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PROFIT OPTIMIZATION OF BATCH-CONTINUOUS PRODUCTION SYSTEMS UNDER STOCHASTIC PROCESSING CONDITIONS BY SIMULATION

É. ORBÁN-MIHÁLYKÓ[1✉], CS. MIHÁLYKÓ[1], B.G. LAKATOS[2], T. SZABÓ[1], A. PAPP[1]

[1]University of Pannonia, Department of Mathematics, H-8200 Veszprém, Egyetem u. 10. HUNGARY
✉E-mail: orbane@almos.uni-pannon.hu
[2]University of Pannonia, Department of Process Engineering, H-8200 Veszprém, Egyetem u. 10. HUNGARY

The properties of a production system working under stochastic processing conditions are investigated. The production system consists of batch units of an input subsystem and a continuously operated deterministic output one which are coupled by an intermediate storage system. The randomness of operation is caused by the uncertainties of batch sizes and the time intervals of arrival to the storage. Taking into account the expenses and the income arising from the production the expectation of the profit is defined and investigated. An integral equation is presented for the expected profit, and is solved using Monte Carlo method. The optimal initial amount of material to be processed, storage volume and withdrawing rate are determined by simulation.

Keywords: Processing system, Intermediate storage, Stochastic operation, Profit, Expectation

Introduction

Intermediate storage plays an important role in improving the operating efficiency of processing systems, and fitting to each other the subsystems with different operational characteristics. Its application increases the availability and variability of systems and reduces the process uncertainties. Such buffering action requires rational sizing of intermediate storages in the design phase [1, 2, 3]. However, beside the important technical aspects, the economical profitability of the processing system is also important and often the main aspect of the design. The aim of the paper is to discuss this problem.

If a processing system consists of batch and continuous subsystems then these parts may be coupled by means of intermediate storage. The batch units form the inputs of the storage system, while at the output the material processed is removed continuously with constant volumetric rate $q(t)=c$ for the continuous subsystem. The removed material is processed and sold, consequently the factory obtains income. Moreover, if the material cannot be stored in the storage because of lack of capacity, it seems rational to sell it immediately. Then an important question arises: how much profit can be obtained as a function of the volume of storage system, the initial amount of the material in the storage and the output rate.

A schematic structure of the processing system is seen in *Fig. 1*.

Published research has demonstrated that modelling the operation of intermediate storage is rather stochastic than deterministic [4]. In this paper we also consider the random behaviour of the processing system and we deal with the maximization of expectation of the discounted profit.

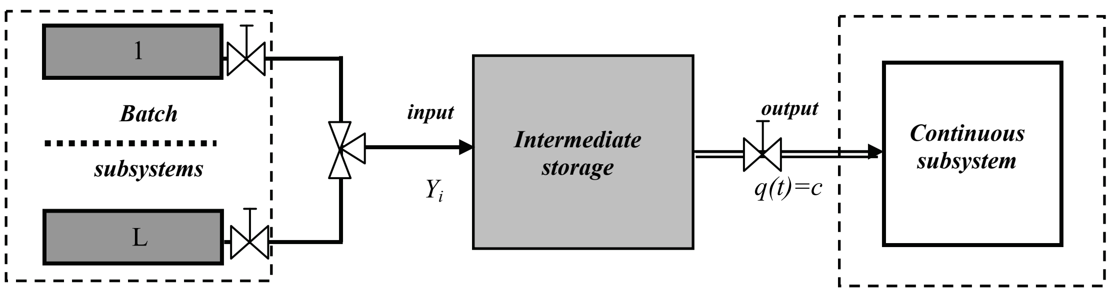

Figure 1: Schematic structure of the processing system

The investigated model: basic notations and assumptions

The material to be processed is filled into the intermediate storage by the units of the input subsystem. Let the starting point of the process be $t_0 = 0$. The time interval between the $(i\text{-}1)^{\text{th}}$ and the i^{th} filling is given by the random variable t_i $i = 1,2,\ldots$. These random variables are supposed to be nonnegative, identically distributed and independent ones with common distribution function $F(t)$, density function $f(t)$ and expectation μ_f. The i^{th} filling happens at the random time point $\sum_{j=1}^{i} t_j$. Let $N(t)$ be the number of fillings up to time t. The amount of material filled into the storage during the i^{th} filling is also random, its value is given by the random variable Y_i, $i = 1,2,\ldots$. These variables are also supposed to be nonnegative, identically distributed and independent with common distribution function $G(y)$, density function $g(y)$, and expectation μ_g. We suppose also that the filling process $N(t)$ and the random variables Y_i are independent from each other.

