2nd Conference for PhD students in Civil Engineering
CE-PhD 2014, 10-13 December 2014,Cluj-Napoca, Romania
www.cephd.ro

Comparative analysis of multi-dimensional heat flow modeling

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Abstract

This paper contains a comparative analysis of multi-dimensional heat flow modeling methods, and examines the calculated results changing by the model's dimension reduction. The possibilities of two heat flow modeling methods are also tested in order to identify the method that should be applied for a complex model at heat conduction problems. In the first part of the paper the fundamentals of thermal bridges and the necessary information about the used numerical calculations are summarized. Afterwards, comparative analyses are shown using the results of two and three dimensional simulated and one dimensional calculated models based on two tasks. The results show that due to the model simplifications needed to FDM, both sections' heat losses are higher than the results given by FEM in a more realistic model. The analyses in this paper can help engineers and architects to solve building structures multi-dimensional thermal conduction problems using the optimal method and tool providing the required result accuracy.

Keywords: Building Physics, Thermal Bridges, Thermal Simulation, Multi-dimensional Heat Flow, Heat Transfer, Building Components, Building Structures, FEM, FDM

1. Introduction

Nowadays, during the design of more and more energy efficient buildings, it is inevitable for structural designers to calculate accurately with heat transfer coefficients of the building envelope as well as the building physical and energy parameters at the design stage. The experts can use several computer simulation software products to calculate thermal bridges (linear thermal transmittance). There are special software products to determine the heat loss at joints of building parts, and there are ones developed for industrial purposes, that are also applicable for buildings. The software products have different constraints, and apply different simplification methods to simulate the reality, in terms of geometry, specifying boundary conditions, and numerical calculation methods used for the differential equation of the heat flow. In this article I compare the widespread simulation techniques and methods, their applicability conditions, as well as the accuracy of their results in two examples.

2. Fundamentals of thermal bridges

The thermal bridge is a part of the building structure, where comparing to average heat-flow of surrounding elements, there is a distinct, multi-dimensional heat-flow, i.e. the gradient of heat-flow changes. The easiest way to recognize a thermal bridge zone is the detection of the change of

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surface temperature and the relative change of slope of isothermal lines within the structure. The most easily identifiable type of thermal bridge, the geometrical thermal bridge is developed, where the geometry or shape of the building structure changes. There is a geometrical thermal bridge at a wall corner, at all overhanging structural elements, at the elements connecting in different angles, and the change of sizes of identical structural elements (fig.1.a). Where the material of a building structure changes but the geometry does not, there is material thermal bridge. Common example is a pillar in a wall made of different material, but with the thickness of the wall (fig.1.b). In case of structural thermal bridge, both mentioned types are simultaneously occurred. Structural thermal bridges are e.g. penetrations, openings, holes. A slab of a cantilevered balcony, or the penetrations of mechanical pipes are examples for this type (fig.1.c).

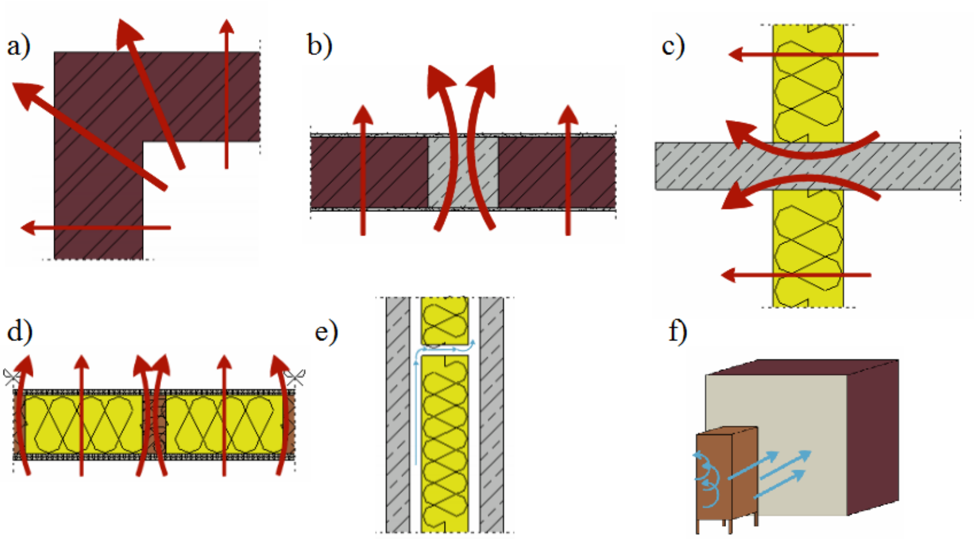


Figure 1. Types of thermal bridges, a) Geometrical, b) Material, c) Structural, d) Periodically repeating, e) Convective, f) Environmental dependent

