# Cross-buoyancy mixed convection from a heated cylinder placed asymmetrically in a channel 

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#### Abstract

Mixed convection heat transfer from a heated circular cylinder asymmetrically placed in a horizontal channel was studied for incompressible Newtonian fluid. A systematic investigation using two-dimensional numerical simulation (Ansys Fluent) was performed for the following control parameters: Reynolds number $R e=50,100$, 150, Richardson number $\mathrm{Ri}=0,1,2$, blockage ratio $\beta=0.2,0.3,0.4$, gap ratio $\gamma=0.25,0.5,1$ and Prandtl number of air $P r=0.7$. Upon changing these parameters, steady and time periodic regimes were identified. The strongest influence on heat transfer (average Nusselt number) was identified when changing the Reynolds number (a $94 \%$ increase between the minimum and maximum $R e$ investigated, with other parameters held constant), followed by Richardson number ( $+18 \%$ ) and blockage ratio ( $+16 \%$ ); the effect of asymmetric placement $(\gamma)$ was almost negligible $(+1 \%)$. The time-mean of the drag coefficient was most influenced by blockage ratio (a $150 \%$ increase), followed by Richardson number ( $+91 \%$ ), gap ratio ( $+19 \%$ ), and $\operatorname{Re}(+10 \%$ ).


| Nomenclature |  |
| :---: | :---: |
| $C_{\text {D }}$ | $\text { total drag coefficient, }=\frac{F_{D}}{(1 / 2) \rho U_{\infty}^{2} d}$ |
| $c_{p}$ | heat capacity, J/(kg K) |
| $d$ | diameter of cylinder, length scale, m |
| $F_{D}$ | drag force per unit length of the cylinder, $\mathrm{N} / \mathrm{m}$ |
| $g$ | acceleration due to gravity, $\mathrm{m} / \mathrm{s}^{2}$ |
| Gr | $\text { Grash of number, }=\frac{g \beta_{v}\left(T_{w}-T_{0}\right) d^{3} \rho^{2}}{\mu^{2}}$ |
| H | half channel height, m |
| $h$ | local heat transfer coefficient, $\mathrm{W} /\left(\mathrm{m}^{2} \mathrm{~K}\right)$ |
| $\bar{h}$ | average heat transfer coefficient, $\mathrm{W} /\left(\mathrm{m}^{2} \mathrm{~K}\right)$ |
| $k$ | thermal conductivity of fluid, $\mathrm{W} /(\mathrm{mK})$ |
| $L$ | dimensionless length of the computational domain in $x$-direction, $=L^{*} / d$ |
| $L_{d}$ | dimensionless downstream length, $=L_{d}{ }^{*} / d$ |
| $L_{u}$ | dimensionless upstream length, $=L_{u}{ }^{*} / \mathrm{d}$ |
| $\mathrm{Nu}_{\mathrm{L}}$ | local Nusselt number, $=h d / k$ |
| Nu | average Nusselt number, $=\bar{h} d / k$ |
| P | dimensionless pressure, $=P^{*} /\left(\rho U_{\infty}{ }^{2}\right)$ |

[^0]```
* dimensional quantity
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## 1. Introduction

The study of fluid dynamics and heat transfer characteristics around various bluff bodies in an enclosure has been given a great deal of attention by researchers in recent times because of its pragmatic relevance. While many studies are available on forced convection flow alone, in real life there is always a combination of free and forced convection. In most situations, free convection is always present, though sometimes small, and this leads to heat transfer through mixed convection. This mixed convection has numerous real world applications such as heat exchangers, transport of fluid in pipelines, cooling towers, etc. Although it is more difficult to analyse and study mixed convection as compared to forced convection alone, both forced convection and mixed convection from an unconfined circular cylinder have been fairly widely studied. In contrast, there are few studies available on mixed convection heat transfer from a cylinder in a confined channel, and to the best knowledge of the authors no results have been published on mixed convection from a confined heated cylinder placed asymmetrically. In this study, cross-buoyancy mixed convection is investigated for incompressible Newtonian fluid from a heated circular cylinder placed in a channel in symmetrical and asymmetrical positions.

## 2. Current status of literature

Following is a summary of previous studies related to the flow of fluids past and heat transfer from a circular cylinder placed at various positions in a confined two-dimensional (2-D) channel. It is presented in the following manner: studies on flow around and forces acting on a cylinder in a confined channel, forced convection studies for a symmetrically and then an asymmetrically placed cylinder, and mixed convection studies for a symmetrically placed cylinder.