The output process is continuous with constant intensity c. Let the volume of storage be denoted by y, and the initial amount of material in the storage be x. Let the random period of process from the beginning to running out of material be called cycle, and first we consider the temporal change of material during such a cycle only. Temporal change of material in the storage is caused by the difference between the material filled into the storage and withdrawn from that. If the volume of the storage would not be a physical bound for the filled material then the function

$$V^*(t) = x + \sum_{i=1}^{N(t)} Y_i - ct \tag{1}$$

could express the amount of material being in the storage as a function of time. Since $N(t)$ and Y_i are random, $V^*(t)$ is a stochastic process. Moreover, this process is controlled as some of the material might not be filled into the storage because of the overflow. The maximum of the amount of material being in the storage is y thus the extra amount as compared to it is not filled into. The total extra amount to time t is given by the random function $R(t)$. Consequently the amount of material being in the storage at time t is

$$V(t) = V^*(t) - R(t)\,. \tag{2}$$

As the inputs arrive at the moments of time $T_i = \sum_{j=1}^{i} t_j$ and they cause finite jumps in the functions describing the amount of the material, therefore the process proves to be one hand side continuous with finite jumps. We consider it right-hand side continuous. If the amount of the i^{th} filling Y_i is so much that it would cause overflow, i.e. the inequality

$$V(T_i^-) + Y_i \ge y \tag{3}$$

holds then the amount

$$V(T_i^-) + Y_i - y \tag{4}$$

is considered to be extra amount which cannot be filled into the storage. This means that at time $T_i = \sum_{j=1}^{i} t_j$ we have

$$R(T_i) - R(T_i^-) = V(T_i^-) + Y_i - y \tag{5}$$

and $V(Y_i) = y$. Over the remaining time the value of $R(t)$ does not change, and $R(0) = 0$. This defines the process in terms of intermediate storage.

Let us now consider the costs of production and the income. Let the cost of the filled material per unit volume be denoted by Λ_k. Consequently the cost of the material during the i^{th} filling equals $\Lambda_k Y_i$. We get a more realistic model if we take into consideration also the effects of inflation. More precisely, we distinguish the nominal and the real value of income discounted by inflation. As the costs and income are computed on discounted values and this cost has to be paid at the moment of time $\sum_{j=1}^{i} t_j$, its present value is

$$\Lambda_k Y_i e^{-\delta \sum_{j=1}^{i} t_j} \tag{6}$$

where δ is the discount factor. Hence the total cost of the material filled into the storage up to time t is given by the function

$$K(t,x) = \Lambda_k \left(x + \sum_{i=1}^{N(t)} Y_i e^{-\delta \sum_{j=1}^{i} t_j} \right) \tag{7}$$

If the extra material is sold at price Λ_1 per unit volume then the discounted income from selling is

$$B(t,y) = \Lambda_1 \int_0^t e^{-\delta s} dR(s)\,. \tag{8}$$

If the income from the production per unit volume is denoted by Λ_2 then the profit from the production up to time t can be given by

$$h(t,c) = \Lambda_2 c \int_0^t e^{-\delta s} ds = \Lambda_2 c \frac{1 - e^{-\delta t}}{\delta}\,. \tag{9}$$

Taking into account that the cost of the start is I and the cost of the investment $Q(y)$ depends on the storage size then the total profit as a function of time can be expressed as

$$H(t,x,y,c) = B(t,y) + h(t,c) - K(t,x) - Q(y) - I\,. \tag{10}$$

The investment cost is computed by the often applied formula $Q(y) = k \cdot y^{0.6}$, where k is a constant depending on the material and operational conditions, mostly on the pressure of the storage [5]. The process is considered while material exists in the storage, i.e. while the inequality $V(t) \geq 0$ holds. The endpoint of the cycle is the time when storage runs out of the material. The random time for this occurrence can be given as follows:

$$T(x) = \begin{cases} \inf\{t \geq 0 : V(t) < 0\}, & \text{if there exists} \\ \quad \text{such } t \geq 0 \text{ for which } V(t) < 0 \\ \infty, \quad \text{if} \quad V(t) \geq 0 & \text{for all} \quad t \geq 0 \end{cases}$$

where x denotes the initial amount of material in the storage.

