



## MATHEMATICAL MODELLING OF CYLINDRICAL CHIMNEY EFFECT IN SOLAR DRYER

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### Abstract

A simple mathematical model of a solar chimney is proposed for the solar dryer. The physical model was made with a cylindrical channel. Outer surface of the chimney is provided with black paint which absorbing solar radiation to heat up the air inside the chimney. Openings provided at the bottom and top of the chimney allow moist air to enter from the drying chamber and leave the channel. Steady state heat transfer equations were set determining the heat transfer coefficient and the air flow in the chimney. The thermal performance of the solar chimney as determined from the surfaces, air temperatures, and air mass flow rate are presented.

### Keywords

buoyancy effect, heat transfer, natural convection, solar chimney

### Introduction

The systematic use of renewable energy reduces the dependency on energy supply from conventional energy sources. Solar energy can be used directly (photovoltaic cell, solar thermal collectors) or indirectly (passive cooling technique) to convert into usable source of energy. The concept of solar energy use in the chimney was proposed in 1960 by Trombe and Michel at the C.N.R.S. laboratory in France [1]. Natural convection solar dryers are normally reported poor in performance than forced solar dryers. The poor ventilation in the natural dryer leads to exceed the required temperature in the drying chamber, which in turns burn the crop instead of drying. This problem is overcome and enhanced by introducing

the chimney on top of the drying chamber. The presence of solar chimney has shown its benefit in accelerated movement of moist air from the drying chamber and thus reducing the drying time for intended products. Ventilation by natural convection makes use of the fact that hot air has lower density than cold air and thus tend to rise.

The solar chimney is designed to sustain a smooth flow of air inside the solar dryer device, which use incident solar radiation. Its performance is primarily described by the induced airflow rate and air temperature in the channel. Fisher and Torrance 1999 [2] reported experimental results of chimney enhanced heat transfer from a vertical parallel plate heat sink. Their result showed that chimneys prove significant heat transfer enhancement. [3] developed a solar chimney-based drying system with the porous absorber to check the heat transfer and air flow in the system. They found a correlation between the airflow speed and the temperature difference between the outlet and the inlet due to the natural ventilation enhanced by the chimney. To enhance the performance, a solar chimney is suggested with possible chimney height, solar radiation, equal inlet and outlet, ambient air temperature, a height/gap ratio, and a solar absorber with larger absorptivity and emissivity [4]. A comparison of thermal behavior of a solar chimney with a conventional one reported by Afonso and Oliveira, 2000 [5]. In their study, thermal model and simulation on a solar chimney were applied with the assumption of unsteady state one-dimensional heat transfer using a finite difference model to the chimney brick wall and allowing heat transfer coefficients to vary along the chimney height. Their results showed that there was a significant increase in ventilation rate with the solar chimney

than a conventional one. A steady state mathematical model for a solar chimney was developed by Bansal et. al. in 1993 [6]. The system consists of a solar collector connected to a conventional chimney. The result showed that a collector area of 2.25 m<sup>2</sup> was able to induce an air flow rate of between 140 (m<sup>3</sup>/hr) to 330 (m<sup>3</sup>/hr) at a solar radiation of between 200 (W/m<sup>2</sup>) to 1000 (W/m<sup>2</sup>), respectively.

According to the passive passive convection application by using a chimney system, determination of heat transfer coefficient and Nusselt number are important in the investigation of free convection, mathematical simulation and application of the system. The determination of the correlation between Nusselt, Grashof and Rayleigh numbers in the vertical duct has been carried out in many research studies [7]. Heat transfer processes in solar chimney involving: internal convection heat transfer, external heat transfer to the surrounding and conduction heat transfer. This paper studies heat transfer processes on solar chimney part of the indirect passive solar dryer applying a simple mathematical approach.