Experimental and numerical research is now accessible in the literature on the fluid flow in the symmetrical placement of a circular cylinder between the walls of a channel (confined domain) [1-8]. Likewise, adequate research on three-dimensional effects in laminar flow around a circular cylinder in the channel can be found in [9-13]. On the other hand, Zovatto and Pedrizzetti [14] studied the effects of wall confinement (blockage ratio $\beta=b /(2 H)$ ) and the flow separation from the cylinder surface. It was reported that the transition from steady state flow to the vortex shedding regime was delayed for higher blockage ratios, where the walls are placed nearer to the cylinder, because of the stability provided by the confining walls to the wake that formed behind the cylinder. Thus, blockage ratio above a certain value led to inhibition of the vortex shedding phenomenon.

In [15-18] a heated circular cylinder is symmetrically placed (gap ratio $\gamma=1$ ) in a channel and forced convection (Richardson number $\mathrm{Ri}=0$ ) is assumed. Progress in research on the forced convection can also be found for the asymmetrical placement of a circular cylinder in the confined domain [19-23]. Mettu et al. [19] explored flow around and forced convection from a circular cylinder placed asymmetrically in a planar channel for Reynolds number Re ranging from 10 to 500, blockage ratio $\beta$ from 0.1 to 0.4 , gap ratio $\gamma$ from 0.125 to 1 and Prandtl number $\operatorname{Pr}=0.744$. A critical Reynolds number that represents the shift from steady flow regime to unsteady periodic flow regime was determined. Drag coefficient was found to increase with a decrease in the gap ratio. The oscillation amplitude of the lift signal was found to have an inverse relationship with blockage ratio.

Champmartin and Ambari [20] studied the effect of disturbances which break the symmetry of the system on heat transfer. An asymmet-
rical configuration of a single cylinder moving in a horizontal confined channel with Dirichlet and Neumann boundary conditions was investigated numerically. The heat transfer was observed to increase with the blockage ratio when Dirichlet condition was used, while an inverse relation was obtained for the Neumann condition. Investigating symmetrical versus asymmetrical confinement, Nirmalkar and Chhabra [21] studied the momentum and heat transfer characteristics from the power-law fluid flow past a heated cylinder in asymmetric confinement for Re ranging from 0.1 to $100, \operatorname{Pr}$ from 1 to $100, \beta$ from 0.2 to 0.4 and $\gamma$ from 0.25 to 1 . They found that Nusselt number did not follow any trend with respect to the asymmetry in the placement of the cylinder. As the cylinder was placed away from the centre of the channel, there was an imbalance of force acting on the cylinder in the vertical direction because of both confining walls, with the wall closer to the cylinder exerting more repellent force than the wall further away from the cylinder, thus leading to some positive value of lift coefficient. However, the drag coefficient and Nusselt number were far more influenced by other parameters like Re and $\beta$ than by $\gamma$. In a recent investigation, Bijjam et al. [23] studied the forced convection from an asymmetrically confined cylinder for $R e=1-40, \gamma=0.375-1, \beta=0.2-0.5$ and $\operatorname{Pr}=1-50$. They found that asymmetrical configuration reduces overall drag, and that the Nusselt number is higher for symmetric cylinder placement than for asymmetric placement.

In contrast, mixed convection $(\mathrm{Ri} \neq 0)$ effects around a symmetrically placed circular cylinder in a channel have not been paid the same attention. Few investigations can be found for a confined symmetrically placed circular cylinder [24-28]. Farouk and Güçeri [24] dealt with mixed convection (aiding buoyancy) from a confined circular cylinder in the steady state at $\beta=0.1667$. Singh et al. [25] examined the mixed convection (aiding/opposing buoyancy) from a confined cylinder for $\operatorname{Re}=100, \operatorname{Pr}=0.7$ and $\mathrm{Ri}=-1$ to 1 at $\beta=0.25$. Vortex shedding is not observed for $\mathrm{Ri}>0.15$. The average Nusselt number increases considerably with Ri. Gandikota et al. [26] explored mixed convection (aiding/opposing buoyancy) effects for a confined cylinder at $R e=50-150$, $\operatorname{Pr}=0.7$ and $\mathrm{Ri}=-0.5$ to 0.5 at $\beta=0.25$. The critical Ri at which vortex shedding stops was determined and it was found to increase with $R e$. The average Nusselt number increases at a higher rate beyond a critical Ri; however, the value of the average Nusselt number remains constant in the negative Ri range. Hu and Koochesfahani [27] studied experimentally the effects of Ri ( $0-1.04$ ) on the flow around a heated cylinder in a vertical water channel at $R e=135$ in a reverse flow situation. At $\mathrm{Ri}<0.31$ the drag coefficient was observed to be slightly smaller than for the unheated cylinder, whereas for $\mathrm{Ri}>0.31$, the drag coefficient increases linearly with Ri. The average Nusselt number decreases almost linearly with increasing Ri. Guillén et al. [28] recently carried out an experimental study to explore the mixed convection (aiding/opposing buoyancy) effects around a symmetrically placed confined cylinder for $R e=170, \operatorname{Pr}=7$ and $\mathrm{Ri}=-1$ to 5 at $\beta=0.287$. They determined how the flow patterns and vortex shedding are modified by $\beta$ and Ri.

