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TESTING THE NOISE REJECTION CAPABILITY OF THE INVERSION BASED FOURIER TRANSFORMATION ALGORITHM APPLIED TO 2D SYNTHETIC GEOMAGNETIC DATASETS

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Abstract: For signal processing, different algorithms can be applied to enhance the quality of measured datasets that contain simple or complex noises during the field survey. Treating these noisy data can be done using the discrete Fourier transform (DFT based noise filtering) which converts the data from time to a frequency domain but in some cases is not preferable due to its low noise suppression capability. Therefore, a robust and effective 2D inversion called the iteratively reweighted least-squares Fourier transformation (IRLS-FT) method is applied. In the framework of this inversion, the continuous Fourier spectrum is discretized using the series expansion to solve our inverse problem in the form of the expansion coefficients. Moreover, the Hermite functions are used as basis functions with the distinguishing feature of the Fourier transform eigenfunctions to facilitate and speed up the calculation of the Jacobian matrix without complex integration. In the robust inversion studied in the article, the Steiner weights are calculated through an internal iteration loop instead of Cauchy weights to overcome the problem of scale parameters. In this paper, the 2D IRLS-FT inversion method is applied to synthetic magnetic datasets and their reduction to the pole. The results demonstrated that the method is very stable during the procedures as well as its robustness, resistance, and effectiveness in the process of noise rejection.

Keywords: *Fourier Transformation, Inversion, Series Expansion, Cauchy Noise, Reduction to Pole*

1. INTRODUCTION

Most of the geophysical geomagnetic datasets measured during a field survey are likely to contain different amounts of noise; some are caused by the effects of the diurnal variation in the Earth's magnetic field on the geophysical equipment, a situation encountered by many geophysicists during data acquisition. These noises strongly affect the processing and interpretation of the measured geophysical data. This demands the search for an effective process to reduce the noise prevalence. Fourier transform based filtering is considered to be one of the most important procedures to achieve such aims.

In signal processing the discrete Fourier transform, which is referred to as DFT, can convert discrete noiseless or noisy time domain data sets to a discrete frequency domain. It can be considered as an implemented procedure of the fast Fourier transform (FFT). Actually, there are several applications of the discrete Fourier transform (DFT) such as calculating the polynomial multiplication, obtaining numerous targets through the radar echoes, correlation analysis, and spectral analysis, as well as estimating the signal's frequency spectrum, which is our main geophysical goal in this paper. For the purpose of noise reduction, Dobróka et al. [1] handled a one-dimensional (1D) inversion based Fourier transformation (S-IRLS-FT) method which was developed and generalized to two-dimensional (2D) inversion, providing accurate and efficient results in the field of the reduction to the pole of the magnetic data sets [2]. Therefore, this study sheds light on the robust 2D inversion method, where the Fourier transformation is considered to be the solution of the overdetermined inverse problem. It is known that the optimal solution in the case of Gaussian noisy data can be estimated by using the simple or weighted least square method. But when dealing with more complicated noisy data such as outliers, the inversion-based FT method, which depends mainly on the iteratively reweighted least-squares Fourier transformation (2D IRLS-FT) method, plays an important role. In this projective filtering, the series expansion method can be used to discretize the continuous Fourier spectrum and calculate the expansion coefficients as a solution of the overdetermined inverse problem. Moreover, the Hermite functions are employed as the basis functions. The Jacobian matrix needs more time to be calculated where integration is required. This problem can be solved by using the Hermite functions as eigenfunctions of the Fourier transformation, which allow quick and accurate determination of the elements of the Jacobian matrix. It is demonstrated that the inversion of iteratively reweighted least square Fourier transformation (IRLS-FT) method can recalculate the Cauchy weights, but in this case the scale parameters should be known a priori. To solve this problem, the Steiner weights are calculated through an internal iteration loop instead of Cauchy weights by using the Steiner most frequent value (MFV) method [3, 4]. This process makes the inversion more robust and resistant. In this paper, the 2D-IRLS-FT method is applied to equidistant 2D synthetic magnetic data sets to test the noise rejection capability as well as the stability of the inversion procedures.