### System description and mathematical model

The indirect passive solar dryer with the chimney under investigation is schematically shown in Fig. 1

(a) and Fig. 1 (b) is the complete structure of a chimney under investigation. The system of 4.20 m high consists of a cylindrical chimney, a drying chamber, and a solar air collector. The chimney was firmly fixed onto sloping metal base and mounted on the drying chamber. A metal cap was fixed onto the top of the chimney to keep out the rain. A sufficient duct length is considered to ensure thermally fully developed condition at the duct outlet. The chimney outside surface was painted matt black (absorptivity 0.87 and emissivity 0.9) to obtain elevated temperature to improve air flow and increase buoyancy or “chimney effect”. This is because the absorber surface absorbing more solar radiation when radiated on the surface [8]. Khanal and Lei in 2010 [9] found that the airflow rate is enhanced with a higher surface emissivity applying numerical study. These two studies showed that the heat transfer by radiation cannot be ignored for a solar chimney. Moist air enters the chimney inlet with a temperature equal to the temperature of air in the drying chamber and assumed constant. Heated moist air at the outlet temperature from the top of the chimney. Temperature of the chimney wall and mean moist air in the chimney are all assumed to be uniform. Resistance to flow due to friction along the surfaces are assumed negligible.

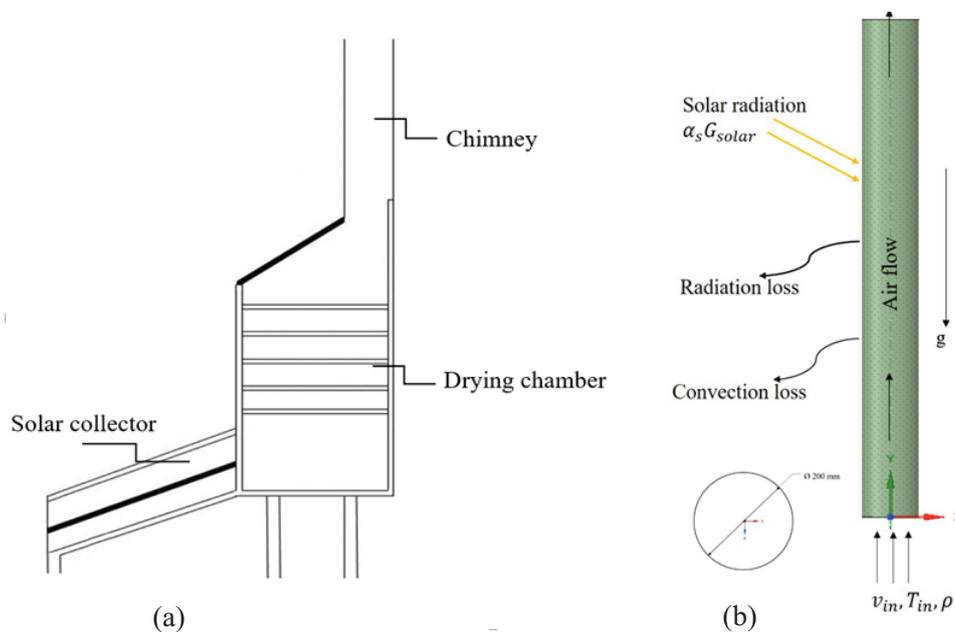


Figure 1. (a) Schematic diagram of a solar chimney dryer; (b) model of a solar chimney.

The value of the heat transfer coefficient for internal and external portion of the chimney treated separately. The heat transfer through the wall of the chimney cause buoyancy-driven flow. Assumption

were made as steady operating condition exist, constant heat flux at the wall, uniform inlet velocity from the drying chamber, and the physical properties of moist air were to vary linearly with temperature

and evaluated for conditions at duct inlet. Sky temperature, ambient temperature and solar daily radiation are taken as average values from environmental weather data. The chimney inlet temperature and airflow rate are also known. The boundary conditions for these difference equations for constant heat flux from the wall to the moist air is given at chimney inlet condition.

$$y = 0; \quad u(0, y) = V_{in};$$

$$T(0, y) = T_{in}; \quad \rho = f(T, x)$$

The mathematical equations governing natural convection flow and heat transfer in a vertical, cylindrical chimney are the Navier-Stokes equations and the energy Eq. (1). Choosing the origin at the outlet of the drying chamber or at the bottom plane of the chimney, the y axis is directed vertically upward along the length of the chimney. The governing equations under the assumptions stated in the last section are:

$$\frac{1}{r} \frac{\partial ru}{\partial r} + \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0, \quad (1a)$$

$$-\frac{v^2}{r} = G_r - \frac{\partial P}{\partial r} + v \left( \nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2} \frac{\partial v}{\partial \theta} \right), \quad (1b)$$

