8th International Congress on Advanced Electromagnetic Materials in Microwaves and Optics – Metamaterials 2014
Copenhagen, Denmark, 25-30 August 2014

3D Imaging With Metamaterials

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Abstract – **Here we present a new kind of 3D display using near zero index (NZI) metamaterial in front of a 2D display. The virtual distance of a pixel is controlled by the refractive index of the metamaterial. As near zero index metamaterials are highly dispersive, a minor shift in the frequency can cause a big change in the virtual distance, hence enabling the control of depth with fine frequency tuning. We derive closed form expression for the virtual distance of a point source in case of lossless isotropic materials and provide a procedure to calculate virtual distance for arbitrary material with given material parameters. A double negative metamaterial working in the microwave regime is demonstrated as proof of the concept.**

I. INTRODUCTION

Although 3D displays are already commercially available, 3D imaging is still intensively researched [1] due to the limitations of current technologies. The most common 3D displays are stereoscopic displays which need the use of headgear and provide image only for one point of view. Promising alternatives are autostereoscopic displays which eliminate the need for headgears and can provide images for more point of view, but they need very high (spatial) definition or very high display frequency. Here we propose a novel kind of 3D display which controls the virtual distance of a pixel with simple optics instead of providing different computed images to different directions.

The concept of the display is based on the phenomena that underwater objects seem to be closer to the observer than they really are due to the refraction of light. Similar way, objects behind a slab with refractive index $n < 1$ seem to be further than they really are. If the index of refraction is small enough, a high virtual distance can be achieved with a thin layer. Furthermore, if the medium is highly dispersive, i.e. the refractive index changes rapidly between 0 and 1, a high change in virtual distance can be achieved with a minor change of frequency. Hence the distance of a pixel can be encoded in the wavelength and frequency tunable pixels can be applied for 3D imaging.

Metamaterials are artificial structures which offer the possibility to engineer nearly arbitrary optical properties, like negative [2] or ultrahigh refractive index. Zero refractive index can also be achieved at certain frequencies in the frequency range where the metamaterial shows resonant behavior. Due to the resonance high dispersion is also ensured, which makes metamaterials suitable for application in the proposed 3D display.

As a first step toward the design of the proper metamaterial structure we make calculations about the image of a point source behind a metamaterial slab. First geometrical optics is considered as an approximation. Then, as metamaterials are lossy and anisotropic, in which case geometrical optics cannot be applied, transfer matrix method is used to determine the image distance. Finally we demonstrate a metamaterial structure working in the microwave regime as a proof of concept.

II. IMAGE OF A POINT SOURCE BEHIND LOSSLESS ISOTROPIC METAMATERIAL SLAB

In case of lossless isotropic materials geometrical optics can be applied to determine the image of a point source. Rays originating from the source propagate according to Snell's law. Once the rays reach the observer they can be traced back as if there were no material in their way. The image is at the intersection of these virtual rays as shown in Fig. 1 (left). Note that the image distance can depend on the observation angle, hence symmetrical pairs of rays have to be considered. Rays can also reflect on surfaces which results in a multiple image as shown on Fig. 1 (right). The distance between the point source and its (first) image can be expressed as:

$$z = \frac{\tan \alpha}{\tan \beta} d - d = d\left(\frac{n_1}{n_2}\sqrt{\frac{1-\sin^2\beta}{1-(n_1/n_2)^2\sin^2\beta}} - 1\right) \quad (1)$$

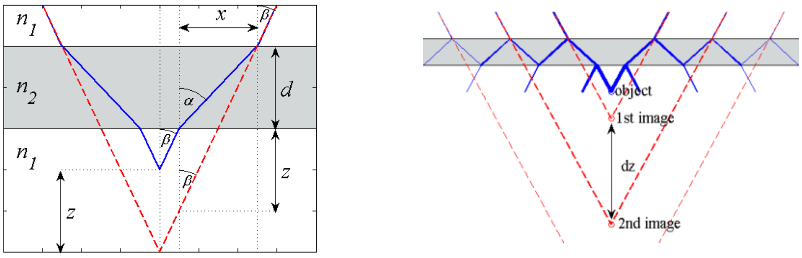

Fig. 1. Real rays (solid line) and virtual rays (dashed line) to calculate the image of a point source (left). Multiple images caused by reflections (right).

