

Ferrofluid Flow in the Presence of Magnetic Field Above Stretching Sheet

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ABSTRACT

Magneto-thermomechanical interaction between heated viscous incompressible ferrofluid and a cold wall in the presence of a spatially varying magnetic field is investigated. A model for two dimensional ferrofluid flows is constructed. The influence of magnetic dipole in a Newtonian ferrofluid is characterized. The dynamic response of heated ferrofluid in a non-uniform magnetic field is studied along a nonlinearly stretching sheet with changing surface temperature. The nonlinearity is considered as power law relationship. Similarity transformation is applied to convert system of partial differential equations into ordinary differential equations and then solved numerically. The impact of the parameters on the velocity and temperature are explored numerically. It is found that the magneto-thermomechanical interaction slows down the motion of fluid particles. At the surface, the skin friction coefficient decreases and the rate of heat transfer increases with increasing ferromagnetic parameter or with increasing power law exponent.

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1. INTRODUCTION

Liquids are intensively investigated due to the necessity of developing high energy storage technologies during the last several decades. There is a high demand to design efficient heating and cooling industrial processes, heat exchangers, single-phase and two-phase thermal management systems. In all such applications it is important to provide the heat exchange between sets of contacting materials. It is required to enhance the heat transfer properties of traditional coolants.

Recently, among researchers several studies can be found in the literature on the application of

nanofluids to enhance the heat transfer rate in fluid flows [1-3].

Nanofluids are homogenous combinations of base fluid and nanoparticles. These suspensions are made of various metal or non-metal particles. Ferrofluid is a special type of nanofluid, where nanoparticles can be magnetized in the suspension. Because of several industrial applications, a number of research works are devoted to the investigation of magnetic fluids, see [4] and the references therein. Nanofluids having magnetic nanoparticles are useful in lubrications, industrial applications, medical treatment and directed drug release [5].

Rosensweig [6] has introduced that magnetization effects on the magnetic fluid flow. In the recent years the flow and heat transfer for Newtonian and non-Newtonian fluids along flat horizontal plates under different hydrodynamic and thermal boundary conditions have been considered extensively [2-4] Magneto hydrodynamic (MHD) flow near infinite plate and heat transfer has been investigated. Andersson and Valnes [8] firstly used ferrofluid flow by a stretching surface with magnetic dipole. It was shown by Andersson [8-9] that an external magnetic field has the same effect on the flow as the viscoelasticity.

MHD flow due to nonlinear stretching or shrinking surface is addressed in [11-13]. In [14] dual solutions were developed for flow past a stretching/shrinking sheet.

The goal of this work is to analyse the flow and heat transport of ferrofluids along a horizontal flat plate subjected to magnetic field. Self-similar solutions are obtained, and the impact of the parameters are investigated numerically

2. PROBLEM FORMULATION

Consider the steady two-dimensional boundary layer in a viscous incompressible electrically non-conducting Newtonian ferromagnetic fluid along an impermeable stretching sheet. The magnetic field is created by two line currents perpendicular to and directed out of the flow plane and equidistant a from the leading edge. The sheet is being stretched with a speed $u_w = U_w x^m$ proportional to the power of the distance from the fixed origin $x = 0$. The wall temperature is a decreasing temperature and is given by $T_w = T_c(1 + Ax^{m+1})$, where A and m are real constants and T_c denotes the Curie temperature as proposed in [16].

The scalar potential can be formulated as

$$\phi(x, y) = -\frac{I_0}{2\pi} \left(\tan^{-1} \frac{y+a}{x} + \tan^{-1} \frac{y-a}{x} \right), \quad (1)$$

where I_0 denotes the dipole moment per unit length and a is the distance of the line current from the leading edge. The negative gradient of ϕ provides the applied magnetic field, i.e. $\underline{H} = -\nabla\phi$. The corresponding field components can be written as follows

$$H_x = -\frac{\partial\phi}{\partial x} = -\frac{I_0}{2\pi} \left[\frac{y+a}{x^2+(y+a)^2} + \frac{y-a}{x^2+(y-a)^2} \right],$$

$$H_y = -\frac{\partial\phi}{\partial y} = -\frac{I_0}{2\pi} \left[\frac{x}{x^2+(y+a)^2} + \frac{x}{x^2+(y-a)^2} \right].$$

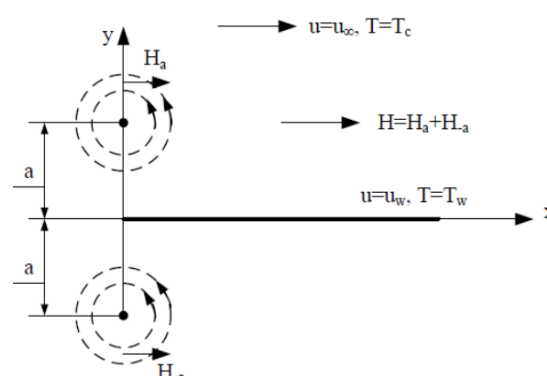


Fig. 1. The physical model in magnetic field.

