

# Tooth Contact Analysis of Straight Bevel Gears in the Function of the Modification of Number of Teeth of the Driving Gear

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## ABSTRACT

The bevel gear is widely used in mechanical structures if we want to change the shaft positions, the transmission ratio and the rotation direction. The aim of the publication is the analysis of the TCA (Tooth Contact Analysis) parameters in the function of the modification of the number of teeth of the driving gear. This analysis is actually a finite element analysis with which the developing normal stress, normal strain and normal deformations could be determined on the contact zone of the gear pairs. This analysis is important for the purpose of the judgment of the goodness of the gear drive. Previously the exact CAD (Computer Aided Designing) modelling of the tested gear drive is needed and after that the exact assembly is necessary. In this publication we want to determine the correlations among the TCA parameters and the number of teeth of the driven gear.

*Keywords:* TCA, CAD, bevel gear, number of teeth

## 1. INTRODUCTION

Straight bevel gears are applied widely in machinery (in vehicles, tool machines, robots, for medical tools etc.). They are used to connect shafts whose axes intersect in some angle, thus the meshing surfaces form a cone on which teeth are shaped (Figure 1, 2) [1, 2, 3, 5, 6, 7, 9, 10, 11, 12].



Figure 1. The application of straight bevel gear in case of moving of molding vat

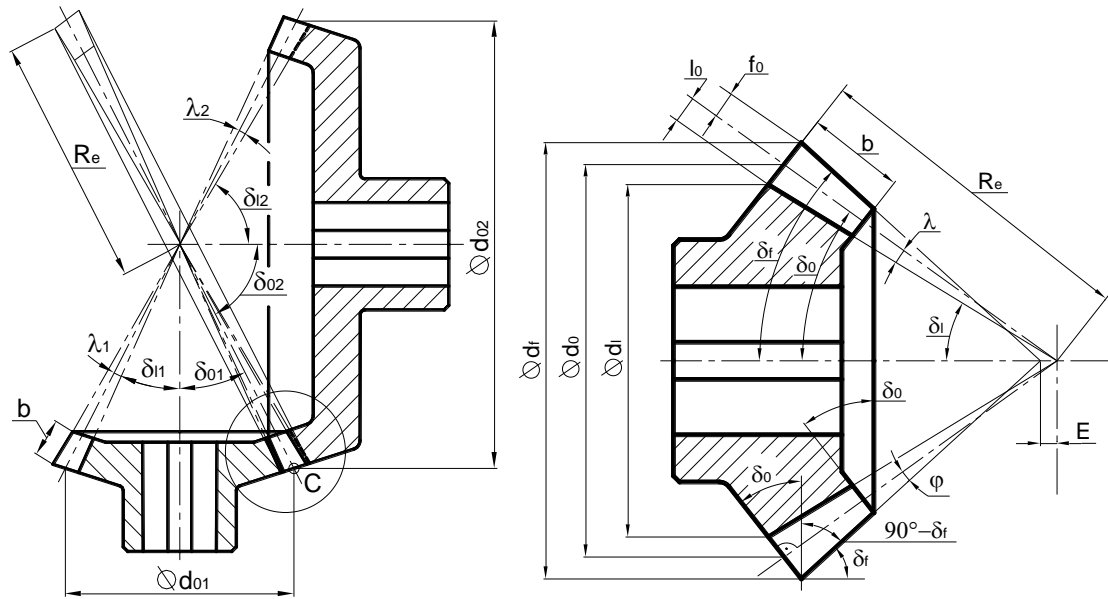


Figure 2. The main parameters of bevel gear pairs

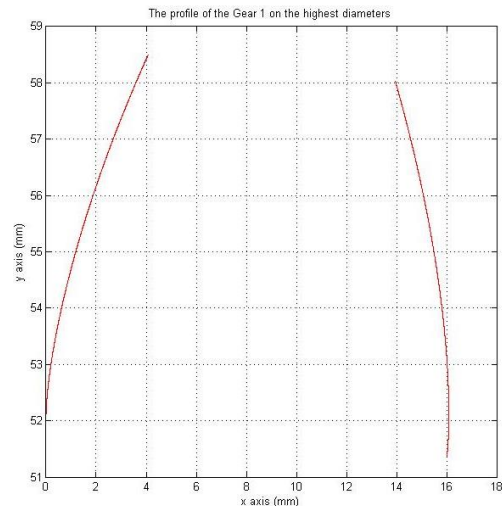
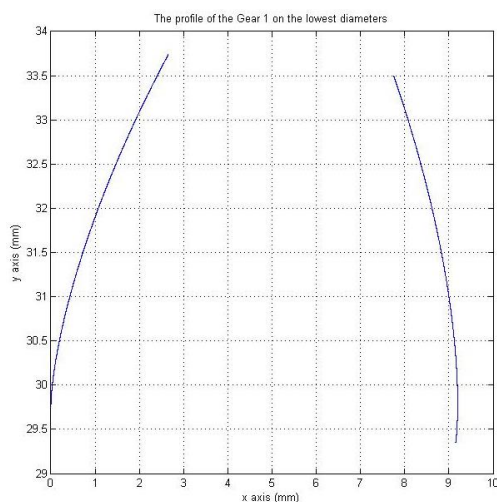
Based on Figure 2 it is visible that many parameters are needed for the exact description of the bevel gear. In case of this type the teeth are situated parallel with the rotation axis of the cone body [2, 9, 10, 11, 12].

The aim of the TCA is the analysis of the developed mechanical parameters which have to be analyzed on the gear connection. The main TCA parameters are the normal stress normal strain and normal deformation values which are determined perpendicularly to the gear surface [1, 6, 8, 9, 13].

The production cost is very expensive because of the complex geometry, the tool costs, device cost, etc. That is why the real production has to be followed after the designing, the CAD modelling and the TCA analysis.

## 2. DESIGNING OF GEAR PAIRS

We have worked out a computer program because of the simplification of the geometric designing process. Previously the suggestions of the references have to be read [4, 9, 10, 11, 12]. Knowing of the designing formulas the computer program could be written.