where α is the angle of refraction, β is the angle of incidence and the angle of observation, n_1 is the refractive index of air ($n_1 = 1$) and n_2 is the refractive index of the slab. The formula shows that a distance from 0 to infinity can be achieved for small angles by varying the refractive index between 1 and 0.

III. Calculation Of Image Distance For Arbitrary Homogenous Material

Metamaterials are mostly lossy or anisotropic, hence geometrical optics cannot be applied. For this case we propose a procedure that utilizes the transfer matrix method [3] to determine the direction of rays reaching the observer. From this point, rays can be traced back again to determine the image distance in function of the observation angle, the same way as discussed above. Note that time reversal method (applying transfer matrix method backwardly in the lack of the slab) cannot be used to determine the distance of the image because of its angle dependence.

Transfer matrix method allows the calculation of the electromagnetic field distribution in an image plane (the observer's side) from the electromagnetic field distribution in the source plane (the plane of the point source that is parallel to the surfaces of the slab). As a first step the electromagnetic field in the source plane is decomposed into Fourier components, each of which defines a plane wave with given tangential wave vector. In case of 2D imaging, where x is the tangential direction and z is the radial direction, the Fourier components are

$$G_{source}(k_x) = \int g_{source}(x) e^{-ik_x x} dx \tag{2}$$

where k_x is the tangential wave vector. For vacuum the radial wave vector (k_z) can be determined from the dispersion relation, while for the metamaterial slab k_z can be retrieved from transmission-reflection data at oblique incidence for any tangential wave vector [4]. The transmission of the slab is then calculated for each Fourier component. In case of TM mode, the transmission for the Fourier components of the magnetic field is

$$T(k_x) = \frac{4\, e^{ik_z^v(d_s+d_i)}}{(\alpha_2+1)(\alpha_1+1)e^{-i\,k_z^s d}+(\alpha_2-1)(\alpha_1-1)e^{i\,k_z^s d}}\;,\;\; \alpha_1 = \frac{k_z^s}{k_z^v\,\varepsilon_r^s}\;,\;\; \alpha_2 = \frac{k_z^v\,\varepsilon_r^s}{k_z^s} \tag{3}$$

where k_z^v is the radial wave vector in vacuum, k_z^s is the radial wave vector in the slab, d_s is the distance from the source to the slab, d is the thickness of the slab, d_i is the distance from the slab to the image plane and ε_r^s is the (effective) relative electric permittivity of the slab which is the function of k_x due to the anisotropy of the slab [4]. ε_r^s or $\alpha_{1,2}$ can be retrieved from transmission-reflection data at oblique incidence as well as k_z.

The magnetic field distribution in the image plane can be obtained by multiplying the Fourier components by the transfer function, and applying inverse Fourier transform for the result. The electric field vector can be calculated as well utilizing that all Fourier components describe a plane and that the image plane is in vacuum. When both electric and magnetic field vector is known in the image plane, the Poynting vector (S) can be determined for each point. As vacuum is homogenous lossless and isotropic, Poynting vectors originating from one point source will coincide with the rays originating from that point providing that there is no interference. As reflection causes interference in the image plane they have to be eliminated. This can be done for each plane wave by multiplying the transfer function by $[1-(\Gamma_2)^2 e^{-ik_z 2d}]$ where $\Gamma_2 = (1-\alpha_2)/\,(1+\alpha_2)$ is the reflection coefficient on the slab-vacuum interface. If reflections are removed, the distance of the (first) image can be determined by considering the direction of the Poynting vectors. The image will be seen by the observer where these directions intersect the x=0 axis as the x coordinate of the source will not change for symmetry reasons.