Neuringer and Rosensweig showed that in ferrohydrodynamic interactions the existence of spatially varying fields is necessary [13]. Further we assume the followings:

- (i) the direction of magnetization of a fluid element is always in the direction of the local magnetic field,
- (ii) the fluid is electrically non-conducting and
- (iii) the displacement current is negligible.

Furthermore, we propose for the variation of magnetization with temperature a linear approximation equation. The dependence of $M = |M|$ on the temperature T is described by $M = K(T_c - T)$, where K is the pyromagnetic coefficient.

According to the above considerations the governing equations (conservation of mass, momentum and energy) of the boundary layer can be given as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{I_0 \mu_0 K}{\pi \rho} (T_c - T) \frac{1}{x^2} + \nu \frac{\partial^2 u}{\partial y^2}, \quad (3)$$

$$c \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2}, \quad (4)$$

where u and v are the parallel and normal velocity components to the plate, the x and y axes are taken parallel and perpendicular to the plate, respectively, ν is the kinematic viscosity and ρ denotes the density of the ambient fluid, which will be assumed constant. The system (2)-

(4) of nonlinear partial differential equations is considered under the boundary conditions at the surface ($y = 0$):

$$u(x, 0) = u_w, \quad v(x, 0) = 0, \quad T(x, 0) = T_w \quad (5)$$

with $u_w = U_w x^m$, $T_w = T_c(1 + Ax^\alpha)$ and

$$u(x, y) \rightarrow u_\infty, \quad T(x, y) \rightarrow T_\infty \quad (6)$$

as y leaves the boundary layer ($y \rightarrow \infty$) with ambient fluid temperature $T_\infty = T_c$, and u_∞ is the exterior streaming speed which is assumed throughout the paper to be $u_\infty = 0$. Parameter m is relating to the power law exponent. The parameter $\alpha = 1$ refers to a linear temperature profile and $m = 0$ to constant exterior streaming speed. In case of $\alpha = 2$, the temperature profile is quadratic and for $m = 1$ the streaming speed is linear. The value of $\alpha = 0$ corresponds to no temperature variation on the surface.

In the temperature range to be considered, the thermal heat capacity c , the thermal conductivity k , and the coefficient of viscosity ν are independent of temperature and are considered constants.

3. RESULTS AND DISCUSSION

Introducing the stream function ψ , defined by $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$, equation (2) is automatically satisfied. For the system (2)-(6) we apply similarity transformation to get similarity solution (see [10], [12-14])

$$\left. \begin{aligned} \psi(x, y) &= C_1 x^b f(\eta), \\ T &= T_c + Ax^\alpha \theta(\eta), \\ \eta &= C_2 x^d y. \end{aligned} \right\} \quad (7)$$

With the choice of

$$b = \frac{\alpha + 1}{4}, \quad d = \frac{\alpha - 3}{4}$$

$$C_1 = \sqrt{\frac{\nu U_w}{\lambda}}, \quad C_2 = \sqrt{\frac{U_w}{\nu \lambda}}$$

equations (3)-(4) and conditions (5)-(6) are simplified to the boundary value problem of the following system of coupled ordinary differential equations for $\alpha = 2m + 1$:

$$f''' - (\alpha - 1)f'^2 + \frac{\alpha + 1}{4}ff' - \frac{\beta}{\lambda^2}\theta = 0, \quad (8)$$

$$\theta'' + Pr \left[\frac{\alpha + 1}{4}f\theta' - \alpha\theta f' \right] = 0, \quad (9)$$

$$f(0) = 0, f'(0) = \lambda, \theta(0) = 1, \quad (10)$$

$$f'(\eta) = 0, \theta(\eta) = 0 \text{ as } \eta \rightarrow \infty, \quad (11)$$

where $\lambda = U_w/C_1C_2$, $Pr = c\nu/k$ is the Prandtl number and $\beta = I_0\mu_0KA/\pi\rho U_w^2$.