The total profit in a cycle is $H(T(x), x, y, c)$. Since it is a random variable we consider its expectation expressed as

$$M(x, y, c) = E(H(T(x), x, y, c)) \tag{11}$$

as a function of the initial amount of material x, the volume of the storage y and the withdrawing rate c.

To illustrate the profit and the change of the amount of material in the intermediate storage we present *Fig. 2* and *Fig. 3*.

In computation of processes shown in Figs. 2 and 3, the time interval and the amount of material filled into the storage were exponentially distributed for which the following parameter values were used: $x = 5$, $y = 8$, $c = 2$, $\mu_f = 2$, $\mu_g = 2.5$, $\Lambda_k = 1$, $\Lambda_1 = 0.9$, $\Lambda_2 = 3$, $\delta = 0.1$, $Q(y) = 0$, $I = 0$.

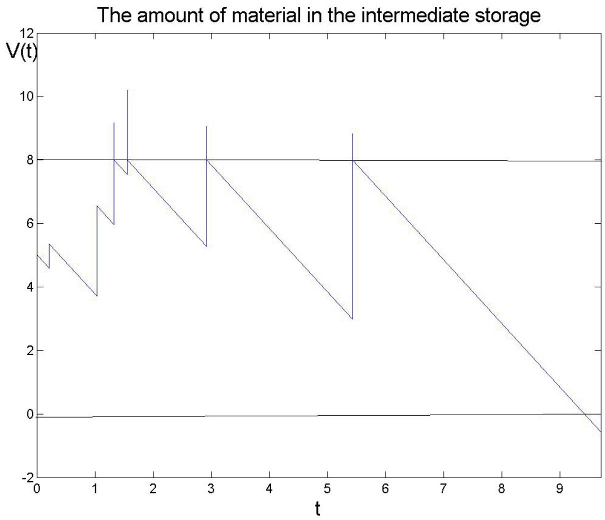

Figure 2: Evolution of the amount of material in the intermediate storage

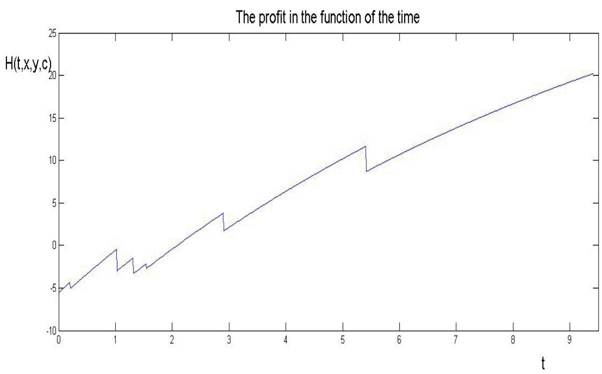

Figure 3: Evolution of the profit as a function of time

Fig. 2 illustrates well that in the generated cycle extra amounts of material were obtained four times, which were sold on market. At the same time, Fig. 3 shows negative profit at the beginning of the process, Naturally, the reasons of that are the expenses of the initial amount of material and, in general case, also the cost of the investment.

Note that this model was investigated previously by Avanzi et al. [6] in insurance mathematics in the special case of $\Lambda_2 = 0$ and $\Lambda_k = 0$. They set up an integral equation for the expectation of the profit and they presented an analytical solution for it in the case of exponential distributions. Actually, our aim is to set up an integral equation satisfied by the expected profit $M(x, y, c)$ when the case $\Lambda_k = 0$ but $\Lambda_2 \neq 0$ holds.

