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Examples for Application of Gravitational Model in the Investigation of Spatial Structure

Áron Kincses and Géza Tóth

Hungarian Central Statistical Office, 1024 5-7. Keleti K. str., Budapest, Hungary

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Abstract: In this paper the authors wish to introduce an application of the gravitational model through two concrete examples. In their investigation the gravitational model was transformed to analyze the spatial structure of Europe, and the impact of accessibility in Hungary. In this analysis not only the size of gravitational forces but their direction can also be measured. Displacements were illustrated by a bi-dimensional regression, which gives a new perspective to the investigation of spatial structure.

Key words: Gravitational model, bi-dimensional regression, accessibility, spatial structure, Europe, Hungary.

1. Introduction

The overall goal of modelling is to simplify reality, actual processes and interactions and on the basis of the obtained data to draw conclusions and make forecasts. Models based on gravitational analogy are the tools of spatial interactions of classical regional analyses. They were first applied in the 19th century [2, 19, 21, 3, 31, 4, 11, etc]

The application of the geographical gravity is confirmed by the theory of experience according to which (just as in time) the things that are closer to each other in space are more related than distant things. This is called the "first law of geography” [25].

There are two basic areas of the application of gravitational models based on physical analogy: the spatial flow analysis [7, 16], and the demarcation of catchment areas [15, 17]. The potential models based on gravitational analogy are the most important groups of accessible models. In general, it can be stated that they are accessible approaches according to which models show potential benefits of the region compared to other regions where the benefits are quantified [20].

The use of accessible models in transport-geographical studies is very common. However, when models are used, it is not entirely clear what is actually modelled; because of their complexity their interpretations may be difficult [13]. It should be stressed that accessibility has no universally accepted definition; in empirical studies different methodological background indicators are used [9, 10, 29, 30].The gravitational theory is a theory of contact, which examines the territorial interaction between two or more points in a similar way as correlations are analysed in the law of gravitation in physics. According to Dusek [5]. despite the analogy, there are significant differences between gravitational models used by social sciences and the law of gravitation used in physics. It is worth bearing in mind that "the gravitational model is not based on the gravitational law”. It is a fundamental statement based on the experience of undeniable statistical character that takes into consideration spatial phenomena. According to this statement, phenomena interact with each other. The phenomena, which are closer to each other in space, are more related than distant phenomena [5 p. 45.].

There are a number of differences between the law and the model. In this study, we wish to highlight a new point of view. As a consequence of the spatial

Corresponding author: Áron Kincses, Ph.D., research field: statistics, regional geography. E-mail: aron.kincses@ksh.hu.

interaction, classical gravitational potential models show the magnitude of potential at spatial points. Regarding the law of gravity in physics, the direction of forces cannot be evaded. In our approach each unit area is assigned an attraction direction. That is, in the case of the gravity model (although such spaces are free of vortex) the space is characterized using vectors.

2. Experimental Section

2.1. Method

The universal gravitational law, Newton's gravitational law, states (1686) that any two point-like bodies mutually attract each other by a force, whose magnitude is directly proportional to the product of the bodies' weight and is inversely proportional to the square of the distance between them [1] (1st formula):

$$F = \gamma \cdot \frac{m_1 \cdot m_2}{r^2} \tag{1}$$

where γ is the proportional factor, the gravitational constant (independent of time and place).

If r indicates the radius vector drawn from the mass point number 2 to a mass point number 1, then r/r is the unit vector drawn from 1 towards 2, thus, the impact of gravitational force on mass point 1 from mass point 2 is in (Eq. 2) (Fig. 1), which reads

$$\vec{F}_{1,2} = -\gamma \cdot \frac{m_1 \cdot m_2}{r^2} \cdot \frac{\vec{r}}{r} \tag{2}$$

A gravitational field is set if the gradient (K) can be specified by a direction and magnitude in each point of the range. Since K is a vector quantity, three numbers (two in plane) are required to be known at each point,

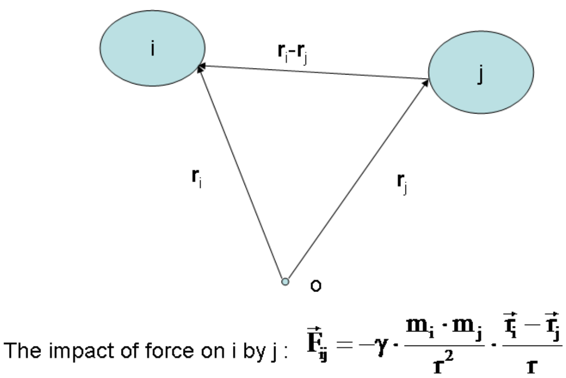

Fig. 1. Gravitational power.

for example the right angle components of gradient K_x, K_y, K_z, which are functions of the site. However, many fields, including the gravitational field, can be characterized in a more simple way. They can be expressed by a single scalar, the so-called potential function, instead of three values.

The potential is in a similar relation with gradient as work and potential energy with force.

Taking advantage of this, gravitational models are also applied in most social sciences where space is usually described by a single scalar function, [13], whereas in gravitational law vectors characterizing the space are of great importance. The primary reason for this is that arithmetic operations calculated with numbers are easier to handle than with vectors. Perhaps, we could say that by working with potentials we can avoid calculation problems in problem solving. The potential completely characterizes the whirl-free gravity gravitational field, because there is a definite relationship between the field strength and the potential:

$$\vec{K} = -\mathrm{grad}U \Rightarrow \quad K_x = -\frac{\partial U}{\partial x}; \quad K_y = -\frac{\partial U}{\partial y}. \tag{3}$$

In other words, the potential (as mathematical functional) is the negative gradient of field strength. Various types of potentials and models, which are different from the ones directly based by the gravitational analogy, but in this case, force effects among space power sources are quite different. In fact, these models differ from each other since the attractive forces remain above a predetermined limit value and within set distances.

The force in a general form is:

$$\left|\vec{F}\right| = C \frac{m_1^{\alpha} m_2^{\beta}}{r^{\gamma}} \tag{4}$$

where C, α, β, γ are constants. [14].