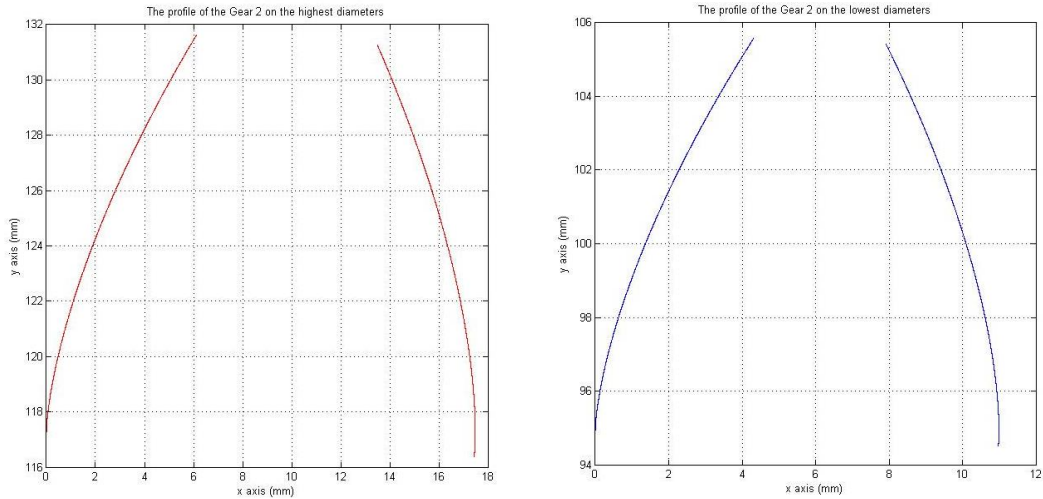


Figure 2. The profiles of bevel gear pairs on the main diameters ( $m=10$ ,  $z_1=20$ ,  $z_2=30$ )

Input data for designing the drive pair are:  $m$  module,  $z_1$  number of teeth of the driver bevel gear wheel,  $z_2$  number of teeth of the driven gear wheel,  $c^*$  root clearance factor and the  $\alpha_0$  angle of contact [4, 9, 10, 11, 12]. Knowing of them the program is calculated all necessary geometric parameters of the gear pairs and drawn the profile curves on the highest and the lowest diameters (Figure 2).

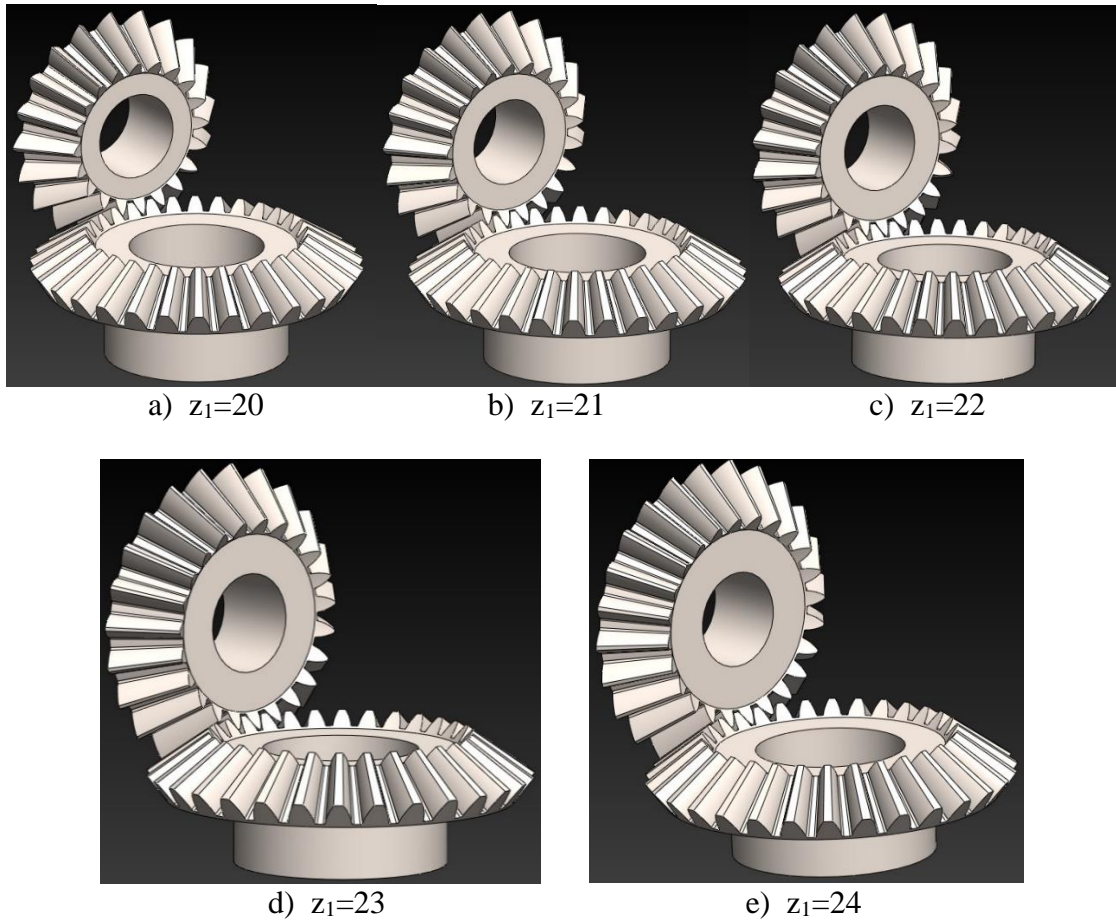


Figure 3. The type of the CAD models of the designed gears

After the geometric calculation the CAD models of the bevel gears could be created by SolidWorks software (Figure 3). Interpolation B-spline has to be fit on the profile points. Knowing of the necessary profile curves the teeth could be created by extrusion.

Table 1. The calculated parameters of the designed bevel gear pairs

<i>The main parameters of the bevel gear pairs</i>	<i>Gear drive I.</i>	<i>Gear drive II.</i>	<i>Gear drive III.</i>	<i>Gear drive IV.</i>	<i>Gear drive V.</i>
Number of teeth of the driving gear ( $z_1$ )	20	21	22	23	24
Number of teeth of the driven gear ( $z_2$ )			30		
Module (m) [mm]			10		
The largest pitch circle diameter of the driving gear ( $d_{01}$ ) [mm]	200	210	220	230	240
The largest pitch circle diameter of the driven gear ( $d_{02}$ ) [mm]			300		
Half pitch angle of the pitch circle of the driving gear ( $\delta_{01}$ ) [°]	56.309	55.008	53.746	52.523	51.340
Half pitch angle of the pitch circle of the driven gear ( $\delta_{02}$ ) [°]	33.69	34.992	36.253	37.476	38.659
Effective pitch surface radius ( $R_e$ ) [mm]	180.277	183.098	186.01	189.01	192.093
Addendum on the largest diameter ( $f_0$ ) [mm]			10		
Dedendum on the largest diameter ( $l_0$ ) [mm]			12		
The largest tip circle diameter of the driving gear ( $d_{f1}$ ) [mm]	211.094	221.469	231.827	242.168	252.493
The largest tip circle diameter of the driven gear ( $d_{f2}$ ) [mm]	316.641	316.384	316.128	315.872	315.617
The largest root circle diameter of the driving gear ( $d_{a1}$ ) [mm]	186.687	196.236	205.807	215.397	225.007
The largest root circle diameter of the driven gear ( $d_{a2}$ ) [mm]	280.03	280.338	280.646	280.953	281.259
Face width (b) [mm]	51.507	52.313	53.145	54.003	54.883
Dedendum angle ( $\lambda$ ) [°]	3.808	3.749	3.691	3.632	3.574
Tip cone angle of the driving gear ( $\delta_{f1}$ ) [°]	60.118	58.757	57.437	56.156	54.914
Tip cone angle of the driven gear ( $\delta_{f2}$ ) [°]	37.498	38.741	39.945	41.108	42.234
Root cone angle of the driving gear ( $\delta_{l1}$ ) [°]	52.501	51.258	50.055	48.891	47.765
Root cone angle of the driven gear ( $\delta_{l2}$ ) [°]	29.881	31.242	32.562	33.843	35.085
Circular pitch on the largest pitch circle diameter (t) [mm]			31.415		
Clearance at flank ( $j_s$ ) [mm]			1.570		
Pitch circle tooth thickness on the largest diameters ( $S_{ax}$ ) [mm]			17.278		
Transmission ratio (i)	1.5	1.428	1.363	1.304	1.25

