A new and finite family of solutions of hydrodynamics: Part III: Advanced estimate of the life-time parameter

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We derive a new formula for the longitudinal HBT-radius of the two particle Bose-Einstein correlation function from a new family of finite and exact, accelerating solution of relativistic perfect fluid hydrodynamics for a temperature independent speed of sound. The new result generalizes the Makhlin-Sinyukov and Herrmann-Bertsch formulae and leads to an advanced life-time estimate of high energy heavy ion and proton-proton collisions.

1. Introduction

This manuscript is the third part of a manuscript series. This series presents various applications of a new, accelerating, finite and exact family of solutions of perfect fluid hydrodynamics, the recently found Csörgő - Kasza - Csanád - Jiang (CKCJ) family of solution of ref. [1]. The first part of this series [2] fixes the notation, summarizes this class of exact solutions and evaluates the rapidity and pseudorapidity density distributions. The second part [3] evaluates the initial energy densities in high energy collisions [1], and provides a fundamental correction to the renowned Bjorken estimate of initial energy density [7].

In this manuscript, we evaluate the Bose-Einstein correlation functions in a Gaussian approximation from the CKCJ solutions [1]. Given that the considered dynamics is a 1+1 dimensional expansion, we evaluate R_L, the Hanbury Brown - Twiss (HBT) radii in the longitudinal (beam) direction. This longitudinal HBT radius parameter is proportional to the mean freeze-out time of the fireball, thus the advanced evaluation of its transverse mass dependence and its constant of proportionality for finite, longitudinally non-boost-invariant fireballs may have important physics implications on life-time determinations.

2. Bose-Einstein correlations and the longitudinal HBT radii

In high energy heavy ion collisions, Bose-Einstein correlation functions (BECF) measure characteristic sizes of the particle emitting source, corresponding to lengths of homogeneity [11]. In high energy heavy ion collisions, the particle emitting source can be approximated as a locally thermalized fireball, surrounded by a halo of long-lived resonances, this is the so-called core-halo picture The momentum dependent intercept parameter λ_* of the two-particle Bose-Einstein correlation function can be interpreted in the core-halo picture of ref. [13] as follows:

$$\lambda_* = \left(\frac{N_c}{N}\right)^2 = \left(\frac{N_c}{N_c + N_h}\right)^2 \tag{1}$$

where $N = N_c + N_h$ is the total number of the emitted particles with a given momentum, adding the contributions from both the core N_c and the halo, N_h. The fireball that undergoes a hydrodynamical evolution corresponds to core [13]. For locally thermalized sources, the lengths of homogeneity are expressible in terms of the derivatives of the fugacity, $\exp\left(\mu(x)/T(x)\right)$ and the locally thermalized momentum distribution, $\exp\left(-k^\mu u_\mu(x)/T(x)\right)$, corresponding to the so called geometrical and thermal length scales [13]. Assuming an effective Gaussian source for the core particles, the BECF can be expressed in terms of the Bertsch-Pratt variables as follows:

$$C(\Delta k, K) = 1 + \lambda_* \exp\left(-R^2_{side}Q^2_{side} - R^2_{out}Q^2_{out} - R^2_L Q^2_L - 2R^2_{out,L}Q_{out}Q_L\right). \tag{2}$$

All the fit parameters (λ_*, R_{side}, R_{out}, R_L and $R^2_{out,L}$) depend on the mean momentum of the particle pair, $K^\mu = 0.5(k_1^\mu + k_2^\mu)$. The four-momentum of a given particle is denoted by $k = (E_k, \mathbf{k}) = (E_k, k_x, k_y, k_z)$. The three-components of the relative and mean momenta are denoted as

$$\Delta\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2, \tag{3}$$

$$\mathbf{K} = 0.5\left(\mathbf{k}_1 + \mathbf{k}_2\right). \tag{4}$$

In the Bertsch-Pratt decomposition of the relative momentum [5, 6], the principal directions are defined as follows: The *out* direction is perpendicular to the beam axis and parallel to the mean transverse momentum of the boson pair; the longitudinal direction (indicated by subscript L) is parallel to the beam axis (r_z), and the *side* direction is orthogonal to the previous two directions. This Bertsch-Pratt decomposition of the relative momentum is defined as follows:

$$Q_{side} = \frac{|\Delta\mathbf{k} \times \mathbf{K}|}{|\mathbf{K}|} \tag{5}$$

$$Q_{out} = \frac{\Delta\mathbf{k} \cdot \mathbf{K}}{|\mathbf{K}|} \tag{6}$$

$$Q_L = k_{1,z} - k_{2,z}, \tag{7}$$

If the Bose-Einstein correlation function is an approximately Gaussian in terms of the relative momenta, the Gaussian HBT radii $R^2_{i,j}$ can be introduced, with $\{i,j\} \,\epsilon\, \{side, out, long\}$. These Gaussian Bertsch-Pratt-radii can be related to the variances of the hydrodynamically evolving core, while the halo of the long-lived resonances is responsible for the effective reduction of the strength of the correlation function:

$$R^2_{i,j} = \langle \tilde{x}_i \tilde{x}_j \rangle_c - \langle \tilde{x}_i \rangle_c \langle \tilde{x}_j \rangle_c. \tag{8}$$