However, how they describe actual power relations between social masses is another question.

Our goal can be reached by using Equation (3) to potentials or directly with the help of forces. We chose the latter one.

In the conventional gravity model [21] D_{ij} is the "demographic" force between i and j where W_i and W_j are the population size of the settlements (regions), d_{ij} is the distance between i and j, and finally, g is the empirical constant (Eq. 5).

$$D_{ij} = g \cdot \left(\frac{W_i \cdot W_j}{d_{ij}^2}\right) \tag{5}$$

In our first study the W_i and W_j weight factors represent the GDP of European regions, d_{ij} is the distance between i and j centres of the NUTS regions.

In our second study the W_i and W_j weight factors represent personal income, which is the base of the income tax in small communities of Hungary, d_{ij} is the actual distance between i and j regional centres measured on road by a minute (regardless of the traffic conditions and only the maximum speed depending on the road type is taken into consideration).

By generalizing the aforementioned formula, we write the following equation (Equation 6 and 7):

$$D_{ij} = \left|\vec{D}_{ij}\right| = \frac{W_i \cdot W_j}{d_{ij}^c} \tag{6}$$

$$\vec{D}_{ij} = -\frac{W_i \cdot W_j}{d_{ij}^{C+1}} \cdot \vec{d}_{ij} \tag{7}$$

where W_i and W_j are the masses, d_{ij} is the distance between them, c is a constant, which is the change of the intensity of inter-regional relations as a function of distance. As the exponent, c, increases and the intensity of inter-regional relations becomes more sensitive to distance, this significance of masses gradually decreases [5]. The minus sign express mathematically, that the masses attract each other (see Fig. 1).

With the extension of the above Equation we cannot only measure the strength of the force between the two regions, but its direction as well.

While performing calculations, it is worth dividing the vectors into x and y components and summarize them separately. To calculate the magnitude of this effect (vertical and horizontal forces of components) the following Equations are required (Equation 8 and 9), which follow from (6):

$$D_{ij}^X = -\frac{W_i \cdot W_j}{d_{ij}^{c+1}} \cdot (x_i - x_j) \tag{8}$$

$$D_{ij}^Y = -\frac{W_i \cdot W_j}{d_{ij}^{c+1}} \cdot (y_i - y_j) \tag{9}$$

where x_i, x_j, y_i, y_j are the coordinates of the i and j regions.

However, if we perform the calculation on all unit areas involved, we will know in which direction their forces exactly act and how strongly they affect the given unit area. (Eq. 10)

$$D_i^X = -\sum_{j=1}^{n} \frac{W_i \cdot W_j}{d_{ij}^{c+1}} \cdot (x_i - x_j)$$

$$D_i^Y = -\sum_{j=1}^{n} \frac{W_i \cdot W_j}{d_{ij}^{c+1}} \cdot (y_i - y_j) \tag{10}$$

It should be noted that while in potential models, the results are modified by the introduction of "self-potential", in the examination of forces we disregard the introduction of "self- forces".

Thus, it is possible to determine the magnitude and direction of force in which other areas affect each territorial unity. The direction of the vector, which is assigned to the region, determines the attraction direction of other unit areas, while the length of the vector is proportional to magnitude of force. For the sake of mapping and illustration, we transformed the received forces into shifts proportional to them in the following way (Eqs. 11 and 12):

$$x_i^{mod} = x_i + \left(D_i^X \frac{x^{max}}{x^{min}} k \frac{1}{\frac{D^{X\,max}}{D^{X\,min}}} \right) \tag{11}$$

$$y_i^{mod} = y_i + \left(D_i^Y \frac{y^{max}}{y^{min}} k \frac{1}{\frac{D^{Y\,max}}{D^{Y\,min}}} \right) \tag{12}$$

where X_i^{mod} and Y_i^{mod} are the coordinates of new points modified by the gravitational force, x and y are the coordinates of the original point set, the extreme values of which are x_{max}, y_{max}, a $x_{min,}$ y_{min}, Di is the force along the x and y axes, k is a constant and in this case it is 0.5. This has the effect of normalizing the data magnitudes.

We assume that in our model the amount of interactions between the „masses" is the same as in Eq. 7, and based on the superposition principle, it can be calculated for a given region by Equation 10. The new model cannot directly be compared with transport -geographical data, but the results compared with traffic data in potential models to verify our model [13].

Our model is a kind of complement to the potential models that ensures a deeper insight into them. In the following sections of this study we intend to communicate some significant results of this model.

2.2 Application of Bi-Dimensional Regression

The point set obtained by the gravitational calculation., is worth comparing with the baseline point set, that is, with the actual real-world geographic coordinates and examining how the space is changed and distorted by the field of force. The comparison, of course, can be done by a simple cartographic representation, but with such a large number of points, it is not really promising good results. It is much better to use a bi-dimensional regression.

The bi-dimensional regression is one the methods of comparing partial shapes. The comparison is possible only if one of the point coordinates in the coordinate systems differing from each other is transformed to another coordinate system by an appropriate rate of displacement, rotation and scaling. Thus, it is possible to determine the degree of local and global similarities of shapes as well as their differences that are based on the unique and aggregated differences between the points of the shapes transformed into a common coordinate system. The method was developed by Tobler, who published a study describing this procedure in 1994 after the precedents of the 60s and 70s [23, 24, 25, 26, 27, 28]. There are many examples using this procedure, which are not necessarily motivated by geographic issues [12, 22, 18].

As for the equation relating to the calculation of the Euclidean version, see [27, 8, 6].

Where x and y are the coordinates of independent shapes, a and b are the coordinates of dependent shapes, and represent the coordinates of dependent shapes in the system of independent shapes. $\alpha1$ determines the measure of horizontal shift, while $\alpha2$ determines the measure of vertical shift. $\beta1$ and $\beta2$ are the scalar difference and (Φ) and (Θ) determine the angle of shifting.

SST is the total square sum of difference. SSR is the square sum of difference explained by regression. SSE is the square sum of difference not explained by the regression (residual). Further details about the background of the two-dimensional regression can be seen in [6 pp. 14-15].