On Table 1 the calculated parameters of the bevel gears could be seen. After the geometric designing the TCA could be followed.

### 3. TCA ANALYSIS OF CONNECTING BEVEL GEAR PAIRS

#### 3.1. The adoption of the finite element mesh

The appropriate selection of the finite element mesh is the basis of the calculation process [8, 13]. One of the main property of the bevel gear is the teeth are continuously narrows from the highest cone's diameters to the lowest cone's diameters. Because of this property the equable mesh subdivision could not be applied.

We have adopted a coordinate system in the middle of the teeth connection zone (Figure 4).

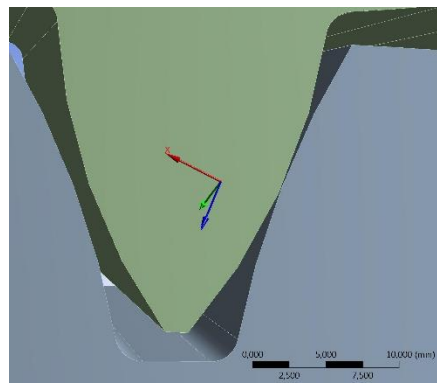


Figure 4. The adoption of the coordinate system into the middle of the connection zone

The x axis of the adopted coordinate system is shown into the normal direction of the connecting surfaces (Figure 4).

Sphere of influence meshing has been applied on the contact zone. The mesh shape is dense triangles. The density is 1.5 mm and the sphere radius is 38 mm. Automatic meshing has been applied in the outside of the contact zone (Figure 5).

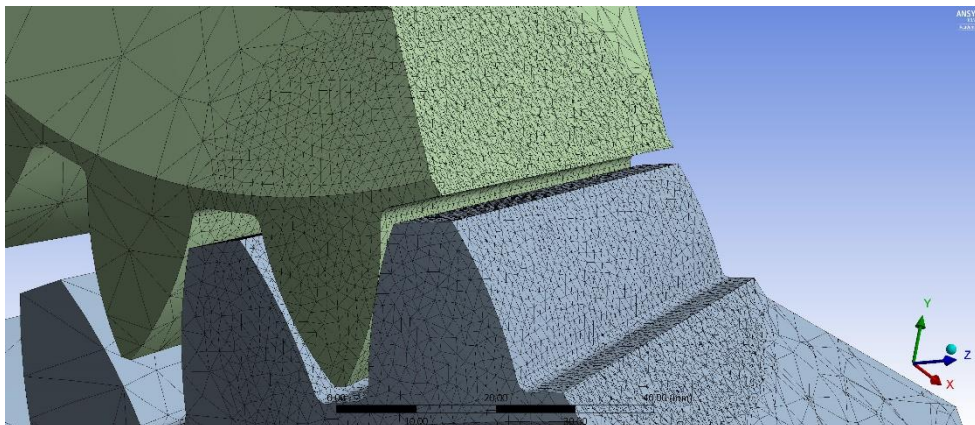


Figure 5. The adoption of the finite element mesh

The type of the applied material has been structural steel (Table 2).



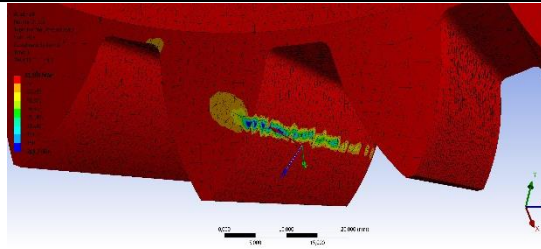
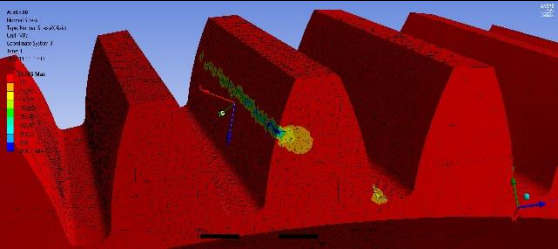
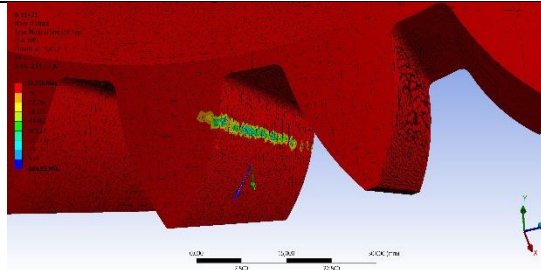
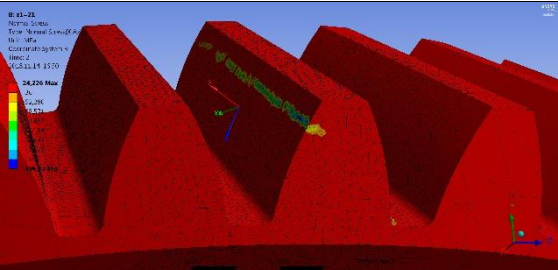
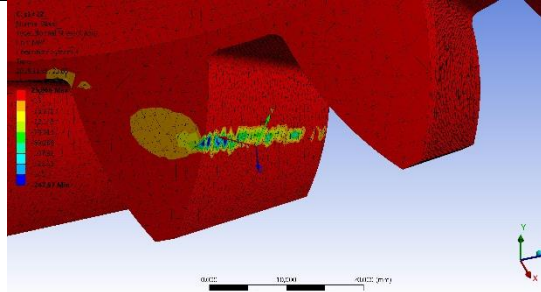
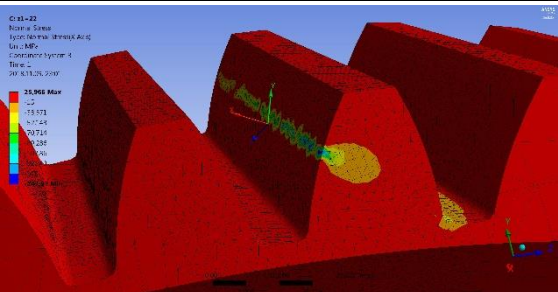
Table 2. The properties of the applied material