Applying the similarity solution f , the components of the non-dimensional velocity $\underline{v} = (u, v, 0)$ can be expressed by:

$$u = \frac{U_w}{\lambda} x^m f'(\eta), \quad (12)$$

$$v = -\sqrt{\frac{\nu U_w}{\lambda}} x^{\frac{m-1}{2}} \left(\frac{m+1}{2} f(\eta) + \frac{m-1}{2} \eta f'(\eta) \right). \quad (13)$$

The physical quantities, of practical concern, that specify the surface drag and heat transfer rate can be given as skin friction coefficient and local Nusselt number interpreted in the following form:

$$C_f = -\frac{2\tau_w}{\rho u_w^2}, \quad \tau_w = \rho\nu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad (14)$$

$$Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}, \quad (15)$$

where τ_w is the shear stress and q_w is the heat transfer at the wall.

Moreover, these quantities are derived by the drag coefficient $f''(0)$ and the $\theta'(0)$, respectively. The case when $m = 0$ corresponds to the case of the sheet moving at a constant speed.

According to our knowledge, the coupled boundary-layer equations for the case when $m = 0$ were first examined by Neuringer [16] when $u_w = 0$ and $u_\infty = U_\infty x^m$. If $m = 0$ and $\beta = 0$, equation (12) is equivalent to the well-known Blasius equation:

$$f''' + \frac{1}{2}ff' = 0, \quad (16)$$

which appears when we analyse a laminar boundary-layer problem for Newtonian fluids [13].

In the mathematical model of the magneto-thermomechanical effect between an incompressible ferrofluid and a flat wall in the presence of spatially varying magnetic field we consider the two-dimensional parallel flow near an infinite plate wall, an external magnetizing field is applied by placing a magnetic dipole pair perpendicularly to the wall. The wall temperature is prescribed by a function depending on the distance away from dipole.

The calculated solutions are valid only for distances greater than a .

4. SOLUTIONS AND RESULTS

The coupled non-linear two-point boundary value problem, eqs. (8) and (9), combined with the boundary condition (10)-(11) are solved numerically using Maple software. The numerical method is used to solve in the interval $\eta \leq \eta_{max}$, where η_{max} is the finite value of the similarity variable η for the far-field boundary conditions.

The effect of the ferromagnetic parameter β , parameters m and λ is represented graphically.

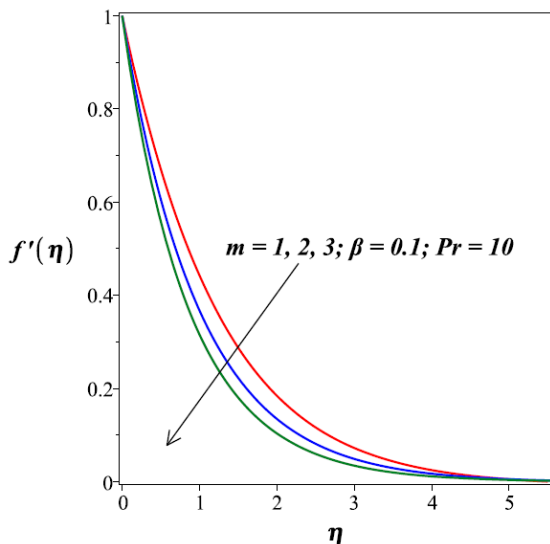


Fig. 2. Velocity distribution for various values of m ($\beta = 0.1, Pr = 10$ with $\lambda = 1$).

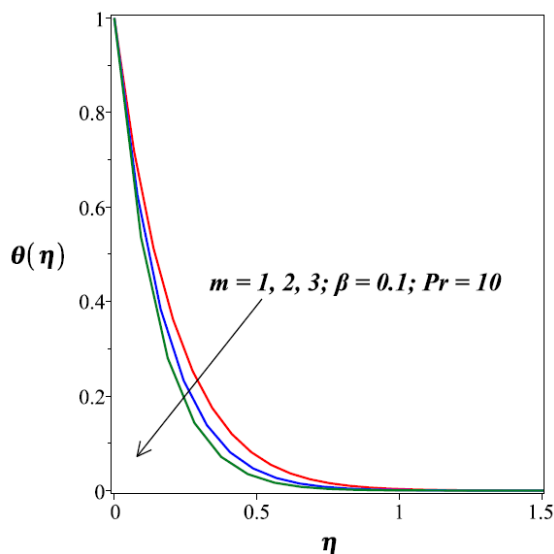


Fig. 3. Thermal distribution for various values of m ($\beta = 0.1, Pr = 10$ with $\lambda = 1$).

Figure 2 displays the velocity profile of fluid and with regard to the parameters of power-law exponent m . From the figure it is apparent that the boundary layer thickness decreases with an increase in the value of m .

The obtained solutions of thermal distribution can be seen on Fig. 4. Both the non-dimensional velocity and temperature solutions for constant wall velocity.