While some literature is available on the mixed convection from a symmetrically confined circular cylinder [24-28], for the topic of flow around and mixed convection heat transfer from an asymmetrically confined circular cylinder there is no literature available, to the best knowledge of the authors. Thus, this study aims to investigate laminar flow and heat transfer for an asymmetrically confined cylinder under the influence of cross-buoyancy mixed convection.

## 3. Problem statement and numerical details

This study investigates the 2-D flow of Newtonian fluid around a circular cylinder of diameter $d$ placed in a channel of height $2 H$ located in an off-centre position. The downstream distance $L_{d}$ and the upstream
distance $L_{u}$ are shown in Fig. 1. The gap ratio $\gamma$ is used to define the position of the cylinder inside the channel, with $\gamma=1$ representing the cylinder at centre and $\gamma=0$ representing the cylinder touching the lower wall of the channel. The blockage ratio $\beta=d /(2 H) \beta=\frac{\mathrm{d}}{2 \mathrm{H}}$ is used to describe the confinement of the channel. An incompressible Newtonian fluid having a fixed temperature of $T_{0}$ enters the channel with a fully developed velocity profile and exchanges heat when it comes in contact with the cylinder, the surface of which is kept at a slightly higher constant temperature $T_{w}$ with a temperature difference $\Delta T \approx 2 \mathrm{~K}$. Because of low $\Delta T$, all thermo-physical properties (heat capacity, viscosity, thermal conductivity, etc.) can be assumed to be the constant except density. Boussinesq approximation is used to overcome the effect of temperature on density, and viscous dissipation is neglected because of the low Reynolds number values investigated.

Considering the above assumptions, the continuity (Eq. 1), momentum (Eqs. (2) and (3)) and energy (Eq. 4) equations can be written in non-dimensional form as
$\frac{\partial V_{x}}{\partial x}+\frac{\partial V_{y}}{\partial y}=0$,

$$
\begin{align*}
& \frac{\partial V_{x}}{\partial t}+\frac{\partial\left(V_{x} V_{x}\right)}{\partial x}+\frac{\partial\left(V_{y} V_{x}\right)}{\partial y} \\
& \quad=-\frac{\partial P}{\partial x}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} V_{x}}{\partial x^{2}}+\frac{\partial^{2} V_{x}}{\partial y^{2}}\right) \tag{2}
\end{align*}
$$

$\frac{\partial V_{y}}{\partial t}+\frac{\partial\left(V_{x} V_{y}\right)}{\partial x}+\frac{\partial\left(V_{y} V_{y}\right)}{\partial y}$
$=-\frac{\partial P}{\partial y}+\frac{1}{\operatorname{Re}}\left(\frac{\partial^{2} V_{y}}{\partial x^{2}}+\frac{\partial^{2} V_{y}}{\partial y^{2}}\right)+\operatorname{Ri} \theta$,
$\frac{\partial \theta}{\partial t}+\frac{\partial\left(V_{x} \theta\right)}{\partial x}+\frac{\partial\left(V_{y} \theta\right)}{\partial y}=\frac{1}{\operatorname{RePr}}\left(\frac{\partial^{2} \theta}{\partial x^{2}}+\frac{\partial^{2} \theta}{\partial y^{2}}\right)$,
where $V_{x}, V_{y}$ are the $x$ and $y$ components of velocity, respectively, which are made dimensionless by $U_{\infty}$, and $P$ and $\theta$ are the non-dimensional pressure and temperature, respectively. The definitions of Richardson Ri, Prandtl Pr and Reynolds Re numbers can be found in the Nomenclature.