Density	7850 kg/m <sup>3</sup>
Yield limit	250 MPa
Ultimate strength	460 MPa

The driving bevel gear has been loaded by 600 Nm moment. Five degrees of freedom have been fixed on the driving gear only the turning motion around the axis of rotation has been let. The driven gear has been totally fixed.

### 3.2. Analysis of the normal stress

The normal stress is interpreted on perpendicular direction of the tooth surface [8, 9, 13]. These direction is the most determinative in aspect of tooth deformation. This normal stress has been calculated for the tooth contact zone (Figure 6).

Driving gear	Driven gear
	
$\bar{\sigma}_n = -5.753 \text{ MPa}$	$\bar{\sigma}_n = -6.104 \text{ MPa}$
$z_1=20$	
	
$\bar{\sigma}_n = -5.888 \text{ MPa}$	$\bar{\sigma}_n = -4.417 \text{ MPa}$
$z_1=21$	
	
$\bar{\sigma}_n = -5.075 \text{ MPa}$	$\bar{\sigma}_n = -4.906 \text{ MPa}$
$z_1=22$	

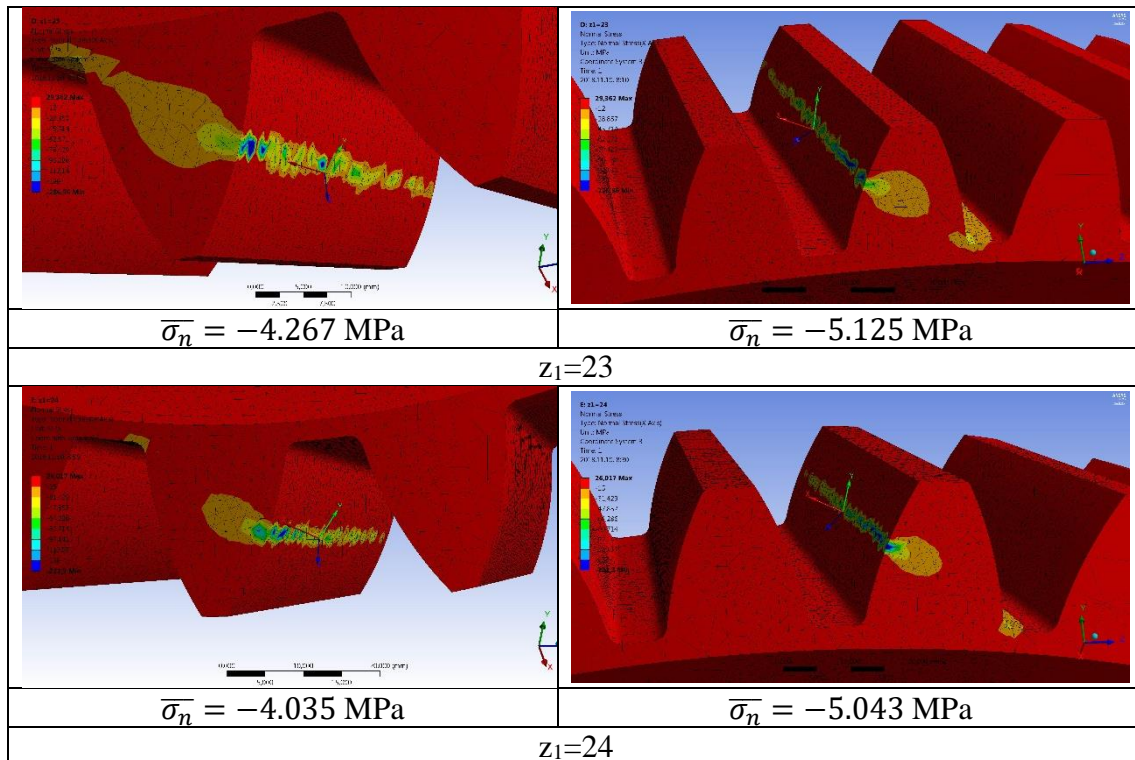
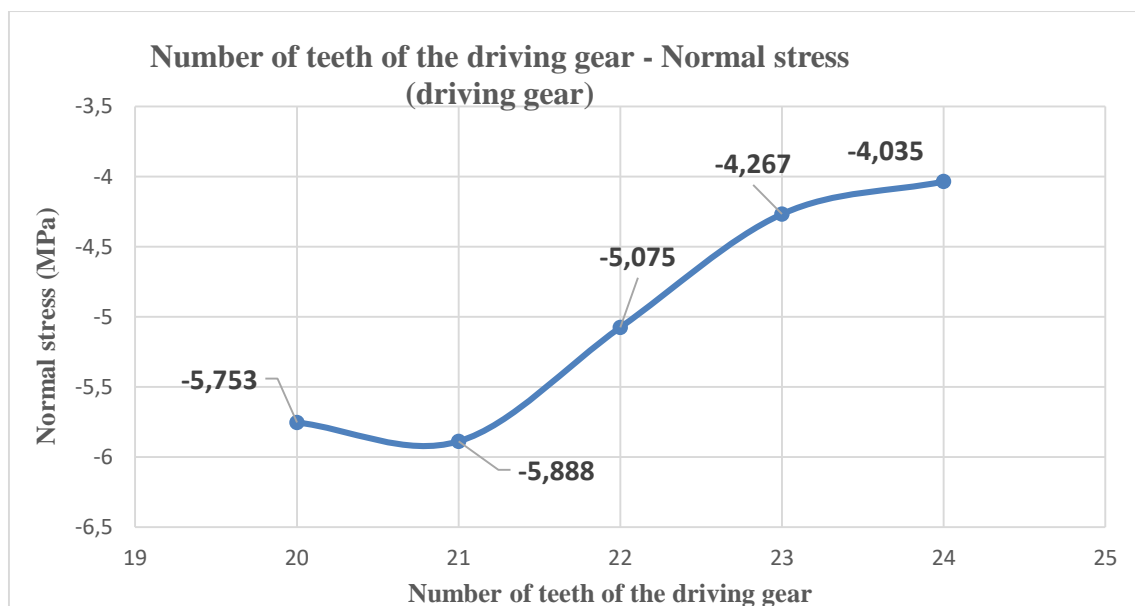