Integral equation for the expected profit

Let c and y be fixed and let we investigate the function $M(x, y, c)$. Obviously, if $y < x$ then

$$M(x, y, c) = x - y + M(y, y, c) \tag{12}$$

On the basis of the renewal theory one can prove that for $0 \leq x \leq y$ $M(x, y, c)$ satisfies the following integral equation:

$$M(x,y,c) = \int_0^{\frac{x}{c}} \int_0^{y-(x-ct)} e^{-\delta t} M(x - ct + z, y, c) g(y) f(t) dz dt +$$
$$+ \int_0^{\frac{x}{c}} \int_{y-(x-ct)}^{\infty} e^{-\delta t} \left(\Lambda_1 (x - ct + z - y) + M(y, y, c)\right) g(z) f(t) dz dt \tag{13}$$
$$+ \Lambda_2 \left(1 - F\left(\frac{x}{c}\right)\right) \int_0^{\frac{x}{c}} e^{-\delta t} dt.$$

This integral equation is inhomogeneous due to the last two expressions on the right hand side. Consequently there is no method to solve it analytically even in special cases. As the upper bound of the integral is infinity, it is difficult to handle it numerically as well. Therefore we elaborated a Monte Carlo simulation method to determine the approximate values of $M(x, y, c)$ in the general case.

Monte Carlo simulation for determination of the expectation of the discounted profit

The basic concept of simulation is the fact that although the process parameters are continuous it is enough to investigate the process in discrete moments of time $\sum_{j=1}^{i} t_j$, $i = 1,2,...$, handling the system as a discrete event. The moments of time are given by $\sum_{j=1}^{i} t_i$ when finite jumps happen and when extra income arrives. The left hand limits of function $V(t)$ in these points show if material runs out. It is also easy to compute the value of

$T_V(x)$ as well because of the linearity of function $V(t)$ in the interval $\left[\sum_{j=1}^{i-1} t_j, \sum_{j=1}^{i} t_j\right)$. Indeed, if

$$V\left(\sum_{j=1}^{i} t_j -\right) \geq 0, \quad i = 1,2,3,..,n \tag{14}$$

and

$$V\left(\sum_{j=1}^{n+1} t_j -\right) = V\left(\sum_{j=1}^{n} t_j\right) - ct_{n+1} < 0 \tag{15}$$

then material has run out in the time interval $\left[\sum_{j=1}^{n} t_j, \sum_{j=1}^{n+1} t_j\right)$ and

$$T_V(x) = \sum_{j=1}^{n} t_j + \frac{V\left(\sum_{j=1}^{n} t_j\right)}{c} \tag{16}$$

Generating the random time points t_j and the inputs Y_j to the storage we can check whether overflow would happen or not and how much extra material arises. Computing the costs and the income and finally determining the total income from production $H(t, x, y, c)$ can be calculated for any realization. As the average of the sample is the best estimator of the expectation we can compute the profit for every realization and take the average. The probability of running out of material is estimated by the relative frequency of this event.

To show the correctness of the simulation we have compared the results getting on the basis of the analytical formula given by Avanzi et al. [6] in special case. The results obtained from the analytical solution and by using Monte Carlo simulation have proved practically identical as it is shown in *Fig. 4*. Here, all simulation runs were carried out repeatedly for N=10000 times.

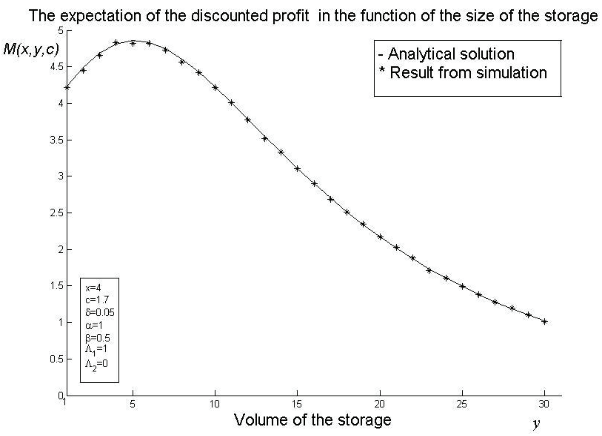

Figure 4: Comparison of the analytical and simulation results

Simulation results for the general case

Let us turn to the investigation of the general model. First we consider it in a single cycle. The expectation of the discounted profit is presented in *Fig. 5*. The expectation of the discounted profit is dependent on the storage volume and the initial amount of material. Again, the investment cost was computed by the formula $Q(y) = k{\cdot}y^{0.6}$. The initial amount of material-storage volume-$M(x, y, c)$ surface, shown in Fig. 5, was computed using the parameter values: $c = 2.5$, $I = 2$, $\delta = 0.01$, $k = 3$, $\mu_f = 0.55$, $\mu_g = 2.5$, $\Lambda_k = 2$, $\Lambda_1 = 1.8$, $\Lambda_2 = 3$.