3. Results and Discussion

3.1 European Analysis

The arrows in Figure 2 show the direction of movement and the grid colour refers to the nature of the distortion. Warm colours indicate divergence; that is, movements in the opposite direction, which can be considered to indicate the most important gravitational fault lines. Areas indicated with green and its shades refer to the opposite, namely to concentration, to the movements in the same directions (convergence), which can be considered to be the most important gravitational centres.

Our analysis can be carried out at the NUTS1, 2, and 3 levels. The comparison of the results (between real and modified coordinates) with those of bidimensional regression can be found in Table 2.

As the results show, the lower the level that is used for the analysis, the smaller the deviation of the gravitational point form is from the original structure.

Table 1 The equation of the two dimensional regression of Euclidean.

1. Equation of the regression	$\begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + \begin{pmatrix} \beta_1 & -\beta_2 \\ \beta_2 & \beta_1 \end{pmatrix} * \begin{pmatrix} X \\ Y \end{pmatrix}$
2. Scale difference	$\Phi = \sqrt{\beta_1^2 + \beta_2^2}$
3. Rotation	$\Theta = \tan^{-1}\left(\frac{\beta_2}{\beta_1}\right)$
4.Calculation of β_1	$\beta_1 = \frac{\sum (a_i - \bar{a}) * (x_i - \bar{x}) + \sum (b_i - \bar{b}) * (y_i - \bar{y})}{\sum (x_i - \bar{x})^2 + \sum (y_i - \bar{y})^2}$
5.Calculation of β_2	$\beta_2 = \frac{\sum (b_i - \bar{b}) * (x_i - \bar{x}) - \sum (a_i - \bar{a}) * (y_i - \bar{y})}{\sum (x_i - \bar{x})^2 + \sum (y \ - \bar{y})^2}$
6. Horizontal shift	$\alpha_1 = \bar{a} - \beta_1 * \bar{x} + \beta_2 * \bar{y}$
7. Vertical shift	$\alpha_2 = \bar{b} - \beta_2 * \bar{x} - \beta_1 * \bar{y}$
8. Correlation based on error terms	$r = \sqrt{1 - \frac{\sum \left[(a_i - a_i')^2 + (b_i - b_i')^2 \right]}{\sum \left[(a_i - \bar{a})^2 + (b_i - \bar{b})^2 \right]}}$
9. Breakdown of the square sum of the difference	$\sum \left[(a_i - \bar{a})^2 + (b_i - \bar{b})^2 \right] = \sum \left[(a_i' - \bar{a})^2 + (b_i' - \bar{b})^2 \right] + \sum \left[(a_i - a_i')^2 + (b_i - b_i')^2 \right]$ SST=SSR+SSE
10. Calculation of A’	$A' = \alpha_1 + \beta_1 (X) - \beta_2 (Y)$
11. Calculation of B’	$B' = \alpha_2 + \beta_2 (X) + \beta_1 (Y)$

Source: [27, 8, 6].

Table 2 Bidimensional regression between gravitational and geographical spaces.

Level	r	α1	α2	β1	β2
NUTS1	0.91	0.19	0.69	0.99	0.00
NUTS2	0.97	0.04	0.15	1.00	0.00
NUTS3	0.99	0.13	-0.04	1.00	0.00

Level	Φ	Θ	SST	SSR	SSE
NUTS1	0.99	0.00	20 430	19 849	582
NUTS2	1.00	0.00	54 121	53 484	638
NUTS3	1.00	0.17	139 884	139 847	37

This is proven by the correlation and by the sum of squared deviations and their components. Because of the mass differences among the regions, the analyses carried out at different territorial levels show results that are different in their nature even if they are similar in many aspects of their basic structure. That is why we decided to carry out the analysis at each territorial level in order to examine the different levels of the spatial structure. Of course, the regression is higher if the examined spatial level is smaller. We visualised our results and drew the following conclusions.

The analysis carried out at the NUTS 1 level contains only the most general relations. These general relations, however, are not sufficient to carry out a deeper analysis of the spatial structure. That is why it is necessary to go on to the NUTS 2 level (Fig. 2). In this case, as shown in Fig. 2, regional concentrations can be seen unambiguously, and we consider them to be the core regions. Based on the analysis carried out at the NUTS2 level, basically three gravitational centres,

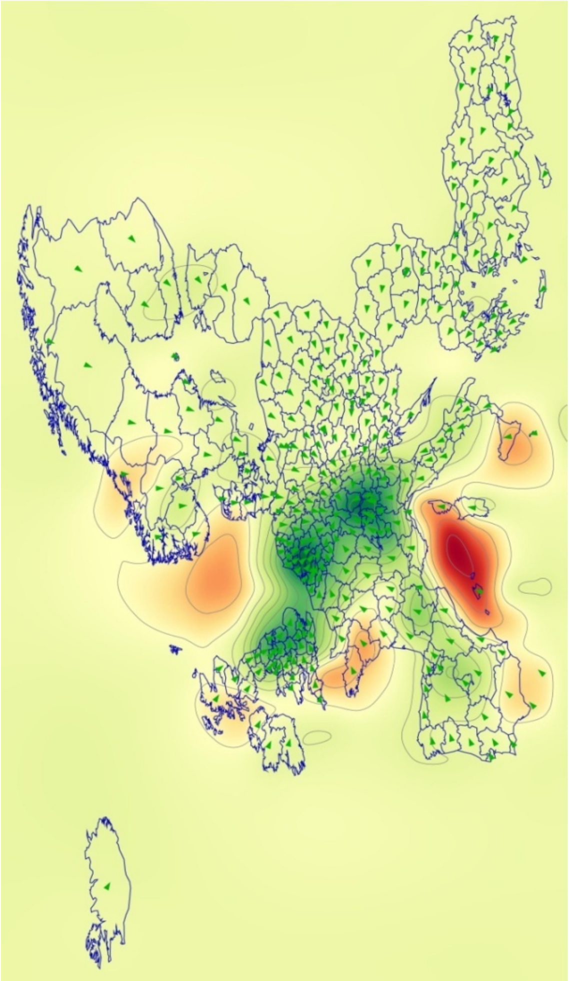

Fig. 2 Directions of the distortion of gravitational space compared to geographical space for the European regions (NUTS2).

slightly related to each other, can be found in the European space. Gravitational centres are the regions that attract other regions and the gravitational movement is toward them. These three centres or cores are (Fig. 2): 1) the region including Baden-Württemberg, the western part of Austria and the eastern part of Switzerland; 2) the region including the Benelux countries and the western part of Nordrhein-Westfalen; 3) the region including most of England. Mainly these core areas have an effect on the regions of the examined area. The three centres also include two concentration spurs. The stronger and without any doubt the more important one extends from the eastern part of Switzerland through south France to Madrid, while the other and somewhat weaker one starts from this point and goes through the Apennine Peninsula.