$$-\frac{wv}{r} = G_\theta - \frac{\partial P}{r \partial \theta} + v \left( \nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2} \frac{\partial u}{\partial \theta} \right), \quad (1c)$$

$$G_z - \frac{\partial P}{\partial z} + v(\nabla^2 w) = 0, \quad (1d)$$

$$u \frac{\partial T}{\partial r} + \frac{v}{r} \frac{\partial T}{\partial \theta} + w \frac{\partial T}{\partial z} = \alpha \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} \right],$$

$$\text{where } \alpha = \frac{k}{\rho c_p}. \quad (1e)$$

From Eq. (2), the net rate of solar energy absorbed by the surface of the chimney is calculated as the difference of heat gain and heat losses from the surface. The heat transfer loss from the outside surface of the chimney to the ambient is due to the convection and radiation transfer. The convective and radiative heat flux are calculated respectively from Eq. (4) according to Newton's law of cooling, and in Eq. (5).

$$q_{net} = q_{gain} - q_{loss} \quad (2)$$

The heat gained from the solar radiation to the surface of the chimney is:

$$q_{gain} = \alpha_s G_{solar}. \quad (3)$$

The convective heat flux from the surface to the ambient air according to Newton's law of cooling:

$$q_c = h_o(T_s - T_a). \quad (4)$$

The radiation heat flux is:

$$q_r = \varepsilon \sigma (T_s^4 - T_{sky}^4). \quad (5)$$

The net rate of energy absorbed by the surface of the chimney is then written as:

$$q_{net} = \alpha_s G_{solar} - [(\varepsilon \sigma (T_s^4 - T_{sky}^4) + h_o(T_s - T_a))] \quad (6)$$

Sky temperature  $T_{sky}$  can be estimated from Eq. (7) [10].

$$T_{sky} = 0.0552 T_a^{1.5}. \quad (7)$$

The analysis was based on the dimensionless numbers commonly used in natural convection processes. The local Rayleigh, Prandtl and Nusselt numbers are calculated, respectively. The solution of the governing conservation equations in their dimensionless form depends on the Rayleigh number based either on the chimney height:

$$Ra_L = Gr_L Pr = \frac{g \beta \Delta T L^3}{\nu^2} Pr, \quad (8)$$

where Prandtl number is given as:

$$Pr = \frac{c_p \mu}{K}. \quad (9)$$

### Heat transfer coefficient calculation

The procedure for calculating local heat transfer coefficient inside and outside the chimney surface Eq. (8) and Eq. (10) are applied. The air density inside the system is depend on air temperature and humidity ratio of moist air entering at the chimney inlet and expressed by Eq. (10)[11]. The nature of heat transfer either forced or natural is checked by the ratio of Grashof number to square Reynold number. During the calculation of the heat transfer coefficient we have to take into consideration that the black painted heat absorbing surface heats the ambient air, instead of direct exposure to solar radiation.

$$\rho = \rho_o \left[ \frac{1}{1 + T/273} \right] (1 + x). \quad (10)$$

The heat transfer processes taking place on the external surface of the chimney are rather more complex than is the case for the inside surface coefficient. Radiation heat transfer does have a part to play. The surface of the chimney will exchange heat by radiation to the surrounding environment, and the convective heat loss will be affected by the prevailing wind speed. The effect of wind speed was neglected in this case.

The local Rayleigh number and more specifically the Grashof number, where  $Gr_L = Ra_L / Pr$  is usually used in heat transfer for the definition of the flow regime to be laminar or turbulent.  $Ra$  is  $< 10^8$  indicate

a buoyancy-induced laminar flow, with transition to turbulence occurring over the range of  $10^8 < Ra < 10^{10}$ . An important characteristic of the flow is the rate of heat transfer through the chimney surfaces. Using

Newton's law of cooling for the local convection coefficient  $h$  the Nusselt number for free convection from outer and inner chimney walls may be expressed by Churchill and Chu [12] Eq.11.

$$Nu_L = \frac{hL}{K} = \left\{ 0.825 + \frac{0.387Ra^{1/6}}{[1+(0.437/Pr)^{9/16}]^{8/27}} \right\}^2 \text{ where } h = \frac{q}{\Delta T} \text{ and } q = \int \frac{\partial T}{\partial x} dy \cdot \quad (11)$$

The air properties in Eq. (8) and Eq. (9) are evaluated at the film temperature written as a function of the bulk temperature, that is, for internal chimney and external chimney written as  $T_{film_i} = (T_s + T_i)/2$ , and  $T_{film_o} = (T_s + T_a)/2$  respectively. The values of thermal conductivity, Prandtl number and kinematics viscosity were reading directly for table given in any heat books.