Due to their frequent occurrence, it is worth dealing with periodical thermal bridges separately, that are repeated regularly according to a well-defined pattern within a structural element. Therefore, in practice, during the calculation of heat losses, it is calculated as it would be one-dimensional heat-flow. The effect of thermal bridges is taken into consideration by increasing the average thermal transfer coefficients of structures. Common example is the frame structure of walls, or the rafters of a pitched roof (fig.1.d). The unplanned air movements within a structure is called convective thermal bridge [6]. The air convection may increase the filtration heat losses, moreover, the thermal conductivity of materials contacting with the airflow having different temperature and humidity can also change. Examples for convective thermal bridges are the gaps within a building structure, inaccuracies of construction joints (fig.1.e). It is possible, that thermal bridges are developed due to not the structure, but different environmental impacts. Due to environmental differences, surface resistances are different, since the air velocity may differ. (It influences the convective heat transfer coefficient.) It is also possible, that a wall section is exposed to different radiations (or at another section, the identical radiation is shaded), consequently the elements with different surface temperatures have different radiation heat transfer coefficients. If a heater device is installed or a furniture is placed in front of a wall section, such thermal bridges can be created easily. These effects are not considered in international standards; however, the Hungarian MSZ-04-140-2:1991 standard, which was withdrawn in 2012 January, recommends 30-50% decrease of the thermal transfer coefficient due to the effect of furniture (fig.1.f).

3. Numerical heat flow simulation methods

3.1 Thermal problem to solve

The problem of a heat transfer during a structural and energy design of a building usually contains solids only, moreover, the model is investigated at steady-state, during which the temperature distribution is sought at constant pressure. The material properties are defined by the average, built-in thermal conductivities according to the concerning standard [3] and data sheet's of the used products, the states within the solid are described by using the Fourier's law of heat conduction. For boundary conditions, the surface (heat transfer) resistances of the boundaries were used according to [4], that describes the convective heat transfer as a function of the wind speed at the surface; and defines the radiative heat exchange as a function of surface emission factor and the air temperature near the surface. The differential equation to solve is the following:

$$\nabla(\lambda \nabla T) = 0 \qquad (1)$$

where: λ Thermal conductivity [W/mK]
T Absolute temperature [K]
∇ One dimensional vector differential operator

In the Hungarian practice, during building energy calculations, linear thermal transmittances are calculated using internal lengths and surfaces, but respecting the proposals of [3].

3.2 Numerical simulation methods and modelling

The two most common methods being applicable to solve heat conductivity problems are the finite element method (FEM) and the finite difference method (FDM) providing numerical solutions for approximated solutions of partial differential equations. The FEM approximates the solutions of the differential equations describing the whole studied domain by simpler equations for the subsets of the whole domain and joining the solutions; while the FDM discretizes the continuous functions in terms of time and space. It describes it for finite number of points and for specified times [1]. It produces difference equations from the differential equations, and approximates the derivatives in the differential equations by difference coefficients.

In practice, in case of thermal simulation software products, the differences resulting from the two methods are noticeable. For the tested FDM software, the model is built of cuboids. Therefore, the resolution of grid is a simple process, since the grid has to be perpendicular to the global coordinate system. As for a disadvantage the local grid compression in FDM products is not solved yet. In some systems, the solution is approximated by an automatic grid resolution, but according to a given factor, in a way, that the end of the simulation, when the iteration process is stopped, determined by the given biggest difference between the result of the simulation and the previous result. In case of software products applying FEM, almost any shape can be investigated, the approximation of solids is occurred by resolution to finite elements with different shapes; and the meshing requires more expertise. The software products developed for heat transfer calculations often do not provide the opportunity for optimizing the grid, they rely on automated mesh generation algorithms. However, the optimization of grid compression, in order to reach the desired accuracy, that obviously influences the calculation time as well, is integrated into every FEM software product. After creating the model, and defining the material properties, that are usually the thermal conductivities in each direction in case of steady-state simulations, the boundary conditions and the heat loads should be given. In some software products the boundary conditions are simulated as separated virtual solids, while in other products they are assigned to the free surfaces. After these steps, if the finite element mesh is done, calculation ran, the results can be evaluated.