Defining the core regions is easy using gravity analysis, provided that these are defined as the regions that have converging spatial movements and that can be considered the main gravitational centres. These regions are shown in green in Fig. 3.

In the following section, we try to take into account the change of the structure. To do so, the gravity calculations are performed for 1995 and 2009. In this calculation, we cannot include the regions of Turkey, so the figures of 2009 will slightly be different from the one described such as Fig. 2. In order to measure changes, we compare and analyse the two gravity sets of points (1995 and 2009). The bi-dimensional regression calculations are shown below (Tables 3 and 4).

Our results show that there is a strong relationship between the two point systems; the transformed version from the original point pile can be obtained without using rotation ($\Theta = 0$). No essential ratio difference between the two shapes is observed.

In term of change from 1995 to 2009, 15 gravity centres can be shown on the map, assigned with red ellipses (Fig. 3). They show a crucial part of the economic potential of big cities. Such hubs are in the surroundings of Rome, Marseille, Madrid, Vienna, Hamburg, Brussels, Olso, Glasgow, etc. A gravity "breakline" can be seen in Northern France, Northern Italy, Switzerland, Hessen in Germany and Northern Saxony.

Based on the obtained results, NUTS3 level regions can be grouped by the direction of force applied to them by other regions. Four groups can be formed. They are as follows:

(1) force from the North and West (blue)

(2) force from the South and West (green),

(3) force from the North and East (red),

(4) force from the South and East (yellow).

Each region can be placed in one of the aforementioned groups. The results for 2009 are shown in Fig. 4. On the map, the East-West segmentation

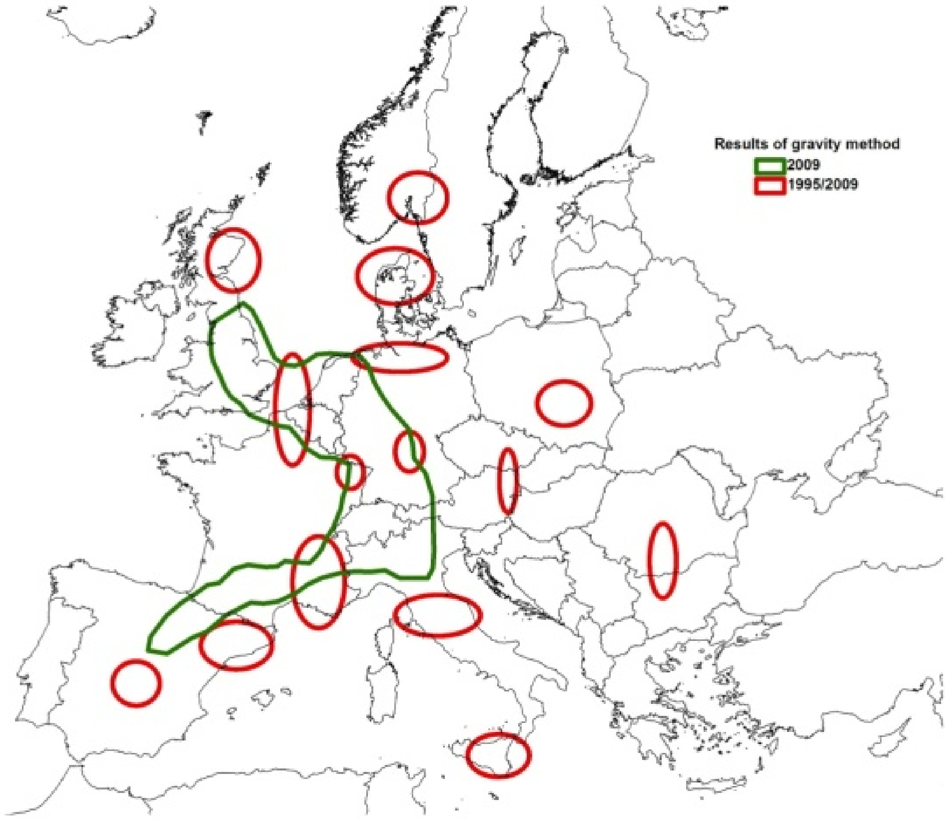

Fig. 3 The results of the Gravity method.

Table 3 Bidimensional regression between gravitational and geographical spaces.

Year	r	α1	α2	β1	β2
1995	0.92	0.07	0.37	0.99	0.00
2009	0.92	0.05	0.26	0.99	0.00

Year	Φ	Θ	SST	SSR	SSE
1995	0.99	0.00	65 446	62 525	2 922
2009	0.99	0.00	65 632	62 811	2 821

Table 4 Bidimensional regression between gravitational spaces.

Year	r	α1	α2	β1	β2
1995/2009	0.99	-0.01	-0.06	1.00	0.00

Year	Φ	Θ	SST	SSR	SSE
1995/2009	1.00	0.00	65 632	65 607	25

appears to be more significant than the North-South. The change from 1995 to 2009 (not shown here, due to lack of space) is insignificant. The impact of regional centres can clearly be seen in the areas where the direction of forces differs from surrounding regions, (i.e. in the neighbouring areas with different colours). These nodes mostly overlap the nodes derived from the examination of change from 1995 to 2009 (see Fig. 3). According to Fig. 3, the barrier lines of different directions of force (see Fig. 3) are similar to those highlighted by the different theories of structural image of Europe.