The following boundary conditions (in non-dimensional form) are used:

- At the inlet plane $(x=0)$ : a fully developed velocity profile is assumed,

$$
\begin{aligned}
V_{x} & =1.5\left[1-(1-2 \beta y)^{2}\right] \text { where } 0 \\
& \leq y \\
& \leq 2 H / d \text { and } \beta \\
& =d /(2 H), V_{y} \\
& =0 \text { and } \theta \\
& =0
\end{aligned}
$$

- At the channel walls: no-slip condition for velocity and adiabatic condition for temperature are used,

$$
V_{x}=0 ; V_{y}=0 \text { and } \frac{\partial \theta}{\partial y}=0
$$

- On the cylinder: no-slip condition for flow and uniform wall temperature condition for temperature are used,
$V_{x}=0 ; V_{y}=0$ and $\theta=1$
- At the outlet plane $(x=40)$ : the outflow setting in Ansys Fluent is used, which is a homogeneous Neumann condition for all dependent variables [8,23],

$$
\frac{\partial \phi}{\partial x}=0, \text { where } \phi=\left(V_{x}, V_{y}, \theta\right) .
$$

The meshing of the whole computational domain is performed using Ansys Workbench. Fine meshing (minimum dimensionless grid size of 0.01 ) was chosen near the cylinder and near the adiabatic walls because variations in the velocity and temperature are expected to be higher in these areas; for the rest of the domain a relatively coarse mesh is used. The momentum and energy equations along with the boundary conditions are solved using Ansys Fluent. The Gauss-Siedel iterative method is used to solve the algebraic equations. The pressure-based 2-D planar solver with SIMPLE algorithm is used. A second order upwind scheme is used for the convective terms, whereas a central difference scheme is used for the diffusive terms in the momentum and energy eqs. A fully developed velocity profile is inserted in Ansys Flu-


Fig. 1. Schematic of the flow situation.
ent with the help of user-defined functions. The absolute convergence criteria for Eqs. (1) to (4) are set to $10^{-15}$.

## 4. Selection of numerical parameters

The choice of numerical parameters is crucial in determining the accuracy as well as the reliability of the solution. Thus, the choice of grid size, domain and time step are very important parameters to obtain accurate results with minimum usage of computational power. The domain is given by the channel height $2 H$, the upstream length $L_{u}$, the downstream length $L_{d}$ measured from the centre of the cylinder (see Fig. 1). The domain independence test is carried out at the minimum $R e$ value investigated because the momentum boundary layer is thick at low $R e$, while the grid independence test is done at the maximum $R e$ investigated, as at increasing $R e$ the momentum boundary layer becomes thinner. As the cylinder is placed asymmetrically inside the channel, computations were carried out on the full computational domain even for steady flow.

Non-uniform grids (with minimum dimensionless grid sizes of 0.01 and 0.006 and with fine meshing in the no-slip zones) are chosen to obtain the optimum grid size. Results for time-mean of drag coefficient $C_{D}$ (hereafter drag coefficient or simply $C_{D}$ ) and average cylinder Nusselt number $N u$ (which is the surface average of the local Nusselt number $\mathrm{Nu}_{\mathrm{L}}$ ) at the highest $R e$ and the extreme value of all other variables in study are presented in Table 1. Based on these estimates, grid G1

Table 1
Variation of $C_{D}$ and $N u$ with grid for $\mathrm{Ri}=0$ and $2, \operatorname{Re}=150, \operatorname{Pr}=1, \beta=0.4, \gamma=1$.

| Grid | No. of cells | Minimum grid size | $C_{D}$ | $N u$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Ri}=0$ |  |  |  |  |
| G 1 | 41,616 | 0.01 | 3.7059 | 9.5006 |
| G 2 | 68,240 | 0.006 | 3.6968 | 9.4773 |
| \% relative variation |  | 0.24 | 0.25 |  |
| $\mathrm{Ri}=2$ |  |  |  |  |
| G 1 | 41,616 | 0.01 | 3.8244 | 9.6604 |
| G 2 | 68,240 | 0.006 | 3.8178 | 9.647 |
| $\%$ relative variation |  | 0.17 | 0.14 |  |

with minimum dimensionless grid size of 0.01 is chosen as the optimum grid size for further study, as not much variation in results occurs between grids G1 and G2.