### Results and discussion

Knowledge of the climatic conditions and availability of solar radiation is a prerequisite in solar drying. Fig. 2 presents the average hourly solar radiation

measured during the four days. It can be seen that higher radiation occurred between 11:00 to 12:00. This gives a higher temperature observed in the solar chimney and air velocity through the dryer was thus improved; which resulted in a lower temperature rise in the drying chamber. therefore, covering of the external surface of the chimney with black painted improved the thermal performance the system. Fig. 3 demonstrate the outlet airflow velocity along the chimney height with CFD analysis based on finite volume method under steady state condition. This result showed the chimney height with thermal input from the sun can enhance the movement of air inside the system.

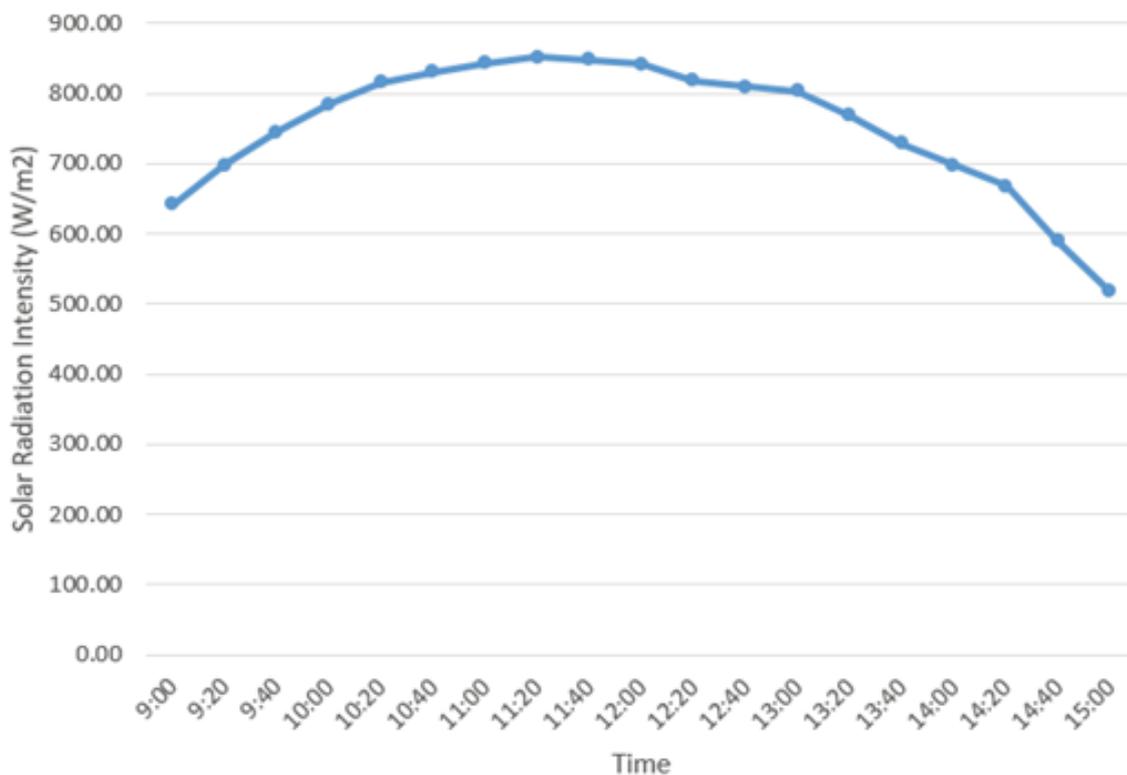


Figure 2. Average hourly solar radiation incident.