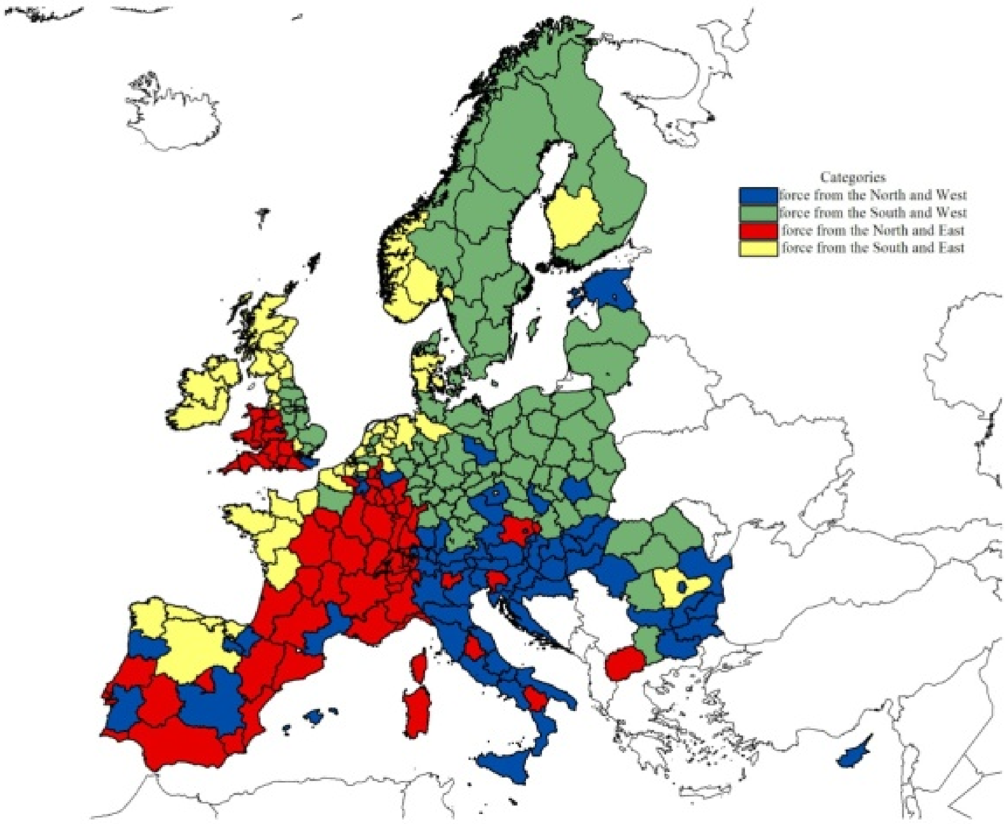

Fig. 4 Regions grouped by the direction of force applied to them by other regions, 2009.

3.2 Hungarian Analysis

Although potential models often characterize concentration on focal points of areas and spatial structures, they fail to provide information in which direction and with how much force the social attributes of other areas attract each delimited area unit. Thus, we attempt to use vectors in order to show the direction where the Hungarian micro regions[1] tend to attract micro regions (LAU1) in the economical space compared to their actual geographical position. This analysis can demonstrate the most important centres of attraction, or discrepancies, and the differences between the gravitational orientations of micro-regions can be displayed on a map after the evaluation of data from 2000, 2005 and 2010 has been performed. In the study the geometric centres of specific micro-regions were the co-ordinates of Hungarian micro-regions, which were determined in the EOV co-ordinate system [2]by (Geographical Information System) GIS software.

Our results show that there is a strong relation between the two point systems; the transformed version from the original point set can be obtained without using rotation ($\Theta = 0$). Essential ratio difference between the two shapes was not observed. Comparing the obtained results, it is obvious that the set of points behaves like a single-centre mid-point similarity, when it is diminished. This means that only the attractive force of Budapest can be determined at a national level.

[1] A micro-region code is the code of a (non- administrative) breakdown covering the whole area of Hungary, which is based on real connections between settlements in terms of work, residence, transport, medium-level provision (education, health care, trade) etc. Through their connections settlements are attracted by one or more central settlements in the system of statistical micro-regions.

[2] The EOV is a plane projection system used uniformly for the Hungarian civilian base maps and, in general, for spatial informatics. Geometric classification: Conformal cylindrical projection in transversal position.

3.3 Map Display and Direction Analyses

The square grid attached onto the shape-dependent coordinate system and its interpolated modified position further generalizes the information received from the participating points.

The arrows in Figs. 5 and 6 show the directions of the shifts, while the colouring illustrates the type of distortion. The warm colours express the divergent forces of the area, which are considered to be the most important gravitational displacements.

The areas illustrated in green and its shades represent just the opposite, namely the most important nodes of gravity.

The data in Table 2 shows that the space shaped by the gravitational model causes only a slight distortion compared to the geographic space. The magnitude of