To fix the computational domain, the dimensionless upstream and downstream lengths were varied from 15 to 20 and from 40 to 45 , respectively, for the extreme values of the parameters, i.e. at $R e=50,150$, $\mathrm{Ri}=2, \beta=0.2$ and $\gamma=0.25,1$. The variations in the values of $C_{D}$ and $N u$ for different cases are shown in Tables 2 and 3. It is clear that there is very little variation when the dimensionless upstream length is increased from 15 to 20 but the computational time required in obtaining these results increases significantly (e.g., 3 to 4 days increase in the computational time). Hence, $L_{u}=15$ was chosen as the upstream length for the rest of the study, as it seems to give a proper balance between the accuracy and the computational cost. Similarly, the variation in $C_{D}$ and $N u$ is reported by varying the dimensionless downstream length from 40 to 45. As there is not much variation in the results, $L_{d}=40$ was chosen as the downstream length. The entire domain independence test was done taking grid G1 with the minimum dimensionless grid size of 0.01 . These distances are also consistent with the literature [5-8,17,19,21,23].

As the governing equations involved in the study are time-dependent, an acceptable value of dimensionless time step $\Delta t$ has to be chosen to solve these equations. As in all the other tests, the values of $C_{D}$ and Nu are compared at the different time steps to find the optimum balance between accuracy and computational time. From the results obtained (Table 4), $\Delta t=0.01$ seems to be the optimal choice for this study.

## 5. Results and discussion

In the current investigation, results are given for the following parameter domains: Reynolds number $\operatorname{Re}=50,100$, 150, Richardson number $\mathrm{Ri}=0,1,2$, blockage ratio $\beta=0.2,0.3,0.4$, gap ratio $\gamma=0.25$, $0.5,1$ and Prandtl number of air $\operatorname{Pr}=0.7$. The values of gap ratio and blockage ratio are chosen based on [21]. The present results ( $C_{D}$ and $N u)$ are compared with those in [8] at various $R e$ values $(50,100,150)$

Table 2
Effect of upstream distance $L_{u}$ on $C_{D}$ and Nu for $\mathrm{Ri}=2, \operatorname{Re}=50$ and $150, \operatorname{Pr}=1$.

| $\beta, \gamma$ | $L_{u}=15$ |  | $L_{u}=20$ |  | \% variation in $C_{D}$ | \% variation in Nu |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $C_{D}$ | $N u$ | $C_{D}$ | $N u$ |  |  |
| Re $=50$ |  |  |  |  |  |  |
| $\beta=0.2, \gamma=1$ | 3.2556 | 5.1121 | 3.2555 | 5.1117 | 0 | 0 |
| $\beta=0.4, \gamma=1$ | 5.5119 | 5.6689 | 5.5119 | 5.6689 | 0 | 0 |
| $\beta=0.4, \gamma=0.25$ | 6.8476 | 4.9607 | 6.8448 | 4.9613 | 0.04 | 0.01 |
| $R e=150$ |  |  |  |  |  |  |
| $\beta=0.2, \gamma=1$ | 2.863 | 9.1958 | 2.8625 | 9.1949 | 0.02 | 0.01 |
| $\beta=0.4, \gamma=1$ | 3.8244 | 9.6604 | 3.8244 | 9.6604 | 0 | 0 |
| $\beta=0.4, \gamma=0.25$ | 5.0697 | 9.1489 | 5.0684 | 9.1491 | 0.02 | 0 |

Table 3
Effect of downstream distance $L_{d}$ on $C_{D}$ and $N u$ for $\mathrm{Ri}=2, R e=50$ and 150, $\operatorname{Pr}=1$.

| $\beta, \gamma$ | $L_{d}=40$ |  | $L_{d}=45$ |  | $\%$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $R e=50$ | $C_{D}$ | $N u$ | $C_{D}$ | $N u$ |  |
| $\beta=0.2, \gamma=1$ |  |  |  |  |  |
| $\beta=0.4, \gamma=1$ | 3.2556 | 5.1121 | 3.2556 | 5.1121 | 0 |
| $\beta=0.4, \gamma=0.25$ | 5.5119 | 5.6689 | 5.5119 | 5.6689 | 0 |
| $R e=150$ | 6.8476 | 4.9607 | 6.8448 | 4.9613 | 0.04 |
| $\beta=0.2, \gamma=1$ |  |  |  |  |  |
| $\beta=0.4, \gamma=1$ | 2.863 | 9.1958 | 2.8629 | 9.1958 | 0 |
| $\beta=0.4, \gamma=0.25$ | 3.8244 | 9.6604 | 3.8244 | 9.6604 | 0 |