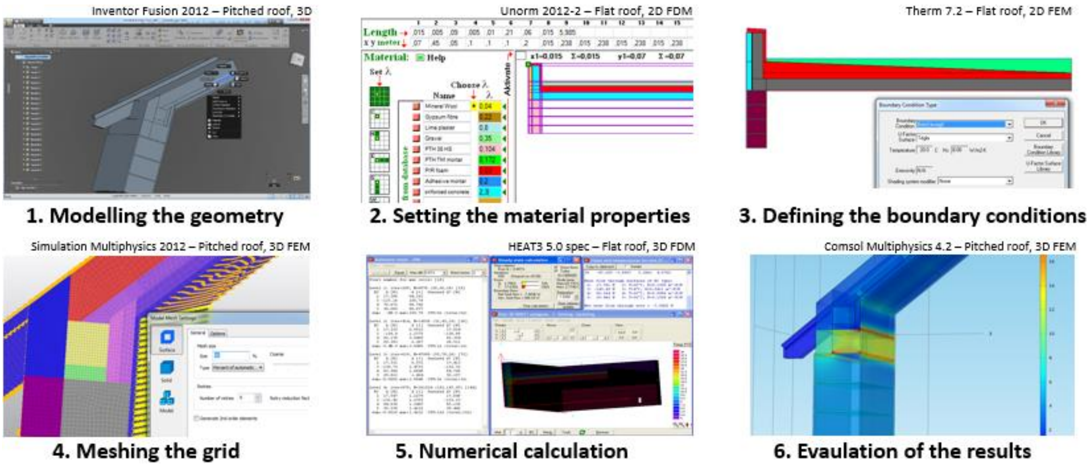


Figure 2. Method of thermal simulation presented with various models and software

4. Analysis of 2D and 3D FEM on periodically repeating pitched roof elements

In the first task in this article a heat flow simulation of a building joint is shown, that is not available in European standards being currently in force. A periodically repeating section of the pitched roof is simulated based on finite element method, the results of two- and three-dimensional simulations, as well as the traditional, one-dimensional simulations based on inhomogeneous cross-section calculations [4] are compared. During modeling the structural joints, choosing the model dimension is a main issue. It may be obvious, that the most detailed results are obtained by three-dimensional modeling; however, if two-dimensional models are used for modeling a structure, the calculation time can be reduced significantly. Creating a three-dimensional model does not require much more time comparing to a two-dimensional modeling. Applying a two- or three-dimensional model is also a financial question, since currently there are free two-dimensional FEM and FDM software products; however, the program packages with three-dimensional calculation capability are quite expensive.

If a structure is simulated in one plane, it is often necessary to simulate several cross-sections in order to follow the changes within the structure; afterwards the results should be averaged according to a method (usually the frequency of a certain cross-section.)

To show this phenomenon a pitched roof with 40° inclination is investigated, during which cross-section in two directions are created, since the distance between the axis of the 10x15 cm rafters is 1,0 meter, and the 15x15 cm purlins are connected to the reinforced concrete ring beam on 20x15x5 cm wood planks. The horizontal cross-sections are necessary (fig.3. 5.-6. sections) due to the 5x5 cm battens placed perpendicularly to the rafters, and the mineral wool thermal insulation between them. Due to the inclination of roof, FEM is applied.