IV. Calculating Virtual Distance For A Double Negative Metamaterial

As a proof of concept a Fishnet structure (Fig. 2, left) is demonstrated working in the microwave regime. The effective parameters (k_z, $\alpha_{1,2}$) are retrieved for every incident angle combining the methods described in [4] and [5]. As it can be seen on Fig. 2 (rigth) the structure has a nearly isotropic dispersion relation for some frequencies and for angles of incidence less than 30°. The retrieved parameters are used in the transfer matrix method, which is applied for a source emitting waves with incident angles less than 30° (Fig. 3, left). Image distances are calculated for some frequencies in the near zero index frequency range (Fig. 2, right). The calculations show, that relatively high virtual distance change can be achieved with a minor frequency shift, while there are no high losses.

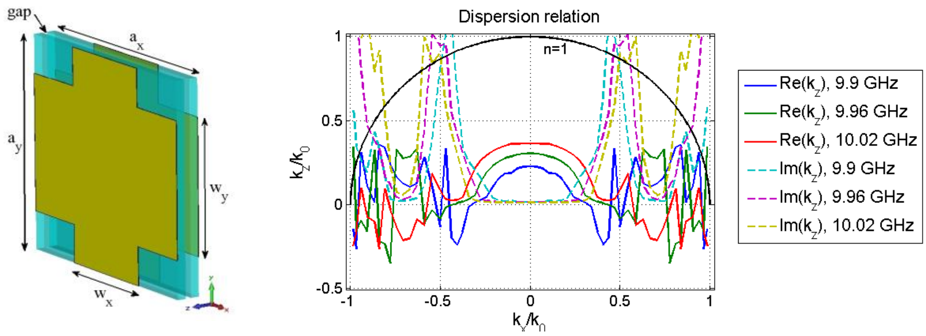

Fig. 2. The unit cell of the Fishnet structure (left). The relation between k_x and k_z in the metamaterial (right).

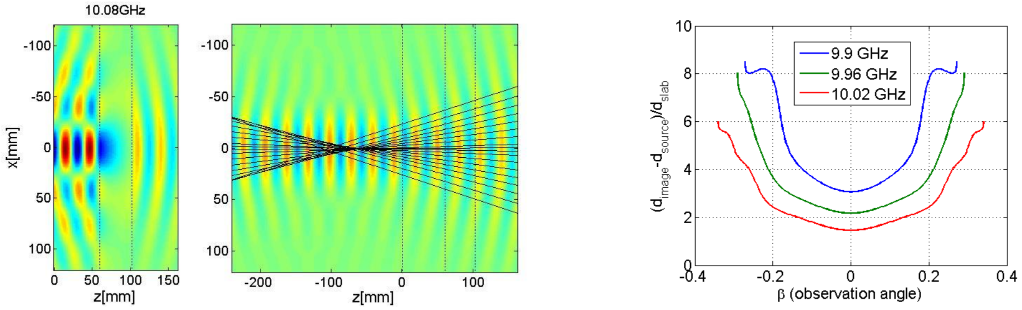

Fig. 3. Wave propagation through the metamaterial slab (left), time reversed waves and virtual rays traced back from the image plane (middle), image distance from the original position relative to the thickness of the slab (right).

V. Conclusion

We proposed a new kind of 3D display using near zero index metamaterial slab in front of a 2D display. As a proof of concept we retrieved the effective parameters of a Fishnet structure for a number of incident angles and calculated the image distance of a point source, placed behind the Fishnet structure, for some frequencies applying a procedure based on transfer matrix method and geometrical optics. It was shown that fine frequency tuning results in the change of image distance as it was expected.

Acknowledgment

This work has been supported by the Bolyai János Fellowship of Hungarian Academy of Sciences, the EUREKA project MetaFer and KMR-12-1-2012-0008 of the National Development Agency Hungary.

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