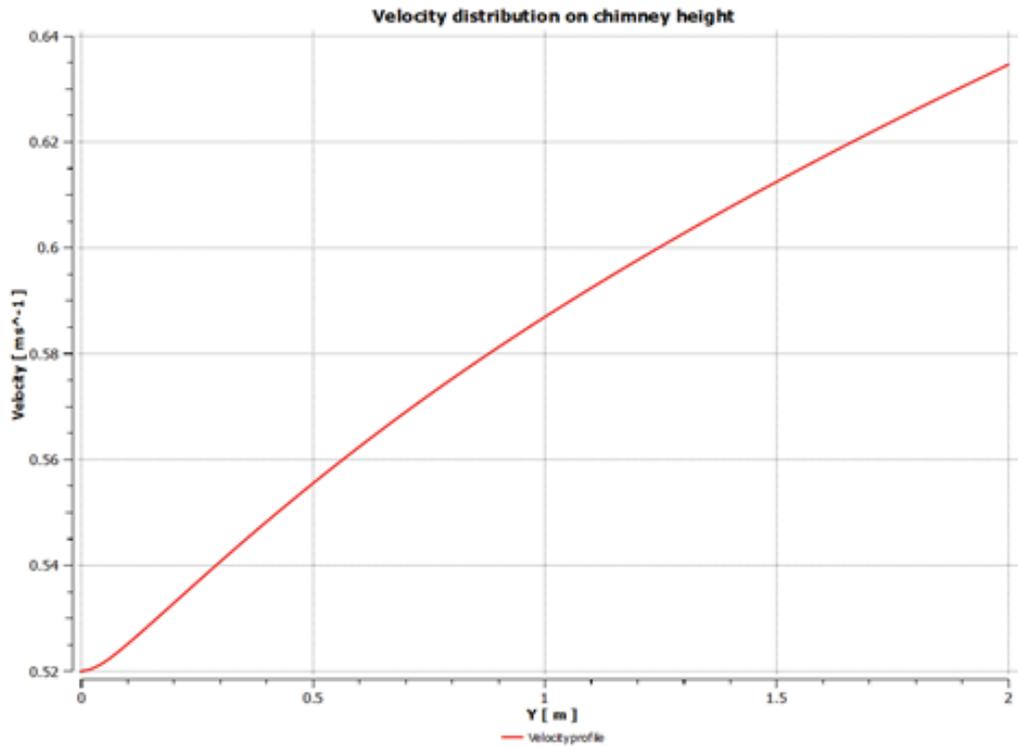


Figure 3. Variation of airflow velocity along the chimney height.

## Conclusions

In this study, the comparative analysis of heat transfer coefficient for a solar chimney applied in solar drying application analysed using a simple mathematical model. The result showed that the chimney with black painted outer surface enhanced the absorbability of solar radiation in the surface. This in turns increased the temperature of the moist air inside the chimney. Moreover, the airflow rate increased along the chimney height. The performance of a solar chimney has been predicted using a simple model. Most heat transfer correlations in natural convection are based on experimental measurements. Therefore, further studies on experimental work is in progress.

## Nomenclature

$V_{in}$	airflow velocity at inlet	$ms^{-1}$
$T_{in}$	moist air temperature at the inlet	K
$G_{solar}$	solar radiation incident	W
$k$	thermal conductivity of air	$Wm^{-1}K^{-1}$
$C_p$	specific heat capacity of air	$Jkg^{-1}K^{-1}$
$T_s$	chimney wall temperature	K
$T_{sky}$	sky temperature	$^{\circ}C$
$T_a$	ambient temperature	$^{\circ}C$
$h_o$	outside heat transfer coefficient	$Wm^2K^{-1}$
$g$	gravitational acceleration	$ms^{-2}$
$x$	humidity ratio	

$N_u$	Nusselt number
$Ra_L$	Rayleigh number
$Gr_L$	Grashof number
$Pr$	Prandtl number

## Greek Letters

$\nu$	kinematic viscosity	$m^2s^{-1}$
$\mu$	dynamics viscosity	$kgm^{-1}s^{-1}$
$\rho$	moist air density	$kgm^{-3}$
$\rho_{air}$	air density	$kgm^{-3}$
$\sigma$	Stephan-Boltzmann constant	$5.67 \cdot 10^{-8} Js^{-1}m^{-2}K^{-4}$
$\beta$	coefficient of volume expansion	$K^{-1}$
$\varepsilon$	surface emissivity	
$\alpha_s$	surface absorptivity	

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