Here the $\langle A \rangle_c$ stands for the average of quantity A in the core, i, j stand for directions (side, out or long) and

$$\tilde{x}_i = x_i - \beta_i t, \tag{9}$$

$$\beta_i = \frac{k_{i,1} + k_{i,2}}{E_1 + E_2}. \tag{10}$$

In this manuscript, we focus on the longitudinal radius, so the radii of the *side* and *out* direction are not discussed, see e.g. ref. [13] for more details on this point. As discussed in [12], for a 1+1 dimensional relativistic source, the longitudinal radius in an arbitrary frame reads as

$$R^2_L = \left(\beta_L \sinh(\eta^s_x) - \cosh(\eta^s_x)\right)^2 \tau^2_f \Delta\eta^2_x + \left(\beta_L \cosh(\eta^s_x) - \sinh(\eta^s_x)\right)^2 \Delta\tau^2, \tag{11}$$

where η_x is the space-time rapidity, and η^s_x is the main emission region of the source, which derived by the saddle-point calculation of the rapidity density, $\Delta\tau$ and $\Delta\eta_x$ are characteristic sizes around τ_f and η^s_x. This formula simplifies a lot in the LCMS (longitudinally co-moving system) frame of the boson pair, where $\beta_L = 0$:

$$R^2_L = \cosh^2(\eta^s_x)\tau^2_f \Delta\eta^2_x + \sinh^2(\eta^s_x)\Delta\tau^2. \tag{12}$$

Our new family of solutions are finite, and limited to a narrow rapidity interval around midrapidity [1]. At mid-rapidity, if $\eta^s_x \approx 0$, the above equation can be simplied even further:

$$R_L = \tau_f \Delta\eta_x. \tag{13}$$

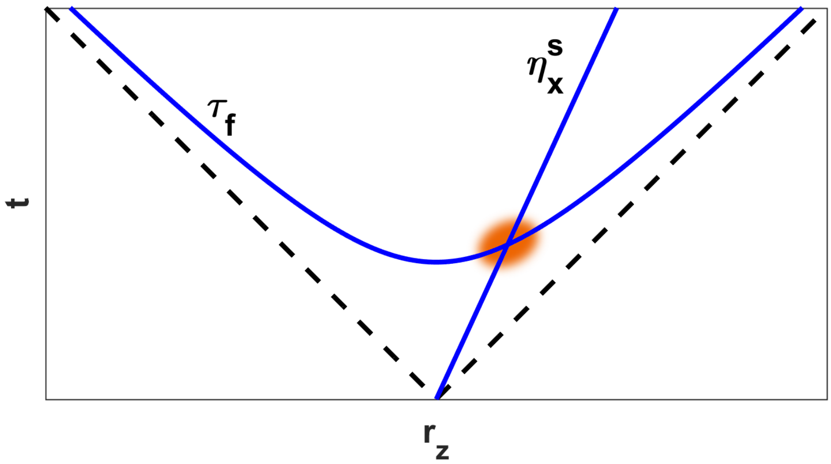

Fig. 1. Space-time picture of particle emission for longitudinally expanding fireballs.

3. Previous results on the longitudinal HBT-radius

For a Hwa-Bjorken type of accelerationless, longitudinal flow [7, 8] Makhlin and Sinyukov determined the longitudinal length of homogeneity in ref. [11] as

$$R_L = \tau_{Bj} \sqrt{\frac{T_f}{m_T}}. \tag{14}$$

In this equation, T_f stands for the freeze-out temperature, m_T is the transverse mass of the particle pair and τ_{Bj} is the mean freeze-out time of the Hwa-Bjorken solution. This result makes it possible to determine the life-time, i.e. τ_{Bj} of the reaction from the measurement of the longitudinal HBT radius parameter, provided that $T_f \approx m_\pi \approx 140$ MeV can be estimated from the analysis of the single particle spectra.