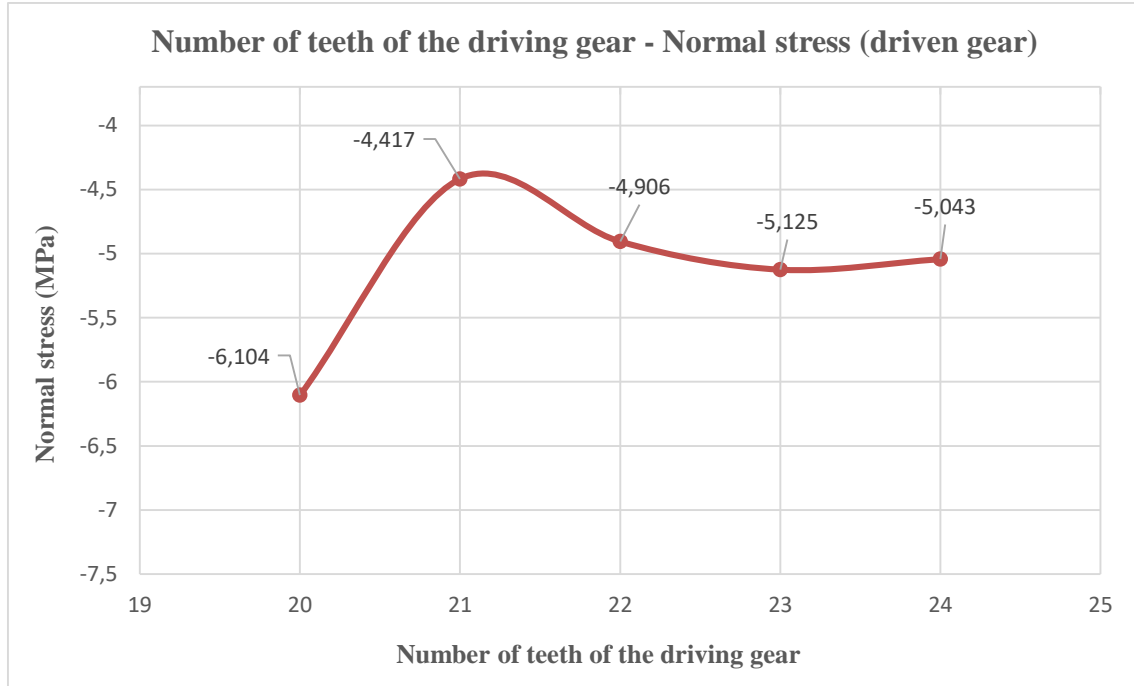
Figure 6. Normal stress results for every bevel gear pairs

Based on the calculations the normal stresses could be seen in the function of the number of teeth of the driving bevel gear on Figure 7.

Based on the results the minimum normal stress of the surface of the driving bevel gear is developed in case of  $z_1=24$  number of teeth in absolute value. The minimum normal stress of the surface of the driven bevel gear is developed in case of  $z_1=21$  number of teeth in absolute value (Figure 7).



a) driving bevel gear

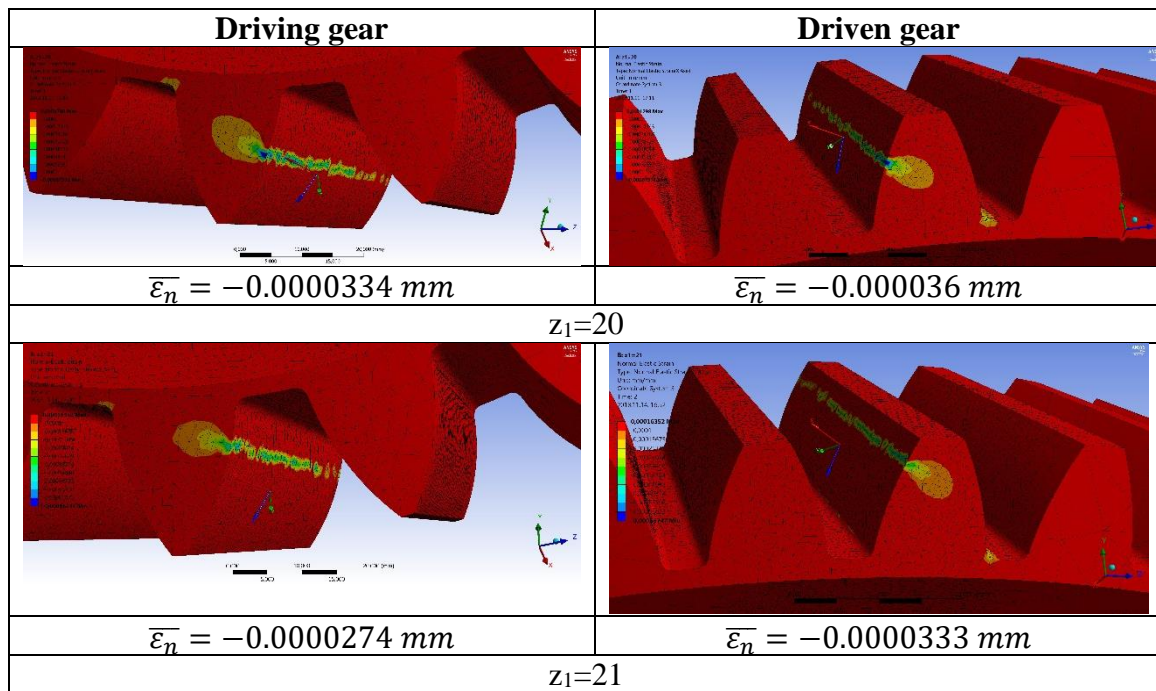


b) driven bevel gear

Figure 7. The developed normal stresses in the function of the number of teeth

### 3.3. Analysis of the normal elastic strain

The normal elastic strain has been calculated for the tooth contact zone (Figure 8). It is defined on perpendicular direction for the tooth surface [8, 9, 13].