2. 2D algorithm of the IRLS Fourier transformation

In the framework of signal processing and interpretation enhancing methods, the algorithm of the Fourier transformation was further developed by the Department of Geophysics, University of Miskolc, Hungary. This algorithm was first handled in one-dimensional and generalized to two-dimensional Fourier transform. To convert from the space domain data to frequency domain data or vice versa, the Fourier transformation and its inverse are applied respectively. The two-dimensional Fourier transform can be written as:

$$U\left(\omega_x, \omega_y\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u\,(x, y)\; e^{-j(\omega_x x + \omega_y y)}\; dx\, dy, \tag{1}$$

where x and y are the spatial coordinates, ω_x and ω_y are the angular frequencies, and j is the imaginary unit. The 2D space domain data sets can be obtained as a result of the 2D inverse Fourier transformation as the following:

$$u\,(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U\left(\omega_x,\omega_y\right) e^{j(\omega_x x+\omega_y y)}\, d\omega_x\, d\omega_y. \quad (2)$$

$U\left(\omega_x,\omega_y\right)$ is the 2D frequency spectrum which has to be discretized to facilitate the procedures of data processing and interpretation. Therefore, the method of series expansion is used to discretize the continuous Fourier spectrum by the following formula:

$$U\left(\omega_x,\omega_y\right) = \sum_{n=1}^{N}\sum_{m=1}^{M} B_{n,m}\, \Psi_{n,m}\left(\omega_x,\omega_y\right), \quad (3)$$

where $B_{n,m}$ are the expansion coefficients that are calculated as the solution to the overdetermined inverse problem and $\Psi_{n,m}\left(\omega_x,\omega_y\right)$ are the basis functions. Inserting *Equation (3)* into *Equation (2)*, the theoretical data can be defined as:

$$u\,(x_k,y_l)^{theor} = \sum_{i=1}^{M} B_{n,m} \frac{1}{2\pi}\int_{-\infty}^{\infty} \Psi_{n,m}\left(\omega_x,\omega_y\right) e^{j(\omega_x x+\omega_y y)}\, d\omega_x\, d\omega_y, \quad (4)$$

where the following term

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \Psi_{n,m}\left(\omega_x,\omega_y\right) e^{j(\omega_x x+\omega_y y)}\, d\omega_x\, d\omega_y$$

is an element of the Jacobian matrix $G_{k,l}^{n,m}$. Therefore, the Jacobian matrix can be considered as the inverse Fourier transform of the basis function and hence the theoretical data can be summarized as:

$$u\,(x_k,y_l)^{theor} = \sum_{n=1}^{N}\sum_{m=1}^{M} B_{n,m}\, G_{k,l}^{n,m}, \quad (5)$$

where $k = 1,2,\dots,K$ and $l = 1,2,\dots,L$ are the sequence numbers of the measurement points in x and y directions, respectively. Because of the fact that the frequency spectrum is given as an integral in the interval (-∞,∞), the basis functions should be given in the same domain, which can be achieved by using the Hermite functions and polynomials. The mathematical background of the Hermite polynomials was established by Laplace [5] and made more sophisticated by Hermite [6] to include the multidimensional polynomials. The scaled Hermite functions which are eigenfunctions of the inverse Fourier transformation can be defined as:

$$H_n(\omega_x,\alpha) = \frac{e^{\frac{-\alpha\omega_x^2}{2}} h_n(\omega_x,\alpha)}{\sqrt{\sqrt{\frac{\pi}{\alpha}}\, n!\,(2\alpha)^n}} \quad \text{where } h_n(\omega_x,\alpha) = (-1)^n\, e^{\alpha\,\omega_x^2} \left(\frac{d}{d\omega_x}\right)^n e^{-\alpha\,\omega_x^2} \quad (6)$$

$$H_m(\omega_y, \beta) = \frac{e^{\frac{-\beta\omega_y^2}{2}} h_m(\omega_y,\beta)}{\sqrt{\sqrt{\frac{\pi}{\beta}}\, m!\, (2\beta)^m}} \qquad \text{where } h_m(\omega_y, \beta) = (-1)^m\, e^{\beta}\, \omega_y^2 \left(\frac{d}{d\omega_y}\right)^m e^{-\beta}\, \omega_y^2, \quad (7)$$

where α and β are the scale factors. In that case, the Jacobian matrix can be written as:

$$G_{k,l}^{n,m} = \frac{(j)^{n+m}}{\sqrt[4]{\alpha\beta}}\, H_n^{(0)} \left(\frac{x_k}{\sqrt{\alpha}}\right) H_m^{(0)} \left(\frac{y_l}{\sqrt{\beta}}\right), \quad (8)$$

where $H_n^{(0)}$ and $H_m^{(0)}$ are the non-scaled Hermite functions. By applying *Equation (8)*, a fast solution to the forward problem can be provided using *Equation (5)*. It is important to note that the total number of the unknown series expansion coefficients can be calculated according to $= N + (M - 1)N = NM$, which is the same for the measured data as $S = K + (L - 1)K = KL$. It can be seen that the Jacobian matrix does not include integration, which makes the inversion procedures faster and less time-consuming. For simplification, the notations $u\,(x_k\,, y_l) = u_s$, $B_{n,m} = B_i$ and $u_s = \sum_{i=1}^{I} B_i\, G_{s,i}$ can be used to estimate the general element of the deviation vector using the following formula:

$$e_s = u_s^{measured} - \sum_{i=1}^{I} B_i\, G_{s,i}, \quad (9)$$

where $i = 1, \dots, I$ and $s = 1, \dots, S$. By calculating the deviation vector, the inverse problem can be continued in a straightforward manner.

3. THEORETICAL ALGORITHMS OF THE INVERSION METHOD

As a method of linearized geophysical inversion, the least-squares method can be used to solve sets of linear equations for quick procedures. Legendre [7] was the first to publish a clear and concise exposition of the least-squares method. It is well known that the least-squares inversion method is more effective when dealing with data of Gaussian noise (regular noise). It can be used to calculate the misfit between the measured and predicted data by estimating their deviation vector from the following formula:

$$E_2 = \sum_{k=1}^{N} e_k^2. \quad (10)$$

This L_2 norm of the deviation vector can be minimized to give the normal equations:

$$\underline{\underline{G}}^{T}\, \underline{\underline{G}}\, \vec{B} = \underline{\underline{G}}^{T}\, \vec{u}^{measured}. \quad (11)$$

Actually, most of the data measured during the field survey include different amounts of irregular noise such as outliers. For this reason, the deviation vector of another norm called the weighted norm is minimized to obtain stable and quick inversion procedures like the following:

$$E_w = \sum_{k=1}^{N} w_k \, e_k^2, \tag{12}$$

where w_k is defined as Cauchy weights, which can be given as:

$$w_k = \frac{\sigma^2}{\sigma^2 + e_k^2}. \tag{13}$$

But in this case, the inversion procedures can be continued only when the scale parameter (σ^2) is known a priori. The solution of this problem can be realized by applying the most frequent value (MFV) method of Steiner to estimate the Steiner weights through an internal iteration loop using the following formula:

$$\varepsilon_{j+1}^2 = 3\,\frac{\sum_{k=1}^{N} \frac{e_k^2}{\left(\varepsilon_j^2 + e_k^2\right)^2}}{\sum_{s=1}^{N}\left(\frac{1}{\varepsilon_j^2 + e_s^2}\right)^2}. \tag{14}$$

It is obvious that at the starting step ($j = 0$), the Steiner scale factor $\varepsilon_0 \leq \frac{\sqrt{3}}{2}\,(e_{max} - e_{min})$ can be used to calculate the Steiner scale factor of the next step ε_{j+1}^2 and this process continues until the stop-criterion is met, which can be also achieved by using a fixed iterations number. Therefore, the Cauchy weights can be replaced by the Cauchy-Steiner weights as the following:

$$w_k = \frac{\varepsilon^2}{\varepsilon^2 + e_k^2}. \tag{15}$$

It is important to note that the inverse problem has become nonlinear because of using the Cauchy-Steiner weights; thus, its solution can be obtained sequentially by implementing the iteratively reweighted least-squares (IRLS) method. First, the elements of the series expansion coefficients at the 0th order step ($\overrightarrow{B}^{(0)}$) are generated from the non-weighted least-squares method to calculate the predicted data from $u_k^{(0)} = \sum_{i=1}^{M} B_i^{(0)}\, G_{ki}$ and in that case, the deviation vector can be estimated from $e_k^{(0)} = u_k^{measured} - u_k^{(0)}$. Then the weights are given as:

$$w_k^{(0)} = \frac{\varepsilon^2}{\varepsilon^2 + \left(e_k^{(0)}\right)^2}. \tag{16}$$