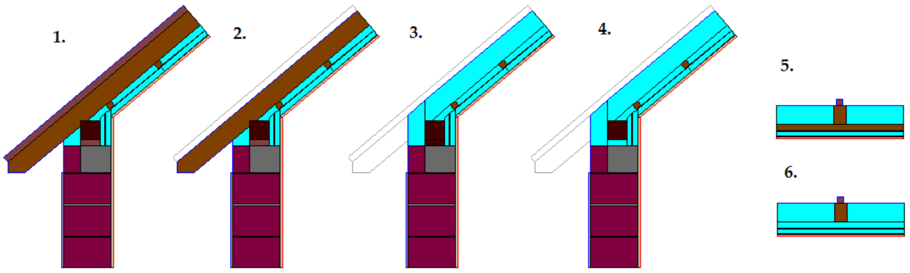


Figure 3. Sections of the pitched roof element for 2D FEM simulation [5]

The material properties applied for calculations and simulations are listed in Table 1. The boundary conditions on the interior side is 20 °C, and R_{si} = 0,131 m^2K/W according to the EN ISO 6946:2007. The surface resistance on the exterior side is declared as R_{se} = 0,041 m^2K/W in case of 4 °C average temperature during the heating season according to the Hungarian regulation of building energy calculations and 4 m/s as an average wind speed.

Nr.	Materials	λ [W/mK]	Nr.	Materials	λ [W/mK]
1.	Mineral wool thermal insulation	0,04	7.	Porotherm 38 HS building block	0,104
2.	Counter battens	0,13	8.	Porotherm therm. ins. ringbeam block	0,101
3.	Gypsum fibre board	0,22	9.	Porotherm TM therm. ins. mortar	0,172
4.	Battens	0,13	10.	Rafter	0,13
5.	Interior lime plaster (1,5 cm)	0,80	11.	Purlin	0,13
6.	Wood plank	0,13	12.	Reinforced concrete	2,3

Table 1. Thermal conductivity of the materials of the pitched roof

The two-dimensional heat flow simulation was carried out with THERM 7.2 software, and the results of the six cross-sectional areas were averaged proportionally. In case of the three-dimensional simulation, the model of the whole building assembly was built, and applying the building materials and boundary conditions listed above, the FEM simulation was carried out with Comsol Multiphysics 4.2 software. The results are summarized in Table 2.

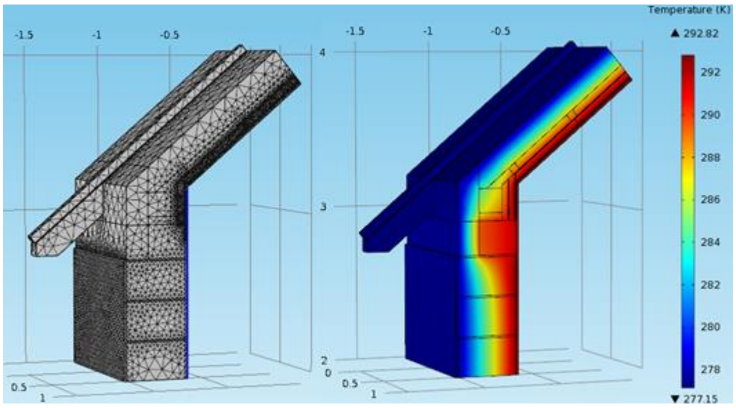


Figure 4. Meshed 3D FEM model and temperature distribution of a pitched roof element

The one-dimensional calculation was carried out according to the manual calculation method described in the EN ISO 6946:2007 standard 6.2. Total thermal resistance of a building component consisting of homogeneous and inhomogeneous layers. During this process the structural element is divided into sections being perpendicular and layers being horizontal to the direction of the heat flow that are considered thermally homogeneous. The total thermal conductivity of a structural element is calculated as the arithmetical average of an upper and lower limit. During the calculation of the upper limit, one-dimensional heat flow, which is perpendicular to the front surface, is assumed, therefore the whole thermal conductivity of each strip can be calculated and summed; while in case of the lower limit, the sections being horizontal to the front surface are considered isothermal surfaces, for each surface an equivalent thermal resistance is calculated and summed. From the total thermal resistance of the structural element calculated from the upper and lower limit of thermal resistances, the interior and exterior temperature, and the size of the surface, the heat flow can be calculated.

During the calculation the same material properties, geometrical sizes and boundary conditions were applied, as during the simulations. The result are listed in Table 2.