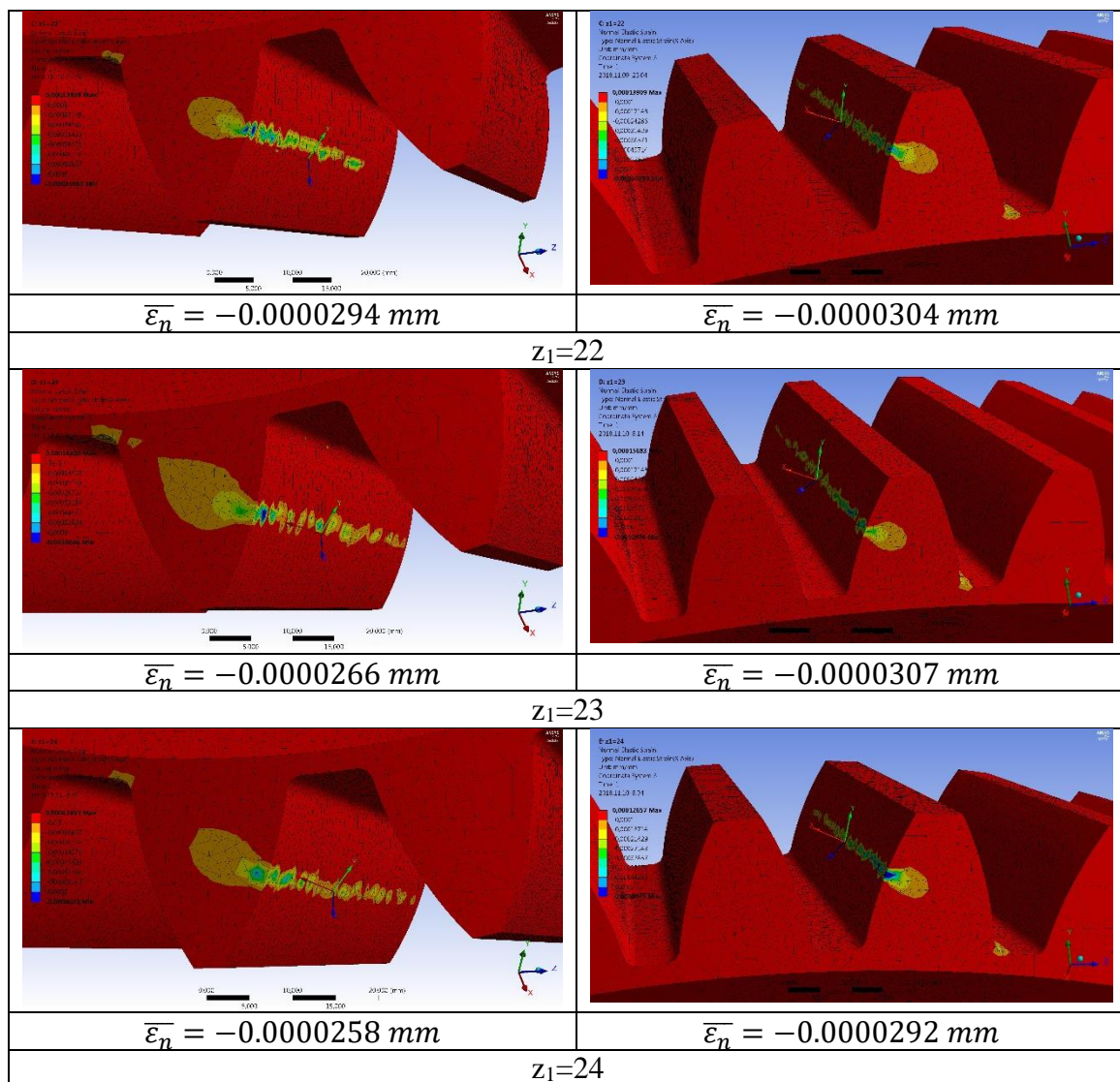
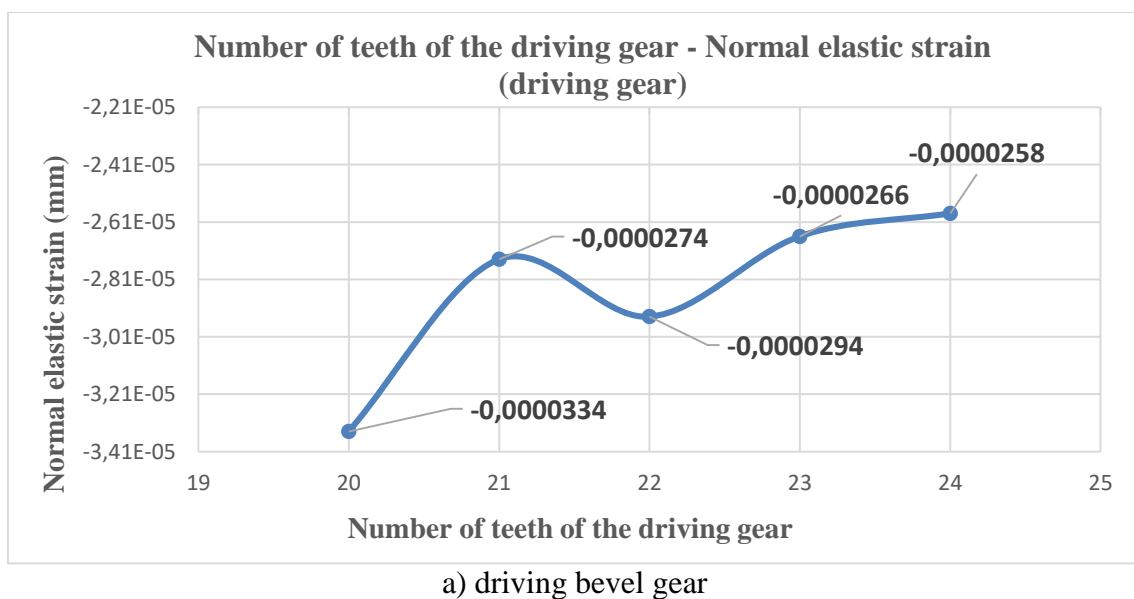
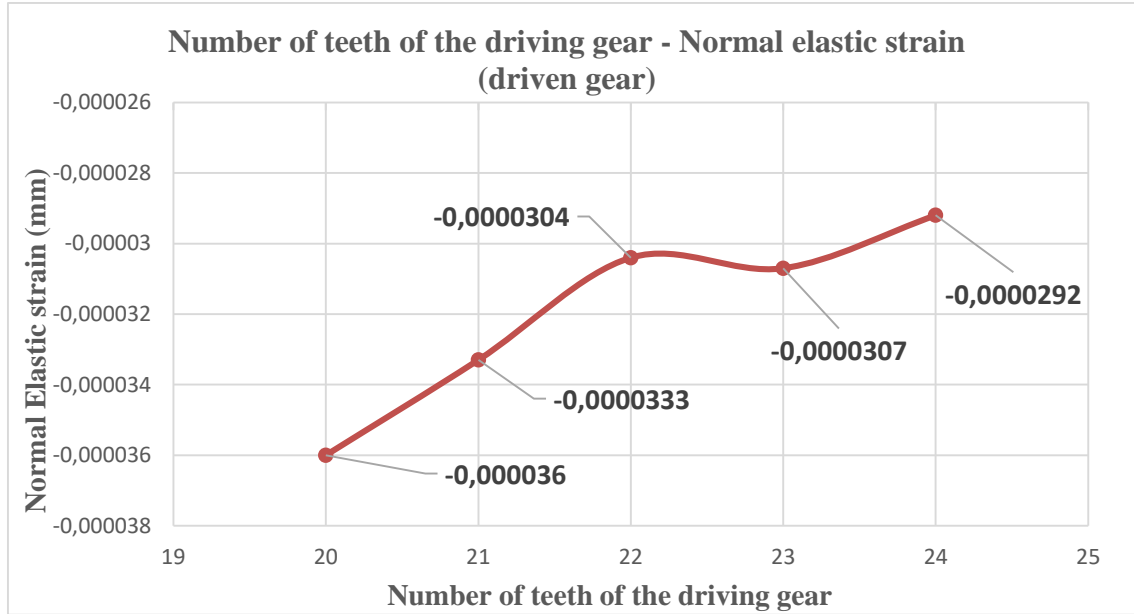