Now the misfit between the measured and calculated data can be determined at the first iteration step as the following:

$$E_w^{(1)} = \sum_{k=1}^{N} w_k^{(0)}\, e_k^{(1)^2}. \tag{17}$$

The minimization of *Equation (17)* leads to the following linear normalized equations as:

$$\underline{\underline{G}}^{T}\, \underline{\underline{W}}^{(0)}\, \underline{\underline{G}}\, \overrightarrow{B}^{(1)} = \underline{\underline{G}}^{T}\, \underline{\underline{W}}^{(0)}\, \overrightarrow{u}^{measured}, \tag{18}$$

where $\underline{\underline{W}}^{(0)}$ is the weighting matrix. It can be seen that the expansion coefficients at the 1st iteration step ($\overrightarrow{B^{(1)}}$) are calculated from *Equation (18)* of linear weighted least-squares method, which can be used to estimate new predicted data from $u_k^{(1)} = \sum_{i=1}^{M} B_i^{(1)} G_{ki}$, and hence the deviation vector is $e_k^{(1)} = u_k^{measured} - u_k^{(1)}$. Again, the weights can be calculated from $w_k^{(1)} = \frac{\varepsilon^2}{\varepsilon^2 + \left(e_k^{(1)}\right)^2}$ which is required to compute the new misfit function $E_w^{(2)} = \sum_{k=1}^{N} w_k^{(1)} e_k^{(2)^2}$ and obtain $\overrightarrow{B^{(2)}}$ through its minimization. This process is continued and repeated over the jth iteration step as the following:

$$\underline{\underline{G}}^{T}\, \underline{\underline{W}}^{(j-1)}\, \underline{\underline{G}}\, \overrightarrow{B^{(j)}} = \underline{\underline{G}}^{T}\, \underline{\underline{W}}^{(j-1)}\, \overrightarrow{u^{measured}} \,. \tag{19}$$

When the criterion is met at the last iteration step, the series expansion coefficients are accepted as a solution to the inverse problem.

4. A GEOPHYSICAL APPLICATION OF THE 2D IRLS-FT INVERSION METHOD

In general, one of the most important applications of the Fourier transformation is to enhance the quality of the measured data through its sensitivity and ability to suppress noisy data. Therefore, this paper concerns a geophysical application to test the traditional Fourier transform (DFT) as well as the two-dimensional iteratively reweighted least-squares (2D IRLS) inversion method on geomagnetic datasets. Before dealing with the geophysical magnetic data, it is necessary to recall the dipolar nature of the Earth's geomagnetic field. Any magnetic body buried beneath the ground, especially located at the intermediate latitudes, can produce an anomaly consisting of two parts (positive and negative). The exact location of this causative body lies between these two parts [8]. This property changes the shape of the magnetic field due to the inclination and orientation of the induced magnetization vector from the magnetic poles to the equator and causes difficulty in detecting the exact shapes and locations of the magnetized causative sources. This problem can be solved only when the total magnetic field is reduced to the northern or southern Earth's poles or equator to avoid the inclination and polarity effect and to locate the anomaly directly above the center of the causative body [9, 10]. Reduction to pole (RTP) enables us to detect the anomaly source position more accurately.

For magnetic data processing, Nuamah and Dobróka [11] applied the algorithms of the DFT and 2D inversion based Fourier transformation (IRLS-FT) methods to non-equidistantly measured magnetic data. In this paper, two-dimensional synthetic magnetic datasets were generated which were then contaminated with random noise to simulate measured data. The Kunaratnam method [12] was applied to calculate these synthetic data above a "U"-shaped magnetic body with a surface ranging from –10,000 m to 10,000 m in both x and y coordinates and a total field anomaly of 200 nT. The values of the magnetic inclination and declination, which selected depending mainly on the geographical locations, were 63° and 3° respectively (for a hypothetical Hungarian location). The measurement points were sampled equidistantly every

500 m in both x and y directions (grid cell size = 500 m), so the total number of the measurements was 1681. The total intensity magnetic map of the noise-free magnetic datasets is shown in *Figure 1*. A close examination of this map shows that the U shaped magnetic body is characterized by the presence of relatively high and low magnetic anomalies of different magnitudes ranging from –250 nT to 100 nT.