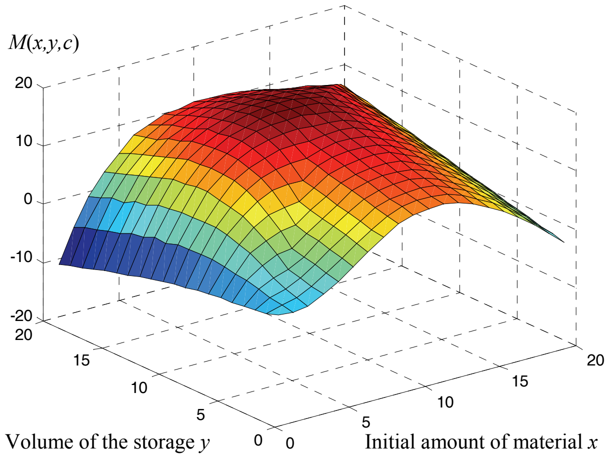

Figure 5: The expected profit as a function of the initial amount of material and the volume of the intermediate storage

If we fix the initial amount of material then the expected profit as a function of the volume of intermediate storage is a single variable function. *Fig. 6* shows this function as a section of the three-dimensional graph of Fig. 5 at $x = 9$. The other parameters were identical with those used in computing the surface in Fig. 5.

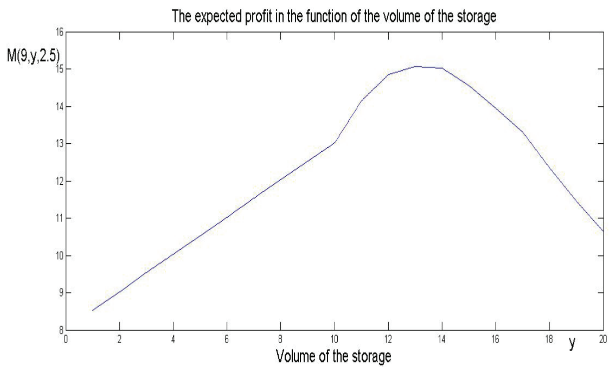

Figure 6: The expected profit as a function of the volume of the intermediate storage

We have determined the maximum of the function shown in Fig. 6 numerically by applying golden section. The maximal value is equal to 14.8 which is achieved at the storage volume $y = 13.3$.

Avanzi et al. [6] proved that in the model considered by them the optimal size of the storage does not depend on the initial amount of the material. We have carried out some numerical experiments obtaining identical result: if $\Lambda_k = 0$ then the optimal size of the storage is approximately constant. Consequently, the value of the maximum profit depends on the initial amount of material but the optimal volume of the storage does not. The simulation results on which this conclusion was drawn are presented in *Fig. 7*.

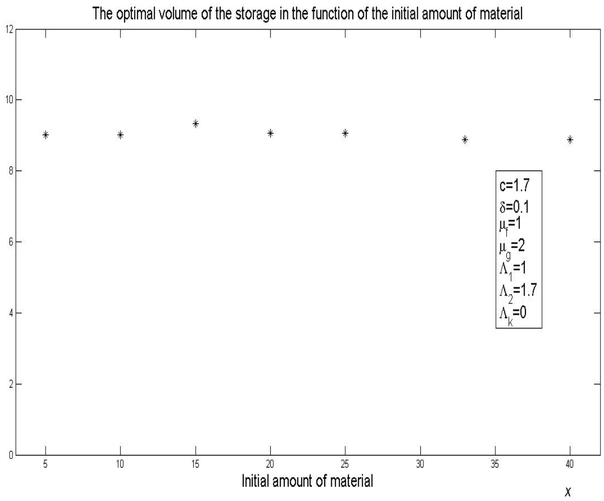

Figure 7: Optimal volume of the intermediate storage as a function of the initial amount of material

We also investigated by simulation the case when the production in the system is restarted following run out of the material. The initial amount of material in the cycle is the filled material after running out. The factory does not have income from production between the time of run out of material and that of restart. It is worth to restart the production only in such cases when the expected profit of a cycle exceeds the cost of the restart. In this case the initial expenses are the costs of start I and the cost of filled amount of material but there are no investment costs.