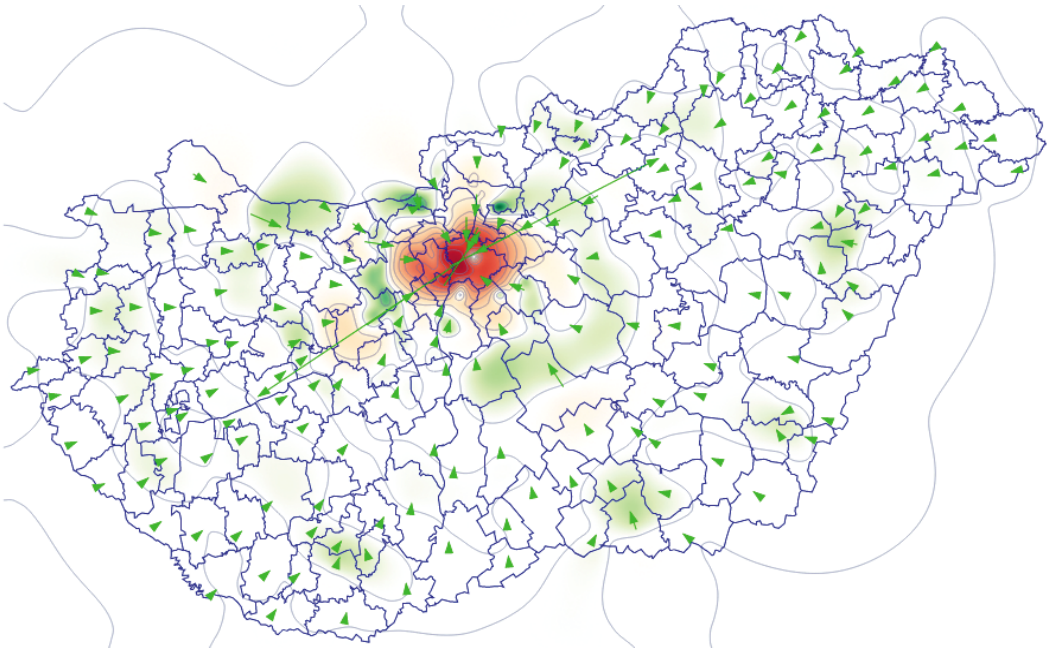

Fig. 5 Results in 2000.

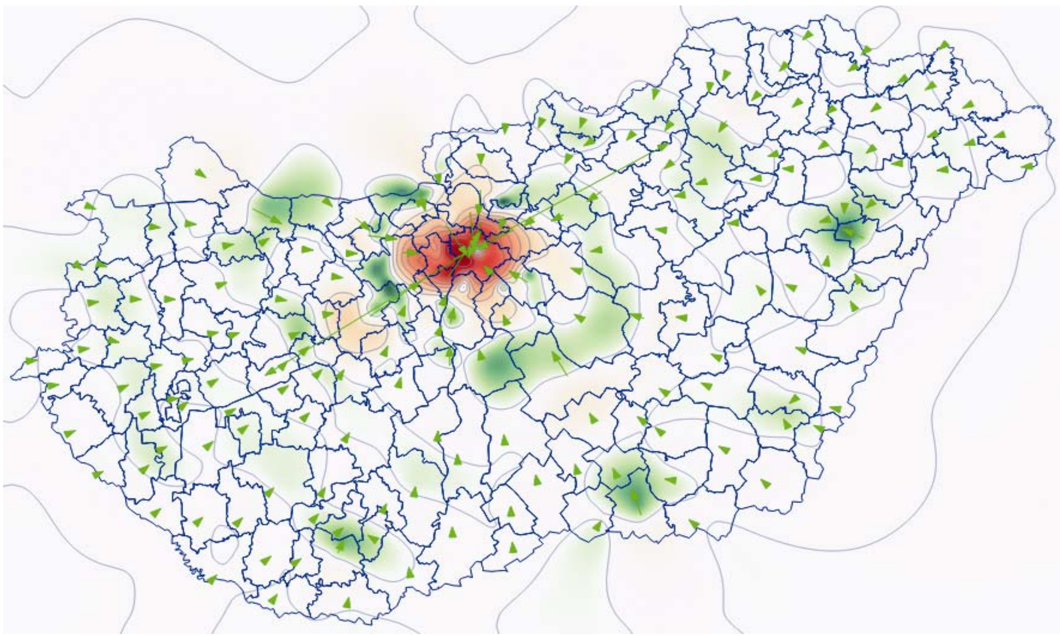

Fig. 6 Results in 2010.

Table 5 Bi-dimensional regression between the gravitational and the geographical space.

Year	r	α_1	α_2	β_1	β_2
2000	0,942	6304,48	2017,44	0,99	0,00
2005	0,942	6030,56	2012,23	0,99	0,00
2010	0,941	8026,79	2632,29	0,99	0,00

Year	Φ	Θ	SST (%)	SSR (%)	SSE (%)
2000	0,99	0,00	100,00	98,73	1,27
2005	0,99	0,00	100,00	98,74	1,26
2010	0,99	0,00	100,00	98,69	1,31

vertical and horizontal displacements increased slightly in 2010.

Practically, the maps produced by Darcy software verify this (Figure 5 and Figure 6). It can be seen that the capital of Hungary is Hungary's main centre of gravity, the centre towards which the largest power is attracted. The regional centres like Győr, Pécs, Szeged, Debrecen are also gravity nodes. The national role of regional centres is weak. In the area of Budapest a gravity fault line emerges.

The reason for this phenomenon is that the Hungarian capital attracts all the micro-regions, while very weak forces are applied to Budapest compared to its mass.

The map also illustrates that the regular force fields are the major transport corridors, namely they are slightly distorted due to highways. Between 2000 and 2010, the significance of green-marked gravitational nodes increased. The comparison of the two maps clearly shows the intensification of regional differences.

Based on the obtained results, micro-regions can be grouped by the direction of force applied to them by other micro-regions. Four groups can be formed in the same way as on Fig. 4.

All micro-regions can be placed in one of the aforementioned groups. The results are shown in Fig. 7 and Fig. 8. On the maps the North-South segmentation appears more significantly than the East-West one. This statement is especially true in Western Hungary where the developed micro-regions of Gyor-Moson-Sopron and Komárom-Esztergom Counties attract other micro- regions of Transdanubia. The effect of the east-west gradient can hardly be demonstrated because of the central location of Budapest and its strong impact on the whole country.

The impact of regional centres is clearly seen in the areas where the direction of forces differs from its environment, i.e., in the neighbouring areas where colours are different from the ones in their environment. According to the results of 2000 and 2010 years, stable local centres can be seen in the micro-regions of Debrecen, Miskolc, Nyíregyháza, Szeged and Pécs

In the advanced territories of northwestern Hungary a similar phenomenon, though less distinctive, is seen when micro-regions differentiate from their environment, since several micro-regional groups tend to be similar in character in these areas.

3.4 Magnitude of force per unit mass

In a micro-region other regions apply different forces. However, the same force strength fails to result the same impact due to the difference of masses. It is possible to calculate the forces acting on a unit of internal mass by the Eq. 13:

$$F = \frac{\sqrt{D_i^{X^2} + D_i^{Y^2}}}{w_i} \tag{13}$$

Where F is the force per unit mass, D_i is the vertical and horizontal forces and w_i is the own weight of point i.