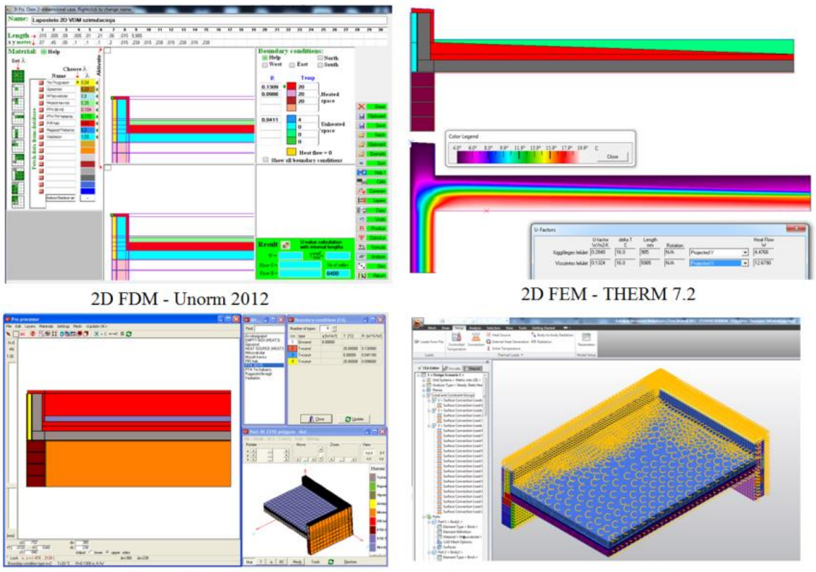
Surface	1D calculation [W]	2D FEM sim. [W]	3D FEM sim. [W]
Jamb wall, vertical, inside [1,184 m^2]	7,67	4,8081	5,4433
Roofing, pitched, inside [0,993 m^2]	2,81	2,8611	2,9404
Total inside surface of component [2,177 m^2]	10,48	7,6692	8,3837

Table 2. Results of the 1D-2D-3D calculations of the pitched roof element

Based on the results, it is concluded, that 1D manual calculation process overestimated the heat loss of the 1 m high section of the pitched roof by 25%; however, the 2D simulation erred “against safety” comparing to the 3D simulation. However, the results at the roof section differ slightly, but at the jamb wall, due to the complexity of that joint, there are large differences between the calculated/simulated heat losses. Based on this investigation, the heat losses of complex joints should be simulated with a 3D FEM software. However, if the heat loss of the roof structure is examined, the manual calculation according to the relevant standard gives a difference less then 4,5% comparing to the 3D simulation, close to the accuracy of the 2D simulation, that differed by 2,7%. However, it is not negligible that in case of more complex structures the manual calculation according to [4] is more time-consuming than performing a 2D simulation.

5. Analysis of 2D and 3D FDM and FEM on flat roof elements

In case of the investigated flat roof, the sloping layer required for drainage is polyisocyanurate foam (PIR foam) thermal insulation. Therefore, the heat transfer coefficient cannot be calculated as one-dimensional heat flow, because heat flows are different at every part of the roof structure (due to the different resistances). The EN ISO 6946:2007 standard, however, specifies a calculation method, which can approximate the average heat transfer coefficient of the insulation layer in such cases. During this task, the methods of the standard are compared with the results of the two or three-dimensional simulations. The simulations are conducted by FEM and FDM as well, and in the latter case, due to the software applying this method, the structure is modelled by the average thickness of the thermal insulation layer, showing the deviation of the average (but in reality varying) heat transfer coefficient to a more realistic model.



2D FDM - Unorm 2012

2D FEM - THERM 7.2

3D FDM - HEAT3 5.0 special

3D FEM - Simulation Multiphysics 2012

Figure 5. 2D and 3D FDM and FEM software interfaces

During the two-dimensional simulation, a 6 meter length section of the flat roof in longitudinal direction, while in the three-dimensional case a corner is investigated, which internal dimensions are 6 m x 4 m lenght. The software products applied are shown in Figure 5.
The surface resistance at the interior side is R_{si} = 0,099 m^2K/W according to the calculation method of the EN ISO 6946:2007 if the constant internal temperature is 20° C, and there is a heat flow upwards, the boundary condition at the exterior side is the same as it is described in 4. above. The material properties are summarized in the following Table 3.