Evaluating the HBT radii from the same Hwa-Bjorken solution [7, 8], Herrmann and Bertsch obtained a more accurate result in ref. [14], using a Gaussain approximation for the longitudinal HBT radius at midrapidity, in terms of Bessel functions $K_1(z)$ and $K_2(z)$, as follows:

$$R_L = \tau_f \sqrt{\frac{T_f}{m_T}} \sqrt{\frac{2K_2(m_T/T_f)}{K_1(m_T/T_f)}}. \tag{15}$$

This formula improves the Sinyukov-Makhlin formula (14) for lower m_T/T_f values, and approaches it in the large m_T/T_f limit.

If the flow is accelerating, the estimated origin of the trajectiories is shifted back in proper-time, thus τ_{Bj} is underestimating the life-time of the reaction. The correction was estimated, based on the modification of the flow-profile, from the Csörgő-Nagy-Csanád (CNC) solution [10] as follows

$$R_L = \frac{\tau_f}{\lambda}\sqrt{\frac{T_f}{m_T}}, \tag{16}$$

where τ_f stands for the freeze-out time. In the $\lambda \to 1$ boost-invariant limit, this formula also reproduces the Makhlin-Sinyukov formula, but for the realistic $\lambda > 1$ parameter values it yields larger life-times as compared to the Makhlin-Sinyukov formula.

4. The longitudinal HBT-radius parameter of the CKCJ solution

Let us evaluate the emission function for the CKCJ solution of refs. [1–3]. The integration of the Cooper-Frye formula is performed by the saddle-point approximation. Near to mid-rapidity, the fluid rapidity is well approximated by a linear function of the space-time rapidity: $\Omega \approx \lambda\eta_x$. Using a saddle-point integration in η_x, we obtain the rapidity distribution:

$$\frac{dN}{dy} \approx \frac{\left(2\pi\Delta\eta_x^2\right)^{1/2}}{2\pi\hbar}\left[k_\mu u^\mu \frac{\tau(\eta_x)}{\cosh(\Omega-\eta_x)}\exp\left(-\frac{k_\mu u^\mu}{T_f(\eta_x)}\right)\right]_{\eta_x=\eta_x^s}. \tag{17}$$

Here η_x^s stands for the saddle-point, which is found to be proportional to the rapidity y: $\eta_x^s \approx \frac{y}{2\lambda-1}$. At midrapidity, the saddle-point vanishes and the emission function can be well approximated by a Gaussian centered on zero. The width of this Gaussian is given by $\Delta\eta_x$ as

$$\Delta\eta_x \approx \sqrt{\frac{T_f}{m_t}}\frac{1}{\sqrt{\lambda(2\lambda-1)}}. \tag{18}$$

At mid-rapidity, these considerations lead to the following longitudinal HBT-radius parameter:

$$R_L = \tau_f\Delta\eta_x \approx \frac{\tau_f}{\sqrt{\lambda\,(2\lambda-1)}}\sqrt{\frac{T_f}{m_T}}. \tag{19}$$

Surprisingly, this result is independent of the equation of state, and it is formally different from the CNC estimate.

Our result thus presents and important step forward: once the parameter λ of the acceleration is determined from the fits to the (pseudo)rapidity distributions [2], this parameter combined with the longitudinal HBT radius measurement can be used to provide an advanced estimate of the life-time of the reaction, solving eq. (19) for the life-time τ_f. The significance of our advanced formula is illustrated on Figure 2.

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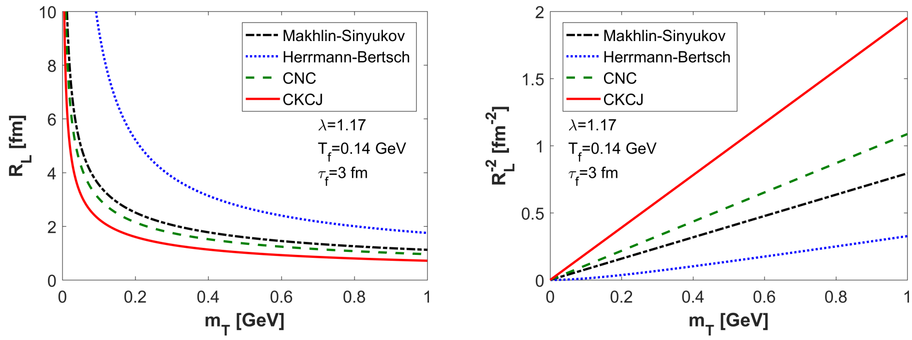

Fig. 2. The HBT radius $R_L(m_T)$ (left) and $1/R_L^2(m_T)$ (right) of the CKCJ solution are shown with solid red lines and compared to earlier estimations. The parameters correspond to fit results of the CKCJ solution to p+p collisions at $\sqrt{s} = 7$ TeV [2].

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