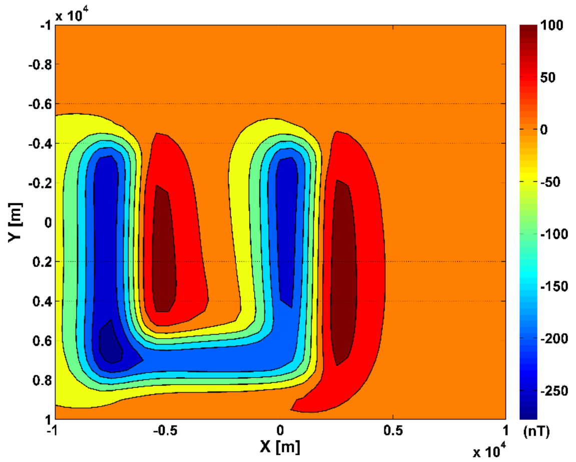

Figure 1. *The noise-free synthetic magnetic dataset*

The two-dimensional traditional Fourier transformation (2D DFT) algorithm is used to convert these magnetic datasets from space domain to frequency domain, resulting in a 2D amplitude Fourier spectrum of noise-free data with the interval (0.5×10^{11} to 4×10^{11}) as shown in *Figure 2*. It is clearly seen that the limits of x and y coordinates were simplified to be from –0.001 to 0.001 (the Nyquist interval of spectrum) to obtain a suitable scale in the wavenumber domain.

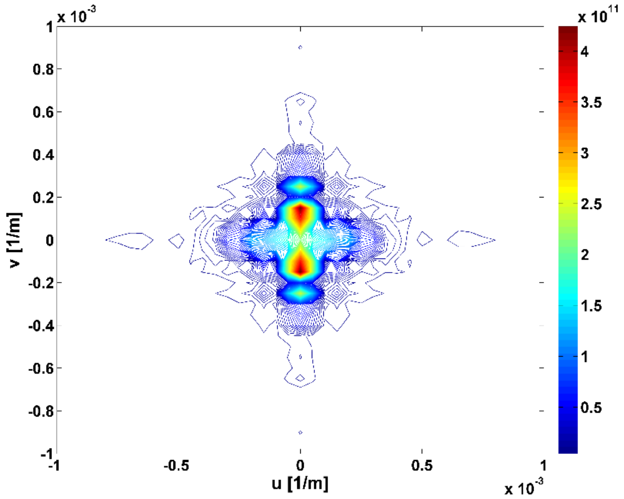

Figure 2. *The 2D amplitude spectrum of the noise-free magnetic data using DFT*

For the purpose of clear and easy magnetic data interpretation, the magnetic measurements were reduced to the pole ($I = 90°$) in the frequency domain using the following formula:

$$R(u,v) = T(u,v)\,S(u,v)\,, \tag{20}$$

where $T(u,v)$ is the 2D Fourier transform of the data and $S(u,v)$ is the frequency domain operator of the pole reduction [10]. To get the calculated data [$R(u,v)$], a 2D inverse Fourier transformation is applied again to the above-mentioned spectrum. This procedure leads to the transformation from frequency to space domain to obtain the calculated data in the form of a magnetic map reduced to the pole, as in *Figure 3*.