As it is shown in Fig. 9, the most significant forces compared to their own mass are the micro-regions located in Budapest agglomeration, especially the

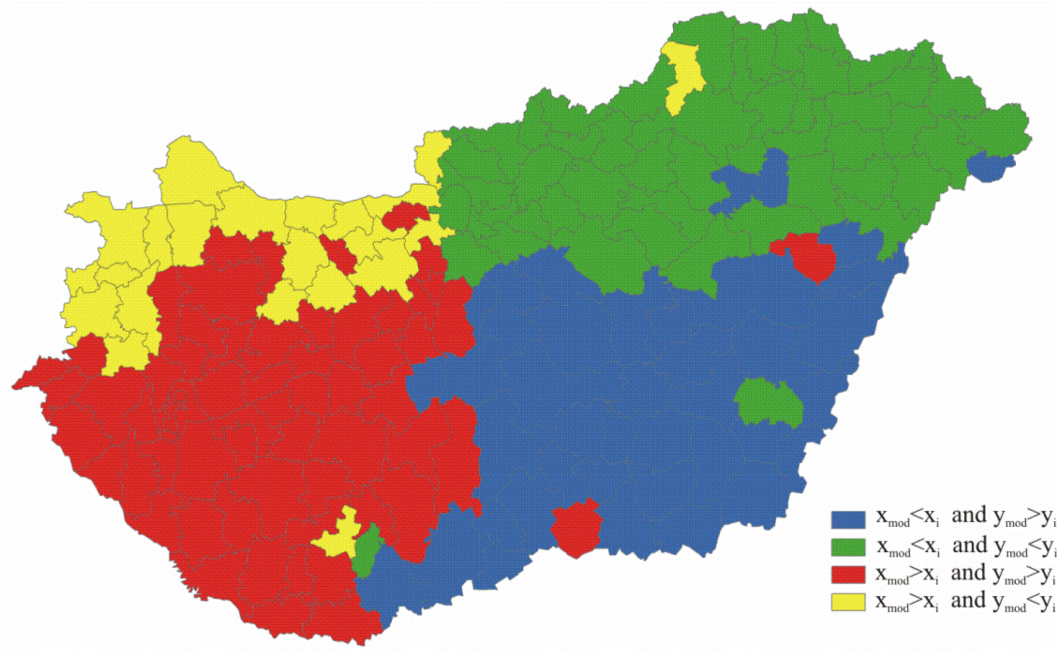

Fig. 7 Results in 2000.

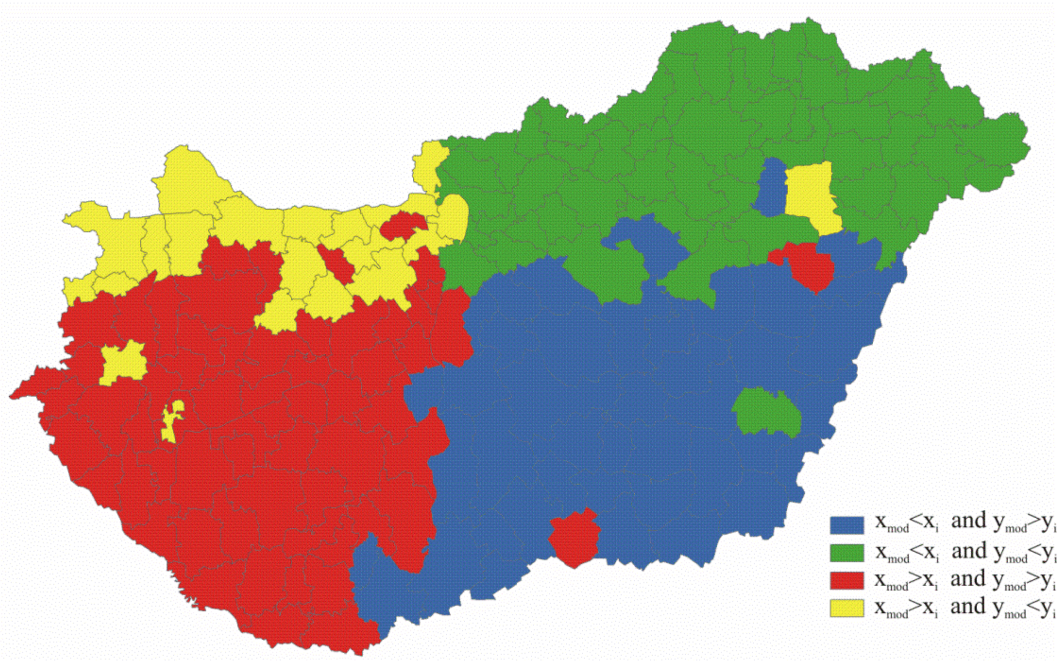

Fig. 8 Results in 2010.

Budaörs micro-region. Outside the agglomeration, Tata micro-region shows an outstanding magnitude. The micro-regions that represent higher than the average value are mainly located further from Budapest, generally along motorways. The long arrows around Budapest were formed due to the impact of the motorways.