In *Fig. 8*, the expected profit is presented as a function of the initial amount of material when modelling the production process with possible restart. In these computations the values of parameters were as follows: $\Lambda_k = 2$, $\Lambda_1 = 1.8$, $\Lambda_2 = 3$, $T = 100$, $y = 10$, $c = 2.5$, $\delta = 0.01$, $I = 2$, $\mu_f = 0.55$, $\mu_g = 2.5$, $k = 3$.

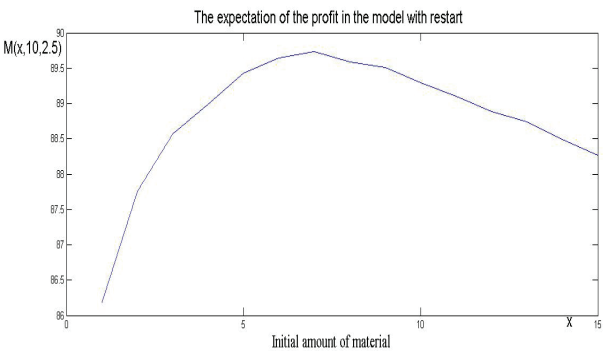

Figure 8: The expected profit as a function of the initial amount of material

It is seen in Fig. 8 the function describing the expected profit increases to about $x = 7$. Here it has a maximum and it decreases when the initial amount of the material exceeds 7. This maximum has been determined numerically.

When the restart of production in the system is possible then it is an interesting question how much is the optimal withdrawing rate to ensure maximum profit up to a certain moment of time. In *Fig. 9*, the expected discounted profit is presented as a function of the withdrawal intensity for the case when the initial amount of material and the size of the storage are fixed.

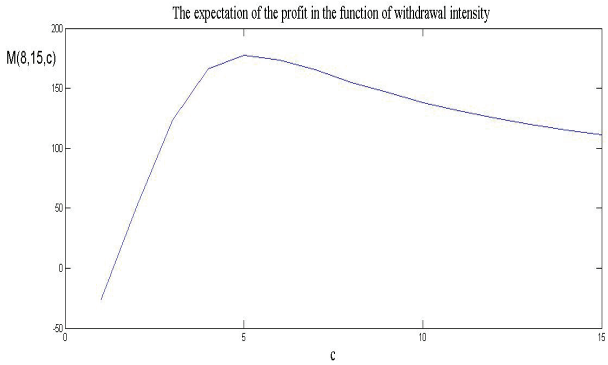

Figure 9: The expectation of the profit as a function of the withdrawal rate

In these computations the values of parameters were as follows: $\delta = 0.01$, $x = 8$, y=15, $k = 3$, $\Lambda_k = 2$, $\Lambda_1 = 1.8$, $\Lambda_2 = 3$, $T = 100$, $\mu_f = 0.55$, $\mu_g = 2.5$.

The maximum value was determined again by golden section and it is proved to be 175.8 at $c = 5.5$.

Conclusions

The properties and behaviour of a production system, consisting of batch units of an input subsystem and a continuously operated deterministic output subsystem, which are coupled by intermediate storage, were investigated. It has appeared that when the uncertainties of batch sizes and the moments of time of filling those to storage are so significant that these cause some randomness of operation then the system can be modelled by means of the tools of discrete event systems.

Taking into consideration the discounted expenses and income from production the expected value of the total profit was formulated. An integral equation, determining the expected profit as a function of the initial amount of material, the storage volume and the withdrawing rate was presented and solved by simulation using Monte Carlo method.

Optimization of the system with the expected profit as the goal function revealed that the maximum profit depends both on the initial amount of the material to be processed by the system and the volume of the intermediate storage. If the production can be restarted when run out of material occurs then the expectation of the profit exhibits optimum also with respect to the withdrawal rate of material for the continuous subsystem.

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