Figure 8. Normal elastic strain results for every bevel gear pairs





b) driven bevel gear

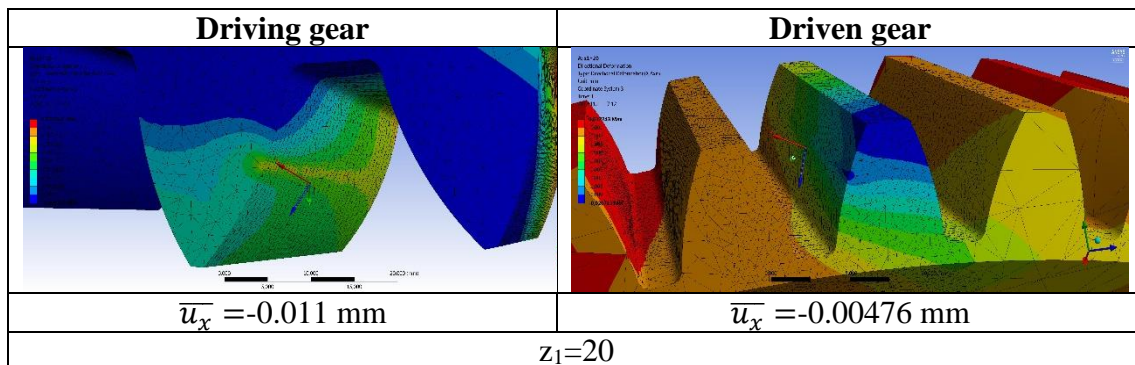
Figure 9. The developed normal elastic strain in the function of the number of teeth

Based on the calculations the normal elastic strain could be seen in the function of the number of teeth of the driving bevel gear on Figure 9.

Based on the results the minimum normal elastic strain of the surface of the driving bevel gear is developed in case of  $z_1=24$  number of teeth in absolute value. The minimum normal elastic strain of the surface of the driven bevel gear is developed in case of  $z_1=24$  number of teeth in absolute value (Figure 9). That is why the increasing of the number of teeth of the driving gear the normal elastic strain will be increased in case of both connecting tooth surfaces.

### 3.4. Analysis of the normal deformation

The normal deformation has been analyzed into the x direction which is perpendicular for the contact surfaces [8, 9, 13]. This direction is the most determinative because the main deformation is applied into the perpendicular direction of the contact surfaces (Figure 10).