Fig.10 shows the changes between 2000 and 2010. The most significant changes are the outcomes of the construction of motorways. A perfect example for this is Tiszavasvári and Komló micro-regions, which are located near M3 and M6 motorways. Somewhat different examples are the famous Sarkadi and Sárvár micro-regions. In their cases, the decrease of their own

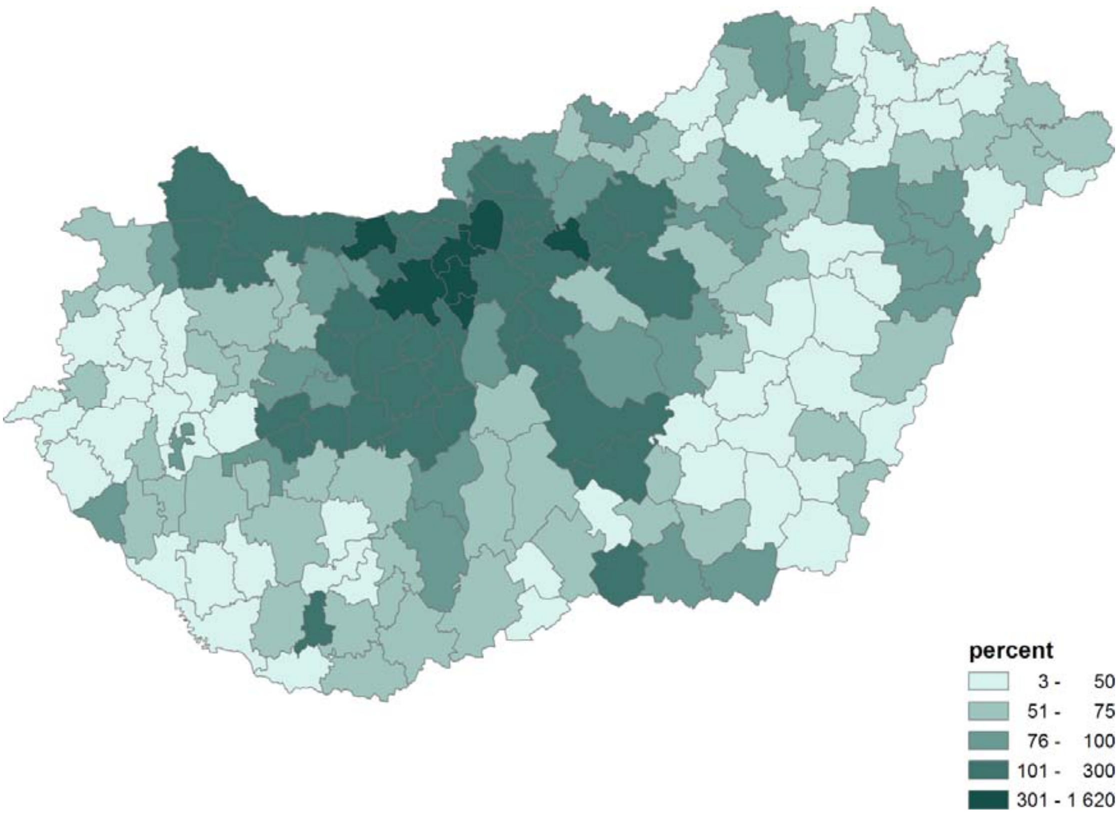

Fig. 9 Forces per unit mass (population) as a percentage of the national average in 2010.

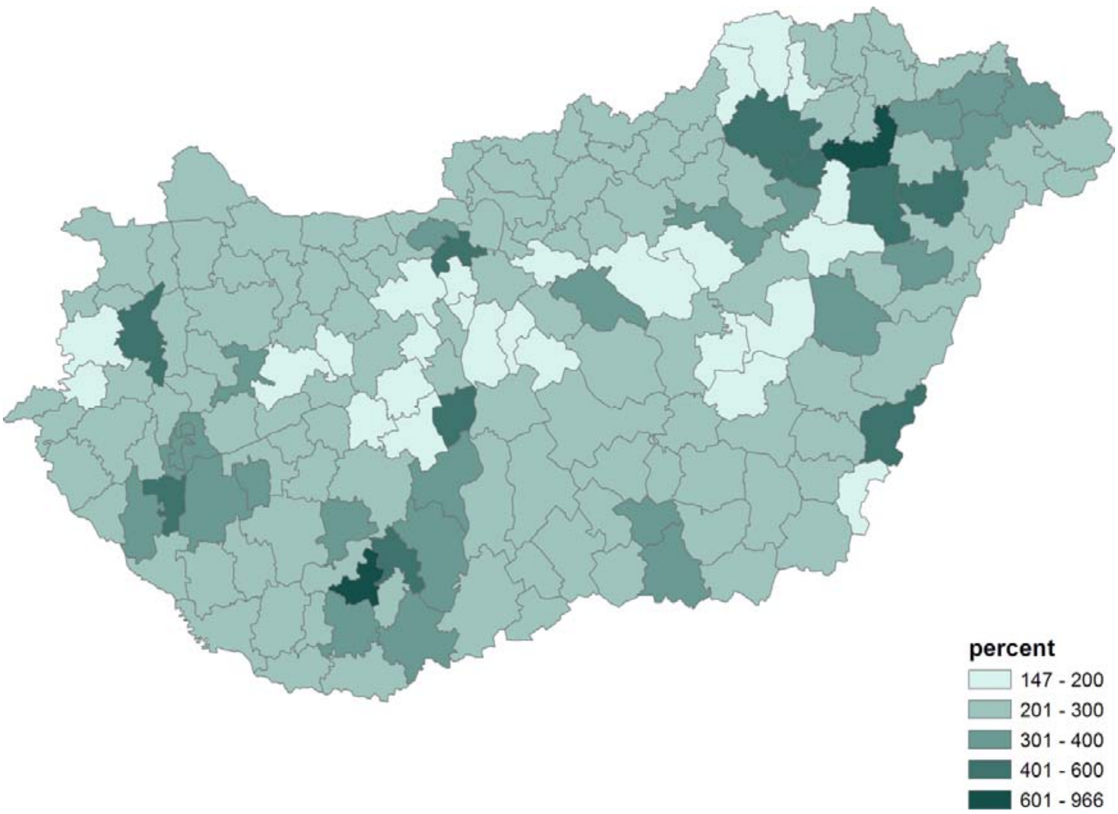

Fig. 10 Changes in forces per unit mass (population) in 2000 and in 2010.

weight caused a specific power growth. There is a micro-region, which emerged primarily due to the population growth in its environment and to its gravitational force growth. It is Pilisvörösvár micro-region.

4. Conclusions

In our study we made an attempt to introduce the potential and unexplored areas of gravitational models and problems of their interpretation by expanding and extending the methodology. As it was seen, with our method several analyses can be done both at macro, and micro regional level. Our results are comparable to the results of other statistical or spatial econometric models which facilitates the preparation of appropriate assessments and analyzes.

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