Table 4
Effect of time step $\Delta t$ on $C_{D}$ and $N u$ for $\mathrm{Ri}=0$ and $2, \operatorname{Re}=150, \operatorname{Pr}=1$.

| $\beta, \gamma$ | $\Delta t=0.01$ |  | $\Delta t=0.005$ |  | \% variation in $C_{D}$ | \% variation in Nu |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $C_{D}$ | Nu | $C_{D}$ | $N u$ |  |  |
| $\mathrm{Ri}=0$ |  |  |  |  |  |  |
| $\beta=0.2, \gamma=1$ | 2.7699 | 8.9349 | 2.7699 | 8.9351 | Negligible |  |
| $\beta=0.4, \gamma=1$ | 3.7059 | 9.5006 | 3.7056 | 9.5006 |  |  |
| $\beta=0.4, \gamma=0.25$ | 3.3913 | 8.1831 | 3.3913 | 8.1831 |  |  |
| $\mathrm{Ri}=2$ |  |  |  |  |  |  |
| $\beta=0.2, \gamma=1$ | 2.863 | 9.1958 | 2.8629 | 9.1957 | Negligible |  |
| $\beta=0.4, \gamma=1$ | 3.8244 | 9.6604 | 3.8244 | 9.6603 |  |  |
| $\beta=0.4, \gamma=0.25$ | 5.0697 | 9.1489 | 5.0697 | 9.1489 |  |  |

and blockage ratio of 0.25 for the limiting case of $\gamma=1$ in Newtonian fluids. Table 5 clearly shows that there are no significant differences between the present results and the results from [8].

### 5.1. Identification of steady and time periodic regimes

Table 6 shows the various flow regimes over the range of conditions in this study. Although this is not the main objective of the study, it is necessary to understand how the flow regimes change on changing various parameters. It is clear (from Table 6) that the farther the cylinder is from the centre of the horizontal channel (i.e. the greater the blockage ratio is), the longer the steady state continues to exist. At the lowest gap ratio of 0.25 , only steady states were found, irrespective of the investigated values of $\mathrm{Re}, \mathrm{Ri}$ and $\beta$. Due to the asymmetric positioning of the cylinder inside the planar channel, the vortex shedding phenomenon is delayed because of the reduction in interaction between the two vortices caused by proximity to the wall. A similar effect is seen for blockage ratio; for instance, for $\gamma=1$ and the highest blockage ratio $\beta=0.4$, the flow was found to be steady for $R e=50$ irrespective of the value of Ri , while at reduced blockage of $\beta=0.2$, the flow is unsteady periodic for $\mathrm{Ri}=0$ and 1 .

### 5.2. Streamlines and isothermal patterns

Fig. 2 displays the instantaneous streamlines at the extreme settings of $R e=150$ and $\beta=0.4$ applied in the current investigation, for different Ri and $\gamma$ values. At low Re the fluid flow tends to remain attached to the cylinder but finally flow separates from the cylinder as we increase the Re, leading to the formation of separated flow regions in the wake of the cylinder. Also, the flow separation significantly depends upon the value of the $\gamma$. For the symmetric case ( $\gamma=1$ ), the separation of flow is represented by the formation of the two asymmetric alternate vortices. On the other hand, in the case of asymmetrical placement of the cylinder $(\gamma \neq 1)$, the flow separation occurs in the lower half of the computational domain and the asymmetric vortices are formed because of the smaller gap between the cylinder and the lower channel wall as compared with the gap between the cylinder and the upper channel wall. Moreover, the vortices formed are smaller compared to the symmetrical case because of the proximity of the confining channel walls (or blockage effect). Similarly, as the blockage ratio increases, i.e. the walls become more confined, wall confinement tends to suppress wake formation and hence, shorter wakes tend to form in such cases. A rise in downstream streamlines (or an asymmetry in the flow field) and a far downstream wake can also be observed because of the effect of the cross-buoyancy mixed convection ( $\mathrm{Ri} \neq 0$ ). This asymmetry in the flow field increases with Ri because higher Ri values strengthen the cross-

Table 5
Validation of present results ( $C_{D}$ and $N u$ ) with [8] at $\mathrm{Ri}=0$ and $\beta=0.25$.

| $R e$ | Present study |  | Ref. [8] |  | $\%$ variation in $C_{D}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |

Table 6
Flow regimes encountered at various conditions (SS - steady state flow, US -unsteady state flow).