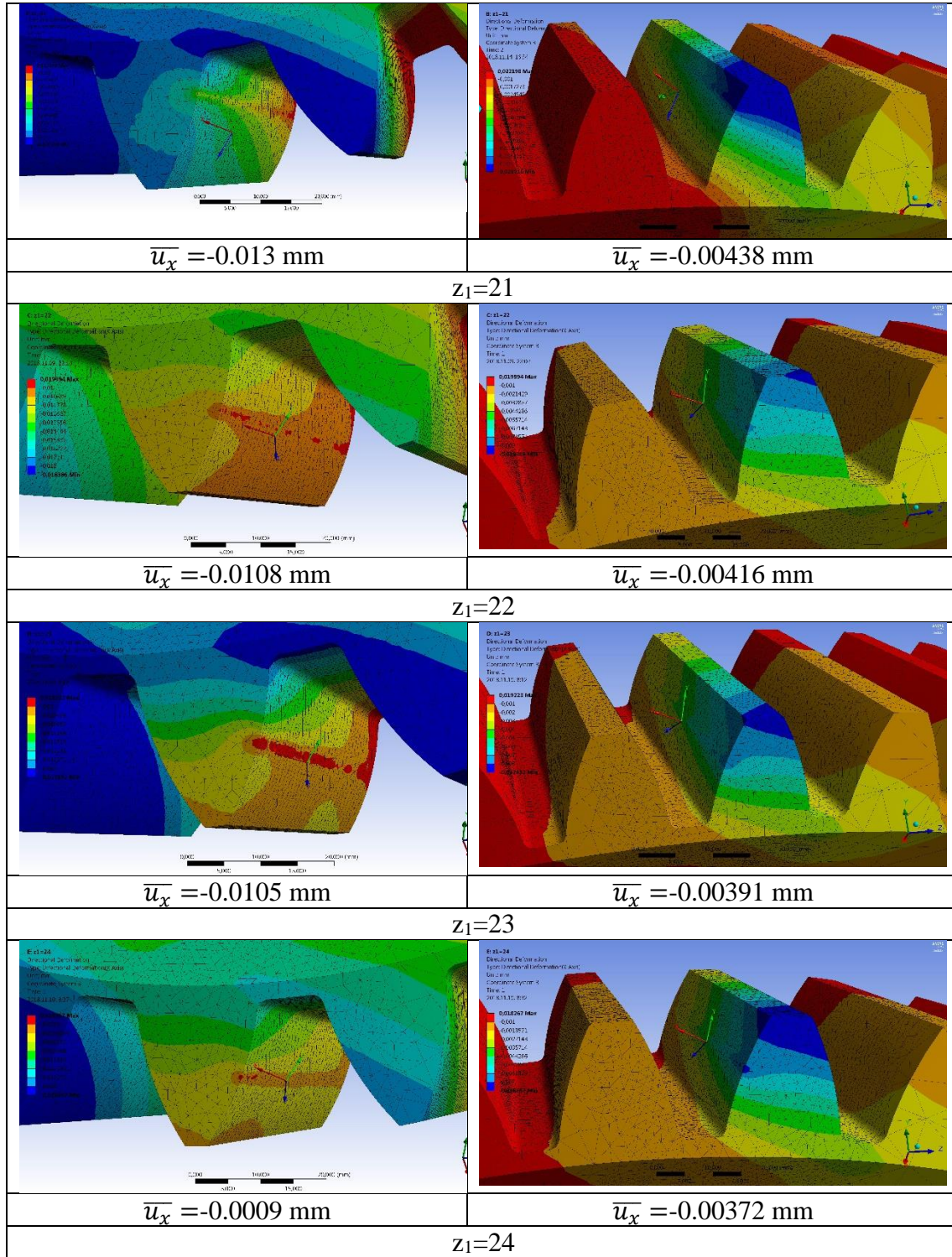


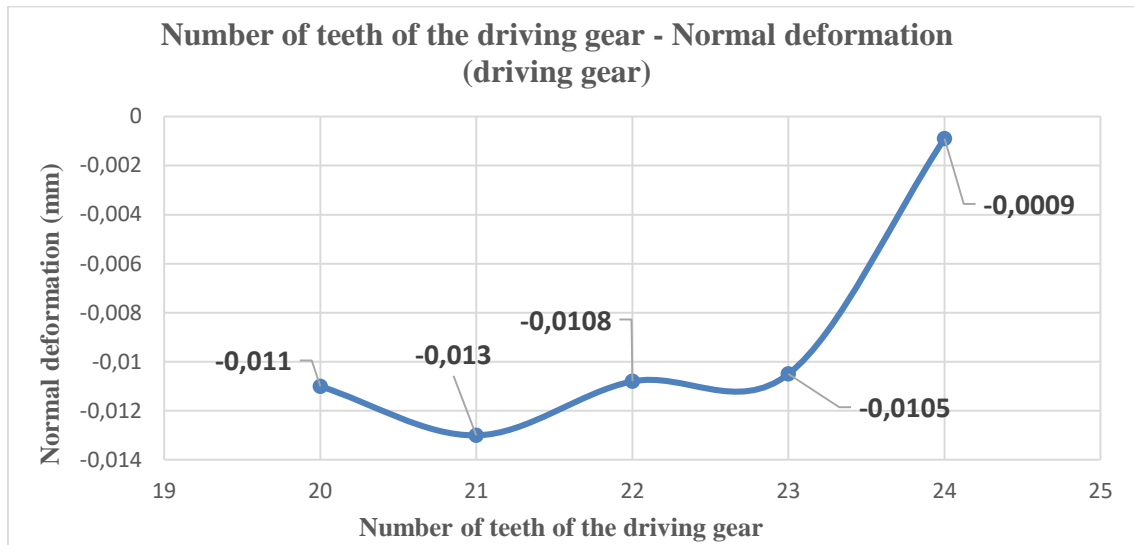
Figure 10. Normal deformation results for every bevel gear pairs

Based on the calculations the normal deformation could be seen in the function of the number of teeth of the driving bevel gear on Figure 10.

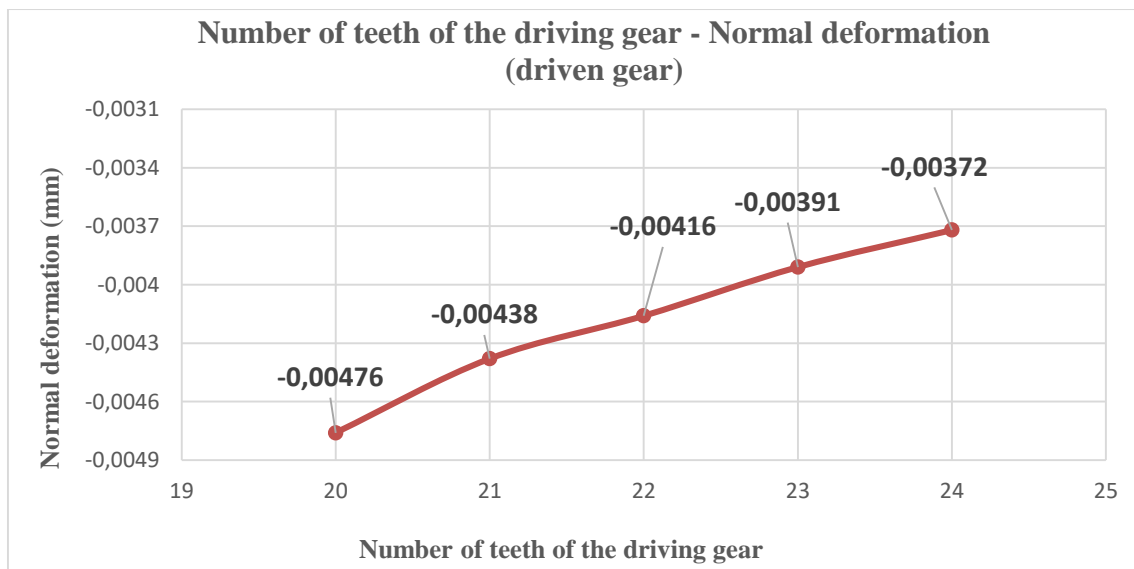
Based on the results the minimum normal deformation of the surface of the driving bevel gear is developed in case of  $z_1=24$  number of teeth in absolute value. The minimum normal deformation of the surface of the driven bevel gear is developed in case of  $z_1=24$  number of teeth in absolute value (Figure 11). That is why the increasing of the number



of teeth of the driving gear the normal deformation will be increased in case of both connecting tooth surfaces.



a) driving bevel gear



b) driven bevel gear

Figure 11. The developed normal elastic strain in the function of the number of teeth

## CONCLUSION

The aim of the research is the analysis of the modification of the number of teeth of the driving gear in the function of the TCA parameters (normal stress, normal, elastic strain and normal deformation).

Before the TCA the determination of the geometric parameters is needed. We have worked out a new-type computer aided software which is helped us for the facilitation of the calculation process. This software is determined the necessary geometric parameters



and the profile curves of the bevel gear pairs. After the process the CAD models of the designed gears have to be created. Knowing of the profile curves an interpolation B-spline is fitted into the calculated points.

Based on the CAD models of the analyzed gears the TCA analysis could be done. We have designed five types of bevel gear pairs. We have done the TCA analysis for every gear pairs.

The effect of the normal stress, normal elastic strain and the normal deformation has been analyzed between the contact surfaces. After the analysis we have evaluated the received results and created the functions separately for the driving gear and the driven gear. We could determine the consequences from this functions.

## ACKNOWLEDGEMENT

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