Nr.	**Materials**	**λ [W/mK]**	**Nr.**	**Materials**	**λ [W/mK]**
1.	Mineral wool thermal insulation	0,04	5.	Porotherm 38 HS building block	0,104
2.	Gypsum fibre board	0,22	6.	Porotherm TM therm. ins. mortar	0,172
3.	Interior lime plaster (1,5 cm)	0,8	7.	PIR foam therm. ins. (10-30 cm)	0,03
4.	Gravel ballast layer (5 cm)	0,35	8.	Reinforced concrete slab (20 cm)	2,3

Table 3. Thermal conductivity of the materials of the flat roof

5.1 Comparison of 2D FEM and FDM at an intermediate flat roof section

During the FDM simulation, since rectangular geometry can be modelled only in the applied software, the varying thickness of the insulating layer have to be considered with its equivalent thickness. The PIR thermal insulation on the slab is 30 cm at the parapet wall, while at the center of the building it is 10 cm, with a slope of 3.3%. The equivalent thickness is determined according to the EN ISO 6946:2007 C.2, that describes the calculation method of the average thermal resistance in case of smoothly varying layer thickness along two opposite edges, for a rectangular form. From the thermal resistance, if the heat transfer coefficient is constant, the equivalent thickness can be calculated. In case of FEM simulations some simplifications were applied, that are common in thermal simulations (i.e. thin technological layers are omitted); however, the designed geometry with varying insulation thickness was used. The results are presented in the following Table 4.

Surface	**2D FDM sim. [W]**	**2D FEM sim. [W]**
Interior wall, vertical intermediate section, interior [5,91 m^2]	28,566	26,8596
Interior horizontal surface, intermediate section, interior [35,91 m^2]	105,264	76,0740
Total inside surface of components [41,82 m^2]	133,83	102,97

Table 4. Results of the 2D simulations of the flat roof element

The 2D FDM simulation resulted in nearly 30% more heat loss. The difference is due to the effect of the geometric simplification, since the application of equivalent thickness of thermal insulation instead of the decreasing thickness from the corners resulted in a significantly larger heat loss at the joint of the interior wall and the slab, overestimating the effect of thermal bridge.

5.2 Comparison of 3D FDM and FEM at the corner of a the flat roof

In case of a three-dimensional FDM simulation, similar simplifications of the geometric model are necessary as in the previous section. However, at the corner the equivalent thickness of a triangle-shape layer with a decreased thickness at the point in front of the leg of the triangle was calculated according to the mentioned standard [4]. The insulating layer’s diagonal slope is 5% and varies with both direction. In case of the finite element method simulation the designed geometry was built, that approximates the reality better, than the finite difference method. The grid resolution was optimized so that the unallocated heat flows are less than 0.2%, which is only one-tenth of the standard’s required value [2], thus it is considered an exact solution. The results are summarized in Table 5.

Surface	**3D FDM sim. [W]**	**3D FEM sim. [W]**
Interior wall, vertical, corner joint, interior [9,82 m^2]	48,240	36,1067
Interior horizontal surface, corner joint, interior [23,85 m^2]	85,187	76,3279
Total inside surface of components [41,82 m^2]	133,427	112,4346

Table 5. Results of the 3D simulations of the flat roof element

In the case of the three-dimensional simulation of a corner junction, FDM simulation resulted in significantly greater heat loss. The difference is 18,7%; therefore, FDM simulation with the geometric simplification (equivalent thickness of layer) according to the standard is not an adequate technique to model the thermal bridge at corners.

6. Conclusions

In this paper, the possibilities of thermal simulation methods with both 2D and 3D geometrical models and standardized calculations are tested on different roof components in order to identify the method that should be applied for a complex model at heat conduction problems.

In the case of a pitched roof component where two-dimensional simulation gives acceptable results, the manual calculations give nearly the same results, but also demonstrated that the one and two dimensional methods do not reach the accuracy of a properly implemented 3D model simulation.

The results of the flat roof simulations show that due to the model simplifications needed to FDM, both analyzed sections' heat losses are higher than the results given by FEM in a more realistic model despite the fact that the variable thickness of the insulation layer calculated by standardized procedure for the FDM simulation.

Acknowledgments

The author is grateful to Mr. Máte Orosz, PhD student, BUTE Dept. of Architectural Engineering for English peer review.

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