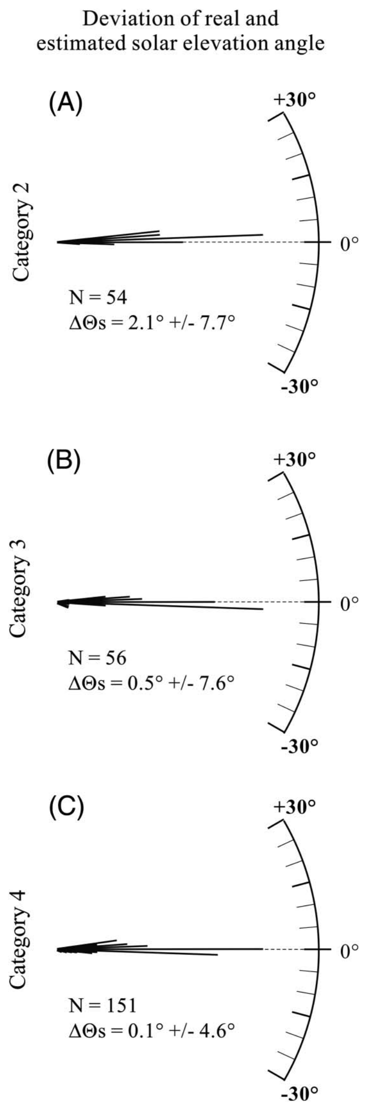

Fig. 6. Histograms of the deviation of solar elevation angles estimated in "fists" from real solar elevations under weather situations A 2, B 3, and C 4. A dashed line marks the deviation of 0°, and positive deviations mark overestimations of the solar elevation angle. The number of individual estimations N and the mean deviation $\Delta\Theta_S$ with 95% confidence limits are given in the lower half of the dials.

More sophisticated or bigger instruments may increase the accuracy of orientation, but the lack of artifacts and described procedures limits the hypothesized inventory of Vikings. A shadow-stick to be used with the Uunartoq dial should be only slightly bigger than 40 mm to support a socket for 1-fist solar elevation, which is about 9°. Such a small object is just perfect for wearing it on the body of the navigator. The distance between the sockets for solar elevations of 3 and 4 fists should be 3.2 mm. Sockets for 4 and 5 fists should be located less than 2 mm from each other; such small space is hardly enough for two separate cavities. With such a small tool the estimation of solar elevation angles with extended fists is just precise enough. The diameter of the Uunartoq dial is only 70 mm. The position of the supposed equinox line marked on this artifact indicates that it operated with a 4.3 mm high gnomon at latitude of 61° N, the supposed latitude where it was used. The shadow tip of such a gnomon falls on the dial only at solar elevation angles greater than 7°. This angle is commensurate with that measured by the arc one sees subtended by the fist with the arms extended.

The Uunartoq dial is small enough to fit into a pocket or small case, but is large enough to be used with a shadow-stick at solar elevations covered by 1–4 fists. Lower solar elevation angles could be reliably estimated, but these angles would require a much larger compass dial. As far as we know, shadow-sticks have not been identified by archaeologists until now. Such small items would badly conserve, and their true identity would be hard to recognize without a concept. But the size of the fists of people are not uniform, and most probably also the compass dials were unique, thus navigators should have used their personal shadow-sticks. Artifact shadow-sticks may exist without recognizing their function. Candidate objects should be about 40–50 mm long, straight, and acute, e.g., a wooden stick, a carved fang, or a metal pendant, and should wear a series of sockets. Distances between the sockets and the tip should follow the ratios given by the function

$$f(\alpha, n) = 1/\sin(n \cdot \alpha/4), \qquad (1)$$

where n is a natural number.

A set composed of a shadow-stick and a Viking sun-compass seems to be a powerful toolkit, although its marine practicability should be tested on board long-haul sail-boats. On the other hand, we showed that calcite sunstones could not be used to locating the occluded sun in the overcast sky, this task requires more sophisticated instruments. If sunstones were used by medieval navigators, they could rather use them in periods when the sun was below the horizon, but clear patches of the sky could be seen [15,16,23].

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