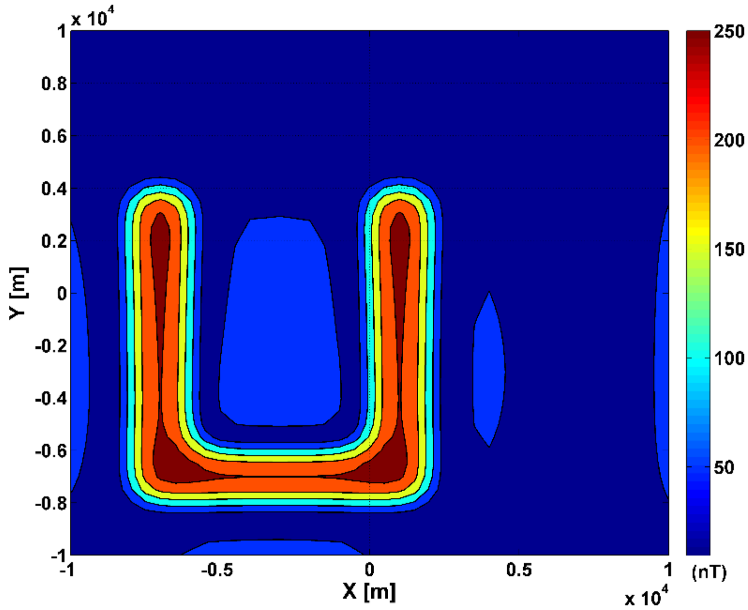

Figure 3. *The reduced-to-pole magnetic dataset using DFT*

It is clearly seen that the noise-free reduced-to-pole magnetic map calculated by the 2D DFT algorithm has values ranging from 50 nT to 250 nT, which is a little higher than that of the noise-free magnetic data in *Figure 1*, but in general both have similar magnetic characteristics. To test the noise sensitivity and the capacity of noise suppression, the generated synthetic magnetic datasets should be contaminated with random noise of Cauchy distribution. *Figure 4* shows the same magnetic data after adding a Cauchy noise of scale parameters 0.03. It is noticed that the noisy magnetic map totally differs from the noise-free datasets in *Figure 1* in both anomaly shapes and magnetic amplitudes. The anomaly contour lines are sharply curved and disconnected as well as displaying more spikes, resulting in quite higher magnetic values (from –500 nT to 400 nT) compared to the noise-free datasets.

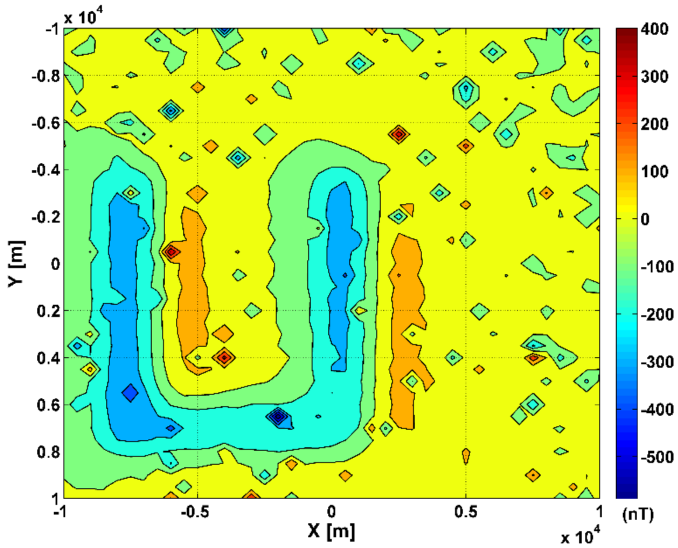

***Figure 4**. The noisy synthetic magnetic dataset*

Similarly, the traditional Fourier transformation (2D DFT) is applied to the above-mentioned noisy magnetic datasets to obtain the 2D Fourier spectrum in *Figure 5*.