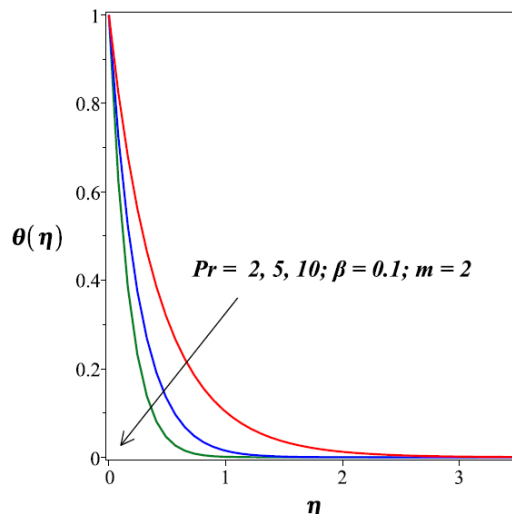


Fig. 4. Thermal distribution for various values of Pr ($\beta = 0.1, m = 2$ with $\lambda = 1$).

Temperature profiles are illustrated in Figs. 3-5, the increase of value of m leads to the decrease of the temperature profile of fluid phase. The impact of the Prandtl number is the same on the thermal distribution. We note that the thermal boundary layer thickness is smaller than the for the velocity distribution.

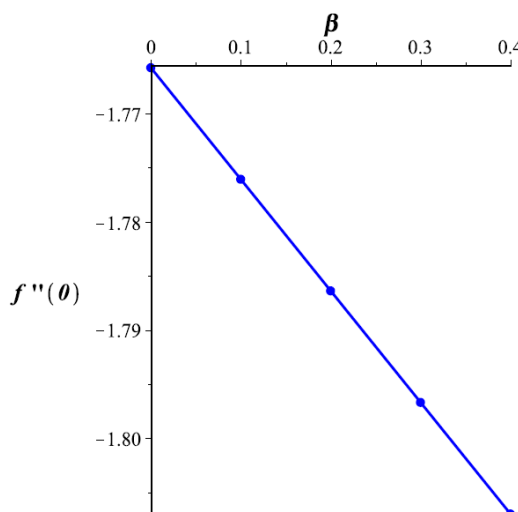


Fig. 5. The effect of β for the skin friction coefficient ($Pr = 10, m = 2, \lambda = 1$).

Figure 5 highlights the effect of the ferromagnetic parameter β on the shear stress at the wall which is proportional to $f''(0)$.

We also note that $f''(0)$ decreases with increasing ferromagnetic parameter or parameters m or λ .

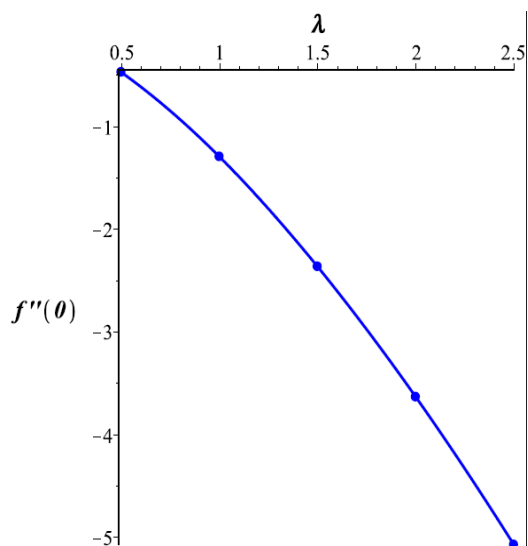


Fig. 6. The effect of λ for the skin friction coefficient ($Pr=10, m=1, \beta=0.1$).

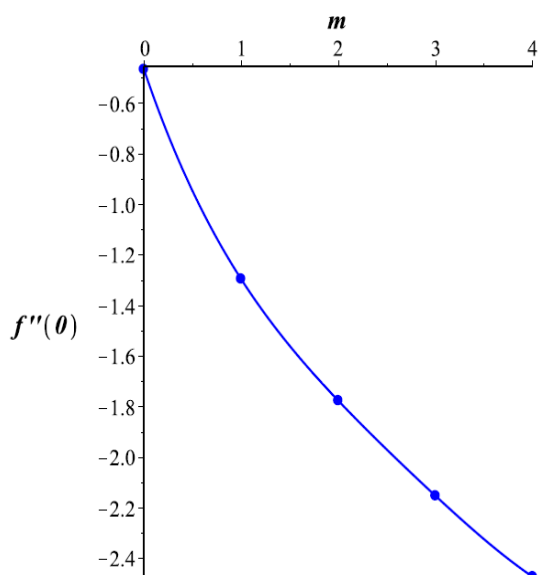


Fig. 7. The effect of m for the skin friction coefficient ($Pr=10, \beta=0.1, \lambda=1$).