| Ri | $\beta=0.2$ |  |  | $\beta=0.3$ |  |  | $\beta=0.4$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $R e$ |  |  | $R e$ |  |  | Re |  |  | $\gamma$ |
|  | 50 | 100 | 150 | 50 | 100 | 150 | 50 | 100 | 150 |  |
| 0 | SS | SS | SS | SS | SS | SS | SS | SS | SS | 0.25 |
| 1 | SS | SS | SS | SS | SS | SS | SS | SS | SS |  |
| 2 | SS | SS | SS | SS | SS | SS | SS | SS | SS |  |
| 0 | SS | US | US | SS | US | US | SS | SS | US | 0.5 |
| 1 | SS | US | US | SS | US | US | SS | SS | US |  |
| 2 | SS | US | US | SS | SS | SS | SS | SS | US |  |
| 0 | US | US | US | SS | US | US | SS | US | US | 1 |
| 1 | US | US | US | SS | US | US | SS | US | US |  |
| 2 | SS | US | US | SS | US | US | SS | US | US |  |


buoyancy effect. However, the asymmetry in the flow field decreases with increasing $\gamma$ and/or $\beta$.

In addition to providing information about the overall heat transfer, isotherm contours also help in visualising the variation in temperature at different parts of the domain, described as hot and cold regions, which can be helpful in the making of various items like food materials and others that may be temperature sensitive. Accumulation of the isotherms just upstream of the cylinder (near the front stagnation point) can be observed from isotherms (Fig. 3), indicating higher local heat transfer (i.e. local values of Nusselt number) at that area. This crowding of the isotherms seems to increase with an increase in $R e$, which explains the increase in Nusselt number with $\operatorname{Re}$ (Section 5.3). Similar to the flow field, more asymmetry in isotherms can be seen in the thermal patterns at higher Ri. This can be attributed to the fact that at high values of Ri, thermal buoyancy effects are higher. These effects are much more pronounced when vortex shedding occurs. The asymmetry in isotherms decreases with $\gamma$ and/or $\beta$.

### 5.3. Drag coefficient and Nusselt number

Variation of the time-mean or overall drag coefficient $C_{D}$ with $\operatorname{Re}$, $\mathrm{Ri}, \beta$ and $\gamma$ is analysed and plotted in Fig. 4 for steady as well as periodic regimes. The overall drag coefficient always decreases with $R e$ at fixed values of other parameters investigated. The drag coefficient follows a direct increasing relationship with Ri for given $R e$ and $\beta$. The maximum percentage of increase in the value of $C_{D}$ of about $91 \%$ (at $\mathrm{Ri}=2$ with respect to $\mathrm{Ri}=0$, i.e. $\left(C_{\left.\left.D, \mathrm{Ri}=2^{-}-C_{D, \mathrm{Ri}=0}\right) / C_{D, \mathrm{Ri}=0} \times 100\right) \text { is seen }}\right.$ at the low blockage ratio $\beta=0.2, \gamma=0.25$ and at the highest $\operatorname{Re}=150$ investigated. For the blockage ratio, it is clear from Fig. 4 that comparing $\beta=0.2$ with 0.4 , the drag coefficient also increases for all the values of Ri and $R e$. As the proximity to the lower channel wall increases, the obstruction to the flow grows, resulting in a steeper velocity gradient and thus higher drag force on the cylinder. This effect is
more significant at low Re because of the thicker boundary layer and higher viscous forces acting on the cylinder. There is not much variation in the drag coefficient with the gap ratio at low $R e$, as also reported by Mettu et al. [19], but higher values of Re are also involved here; in most of these cases, the higher the gap ratio, the smaller the drag coefficient. However, this is not true for all cases, for instance when the cylinder is placed symmetrically at the centre of the channel, much less variation is seen, especially for small blockage ratios.

The average Nusselt number is calculated to estimate the rate of heat transfer from the cylinder to the ambient fluid. It seems to increase steadily with the value of $R e$ when all other parameters are kept constant, as can be seen in Fig. 5. This can be attributed to the higher chance for vortex shedding, which increases the heat transfer. Moreover, the thermal boundary layer keeps becoming thinner with $R e$, thus increasing the heat transfer rate. The maximum heat transfer change is found to be around 94\% comparing $\operatorname{Re}=150(\beta=0.4, \gamma=0.25, \mathrm{Ri}=0)$ with $\operatorname{Re}=50(\beta=0.4, \gamma=0.25, \mathrm{Ri}=0)$.