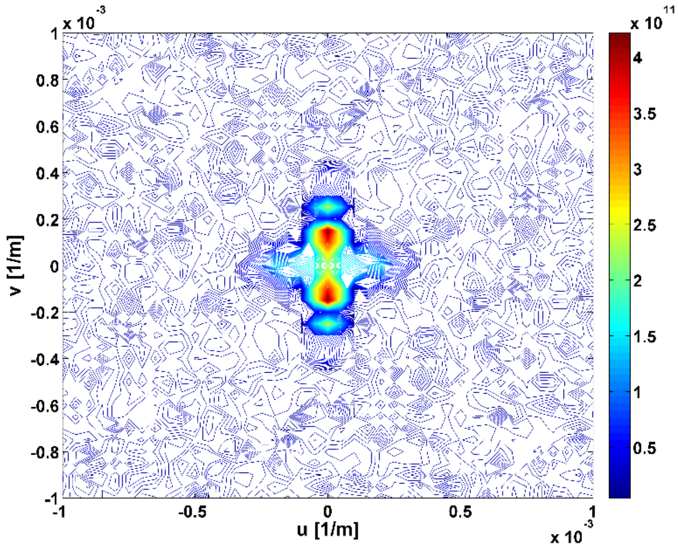

***Figure 5.** The 2D amplitude spectrum of the noisy data using DFT*

Compared to the 2D amplitude spectrum of the noise-free data in *Figure 2*, the amplitude Fourier spectrum of the 2D noisy data is totally deformed, which reflects the quite high sensitivity of the traditional Fourier transformation (2D DFT) algorithm to the added noise. Again the 2D inverse Fourier transform is applied to the noisy data, demonstrating to what extent the traditional Fourier transformation treats or enhances the highly deformed noisy data. The reduced-to-pole space domain data (RTP) using DFT are calculated in *Figure 6*.

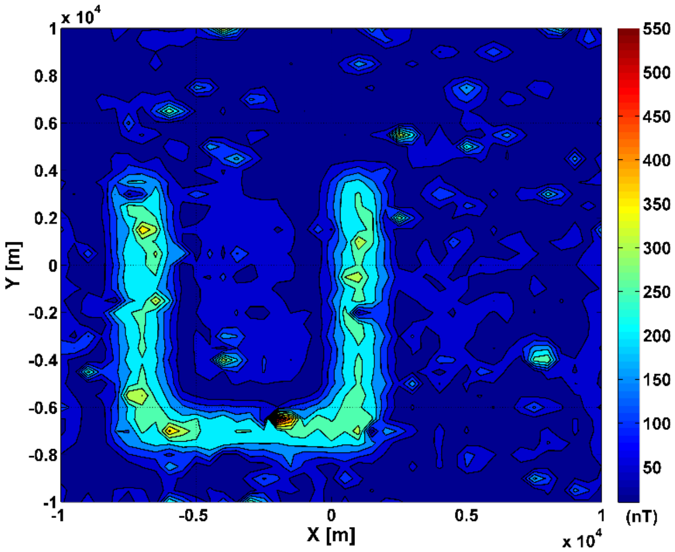

Figure 6. *The reduced-to-pole noisy magnetic dataset using DFT*

It is clearly seen that the calculated pole reduction of the noisy magnetic datasets using the familiar Fourier transformation (DFT) is extremely distorted. The evidence of the distortion obviously appears in the remains of the added Cauchy noise simulating the outlier effect. In addition to that, the contour lines became more curved and highly deformed compared to the generated noisy magnetic datasets in *Figure 4*, causing very high magnetic values ranging from 50 nT to 550 nT. The results are compared to those found by Nuamah and Dobróka [11] demonstrating the low noise rejection capability of the conventional Fourier transformation (2D DFT). Therefore, this is the proper stage to implement an algorithm of a robust Fourier transformation inversion method to remove and suppress the outlier's effects. This can be achieved by using the iteratively reweighted least-squares Fourier transformation (2D IRLS-FT) method. For comparison, the same generated noisy data in *Figure 4* are subjected to the 2D IRLS Fourier transformation to obtain the 2D amplitude spectrum in *Figure 7*. The results show sufficient improvements by using the inversion-based FT method compared to the 2D amplitude spectrum obtained by the traditional Fourier transformation (DFT) in *Figure 5*. In the new spectrum with IRLS, most of the defects that appeared with DFT have been removed, and it is very similar to the 2D amplitude spectrum of the noise-free magnetic datasets calculated by DFT (see *Figure 2*).