Figures 8-10 provide the correlation of $\theta'(0)$ with various values of β, m and λ respectively. It shows the impact of these parameters on the local Nusselt number.

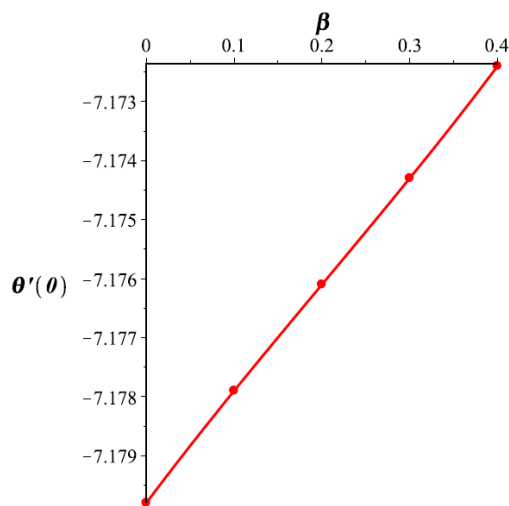


Fig. 8. The effect of β for the heat transfer at the wall ($Pr=10, m=2, \lambda=1$).

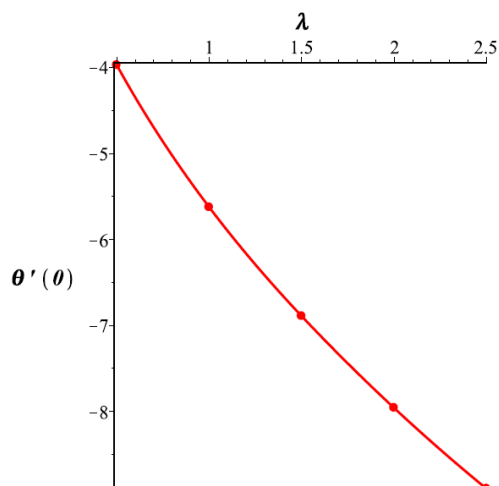


Fig. 9. The effect of λ for the heat transfer at the wall ($Pr=10, m=1, \beta=0.1$).

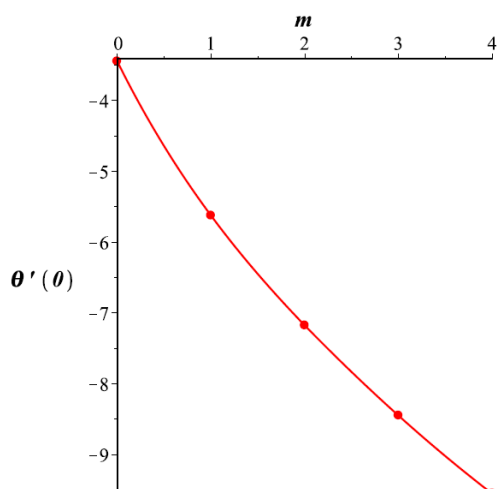


Fig. 10. The effect of m for the heat transfer at the wall ($Pr=10, \beta=0.1, \lambda=1$).

5. CONCLUSION

Here we studied a ferrofluid flow in magnetic field due to a stretching sheet which coincides with the plane $y = 0$ under boundary conditions. The stretching sheet has nonuniform temperature. We studied the magneto-thermomechanical effect between an incompressible ferrofluid and a flat plate wall in the presence of spatially varying magnetic field. An external magnetizing field is applied by placing a magnetic dipole pair perpendicularly to the wall. The sheet is stretched with a non-constant speed. The wall temperature is prescribed by a function depending on the distance away from dipole.

The coupled, nonlinear partial differential equations of the ferrofluid flow was transformed into a system of coupled and nonlinear ordinary differential equations by similarity analysis. Then, the boundary value problem was solved in Maple and it was presented how the modified magnetic field and stretching parameter influence together with boundary condition on the velocity and temperature profiles of fluid.

The findings of our study are as follows:

- A decrease in the momentum boundary layer thickness and in the thermal boundary layer thickness was observed for increasing values of m (Figs. 7 and 10).
- Increasing Prandtl number reduces the temperature (Fig. 4).
- Increasing the ferromagnetic parameter slows down the velocity and slightly increase the temperature in the boundary layer (Figs. 5 and 8).
- An increase in power law exponent m corresponds a decrease in the temperature (Fig. 3).

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