The average Nusselt number also increases with Ri. Basically, Ri is the representation of free convection (due to buoyancy) relative to the forced convection. As Ri is increased, buoyancy forces responsible for the movement of the fluid due to mixed convection increase and thus the heat transfer rate also increases. The maximum heat transfer change is found to be around $18 \%$ at $\mathrm{Ri}=2(\operatorname{Re}=50, \beta=0.2, \gamma=0.25)$ with respect to $\mathrm{Ri}=0(R e=50, \beta=0.2, \gamma=0.25)$.

As can be seen in Fig. 5, the average Nusselt number increases with the blockage ratio when all other parameters are kept constant. This is because as the blockage ratio is increased and the confining walls are set closer to the cylinder, a steeper temperature gradient occurs, leading to an increased heat transfer rate. Another result is when the cylinder is placed in the centre of the channel $(\gamma=1)$, at the highest value of blockage ratio ( $\beta=0.4$ ) investigated the variation in the Nusselt number seems to be somewhat less significant than for asymmetric cylinder placement. The maximum heat transfer change is found to be


Fig. 3. Instantaneous isotherm contours at $\operatorname{Re}=150$ and $\beta=0.4$ for different Ri and $\gamma$ values.


Fig. 4. $C_{D}$ versus $R e$ at various Ri, $\beta$ and $\gamma$ values.
around $16 \%$ at $\beta=0.4(\operatorname{Re}=150, \gamma=0.25, \mathrm{Ri}=0)$ with respect to $\beta=0.2(\operatorname{Re}=150, \gamma=0.25, \mathrm{Ri}=0)$. Asymmetric placement of the cylinder in the channel has an adverse impact on the heat transfer rate. Due to the asymmetric placement of the cylinder, the fluid flow can be
divided into two regions. While the fluid flowing below the cylinder near the bottom wall increases the local rate of heat transfer in the lower region, this is mostly offset by the poor circulation in the upper region of the flow, which overall decreases the average heat transfer

rate and thus the average Nusselt number (although not significantly). The maximum heat transfer change is found to be only around $1 \%$ at $\gamma=0.25(\operatorname{Re}=50, \beta=0.2, \mathrm{Ri}=2)$ with respect to $\gamma=1(\mathrm{Re}=50$, $\beta=0.2, \mathrm{Ri}=2$ ).

Further comments can be made on the ratio of drag coefficient and average Nusselt number. Since the drag coefficient is a decreasing function of the $R e$, while Nusselt number is an increasing function, it is observed that the ratio of both of these quantities decreases with Re. Both the drag coefficient and Nusselt number increase with Ri, and the ratio of these also increases, which shows that the drag increases with Ri at a slightly faster rate than Nusselt number does. This implies that heat transfer can be increased with Ri, but at the expense of higher drag forces on the cylinder. Increasing the blockage ratio tends to increase drag forces on the cylinder more than it increases heat transfer. Moreover, when the cylinder is placed at the centre of the channel, it can be seen that there is not much change in the values of the ratio of drag to Nusselt number at various Ri, which means that the drag coefficient and the Nusselt number increase at roughly the same rate. However, when the cylinder is moved away from the centre of the channel the ratio increases, indicating that asymmetrical cylinder placement affects drag forces more strongly than it influences heat transfer.

## 6. Concluding remarks

The present study deals with the two-dimensional flow and heat transfer of an incompressible constant-property Newtonian fluid in a horizontal plane channel with a circular cylinder placed in a symmetric or asymmetric position. The aim here is to investigate the cross-buoyancy mixed convection from a confined asymmetric cylinder. We carried out domain-, grid- and time-step-independence studies. Steady and time periodic regimes were identified by changing various parameters. The drag coefficient is found to be more strongly influenced by the Reynolds number, Richardson number and blockage ratio than by the gap ratio. Similarly, the average Nusselt number increases with each
parameter with the exception of the gap ratio, where the heat transfer rate decreases due to proximity of the cylinder to the wall. Moreover, the asymmetric placement of the cylinder increases the drag forces acting on the cylinder at a higher rate as compared to the increase in the heat transfer. The maximum heat transfer change is found to be around $94 \%$ at $R e=150(\beta=0.4, \gamma=0.25, \operatorname{Ri}=0)$ with respect to $R e=50$ at the same parameters. Computations for higher Ri numbers and reduced $\gamma$ are planned for future work.

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