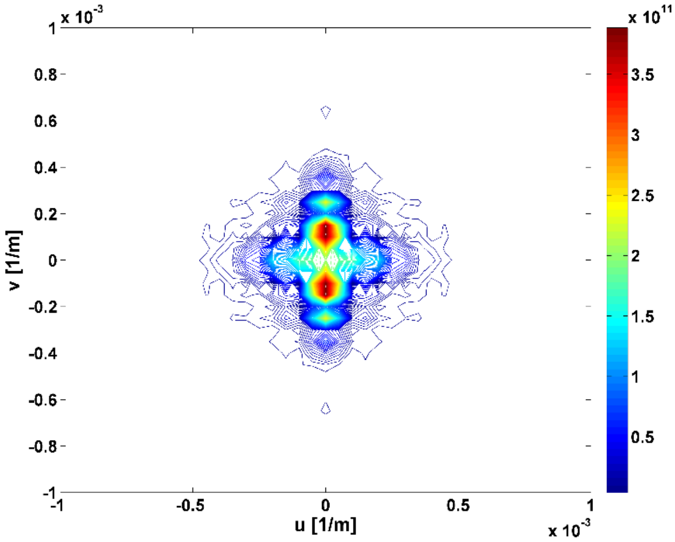

Figure 7. *The 2D amplitude spectrum of the noisy data using IRLS-FT*

To prove the success of the inversion-based FT method, the reduction to the pole of the same noisy magnetic datasets is calculated using the inverse Fourier transformation of the IRLS, as shown in *Figure 8*.

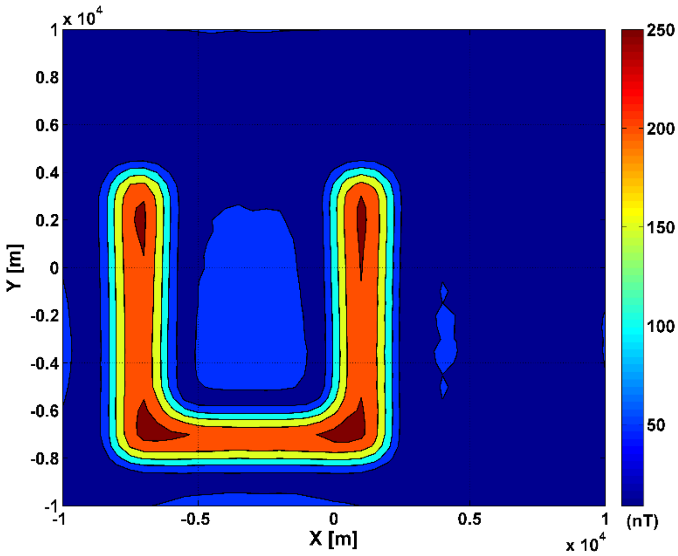

Figure 8. *The reduced-to-pole noisy magnetic dataset using IRLS-FT*

It is clearly seen that the quality of the proceeded data obtained by the 2D IRLS-FT method has been greatly enhanced. A close examination of the reduced-to-pole space domain data calculated by the IRLS method in *Figure 8* indicates that smooth and continuous contour lines are established compared to the previously curved and discretized contours of the reduced-to-pole noisy data calculated by the conventional Fourier transformation (DFT) in *Figure 6*. In addition, the applied inversion method solved the problem of outliers that appeared clearly in the noisy RTP magnetic map of the DFT method. Moreover, the reduced-to-pole magnetic map estimated by the 2D IRLS-FT inversion method is extremely similar to that of the noise-free reduction to pole space domain data *(Figure 3)* in both anomaly shape and magnetic amplitudes (50 nT to 250 nT). All results are in agreement with those obtained by Nuamah and Dobróka [11]. According to the results shown in this paper, the iteratively reweighted least-squares Fourier transformation (2D IRLS-FT) method is outlier-resistant and quite robust, demonstrating great success in the framework of noise rejection capability for processing magnetic data measurements.

5. Conclusions

In signal processing, the discrete Fourier transform (DFT) is one of the most frequently used methods for processing datasets but when dealing with complex noisy data containing outliers, it is not the proper solution. Therefore, this paper applied a robust inversion method called the iteratively reweighted least-squares Fourier transformation (2D IRLS-FT) to achieve such an aim. In the method, the Fourier transformation was considered as the solution of the inverse problem with the help of Steiner weights for scale parameters determination. For high quality and quick procedures of our inversion, the Hermite functions were defined as basis functions using the special character of the Fourier transform eigenfunctions. The expansion coefficients were calculated through the discretization process of the continuous Fourier spectrum by using the series expansion. Based on the results from reduction-to-pole magnetic datasets shown in this paper, which confirmed previous work, it can be concluded that in the case of Cauchy noise the 2D IRLS-FT method is extremely effective, robust, and has a higher noise rejection capability than the DFT method.

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