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ORIGINAL RESEARCH
PAPER



Method of calculation of three-layer plates with discrete joint

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ABSTRACT

The paper presents the basic principles of static calculations of plates with discrete filling-ribs; the method is based on the analytical solution of the problem of the stress state of a ribbed plate. The bottom line is to use special discontinuous functions at the stage of writing the initial assumptions of differential equations, as well as in the process of solving them due to the nature and physical meaning of the problem.

KEYWORDS

sandwich panels, stress state, discrete functions, discrete filling

1. INTRODUCTION

In the practice of designing enclosing structures of buildings and structures, three-layer panels with discrete filler occupy a large place. In these structures, the outer layers and the ribs connecting them are load-bearing elements, with insulation placed in the middle layer. The thickness of the outer layers and the dimensions of the ribs are selected from the strength conditions, and the thickness of the middle layer is selected according to the thermophysical calculation.

This article investigates the stress-strain state of three-layer plates with discrete filler. Various scientific approaches to plate computation can be found in publications [1–9]. A comprehensive review of the literature in the field of plate calculation can be found in the work [10]. The paper [11] presents a three-dimensional static analysis of single-layer and multi-layer plates and coatings. The study [12] analyzed the behavior of laminated composites using high-order or layered finite element calculations. The article [13] presents plate theories with the use of trigonometric series, exponential functions, and various polynomials. The paper [14] presents an analysis of three different sandwich panels for compression and out-of-plane shear. The paper [15] presents bending of layered beams with a core made of discrete ceramic tiles.

The use of special discontinuous functions, both at the stage of recording the initial relationships and in the desired solutions, made it possible to build an effective calculation algorithm that allows obtaining the discontinuous nature of the components of the stress-strain state of the structure with a minimum time cost of calculations.

2. CHARACTERISTICS OF THE APPROACH TO THE CALCULATION OF A THREE-LAYER PLATE

It is suggested that the structure consists of two plates - two-dimensional elements for which the Kirchhoff-Love hypotheses are valid, and ribs - one-dimensional elements for which the hypotheses of flat sections are valid. In the design scheme, the original structure is replaced by two ribbed plates loaded with an external load and unknown forces and moments applied along the lines of separation of the ribs, as it is shown in Fig. 1.

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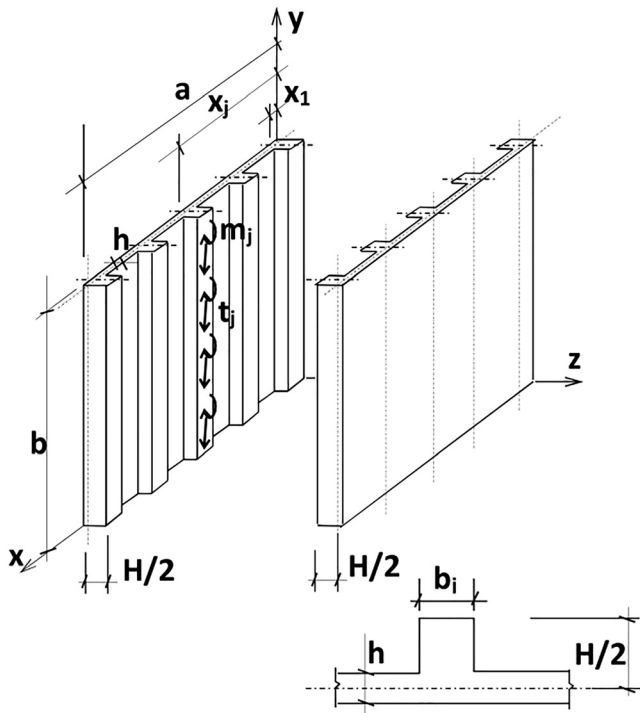


Fig. 1. Calculation scheme for panels with discrete filler in the form of ribs

According to (1), and taking into account the loads applied along the cutting lines of the ribs, the generalized forces and moments are represented as:

$$\begin{aligned} T_2^* &= T_2 + \sum_{j=1}^N [E_j F_j \epsilon_2 + t_j b_j] \delta(x - x_j), \\ M_2^* &= M_2 + \sum_{j=1}^N \left[E_j I_j \chi_2 + \frac{t_j b_j H}{2} + m_j b_j \right] \delta(x - x_j), \end{aligned} \quad (1)$$

where T_2 , M_2 are the tangential forces and bending moment in the smooth part of the plate; E_j , F_j , I_j are the modulus of elasticity, area and moment of inertia of the j -th rib; ϵ_j , χ_2 are the deformation components of the middle surface of the plate; $\delta(x - x_j)$ is the Dirac delta function; b_j is the width of the j -th edge; H - is the height of the rib; t_j is distributed force, m_j is the distributed moment.

Substituting expressions (1) into the system of equilibrium equations and strain continuity relations, using the known relations of the theory of plates, we obtain a resolving system of differential equations with respect to the normal displacement function ω and the force function F , of the form:

$$\begin{aligned} \Delta^2 \omega &= \frac{q_3}{D} - \frac{1}{D} \sum_{j=1}^N \left(E_j I_j \frac{\partial^4 \omega}{\partial y^4} + \frac{H b_j}{2} \frac{\partial^2 t_j}{\partial y^2} + b_j \frac{\partial^2 m_j}{\partial y^2} \right) \delta(x - x_j), \\ \Delta F^2 &= \sum_{j=1}^N \left[\frac{E_j F_j}{Eh} \left(\frac{\partial^2 F}{\partial x^2} - \mu \frac{\partial^2 F}{\partial y^2} \right) + t_j b_j \right] \delta''(x - x_j) \\ &\quad - \mu \sum_{j=1}^N \left[\frac{E_j F_j}{Eh} \left(\frac{\partial^4 F}{\partial x^2 \partial y^2} - \mu \frac{\partial^4 F}{\partial y^4} \right) + b_j \frac{\partial^2 t_j}{\partial y^2} \right] \delta''(x - x_j) \\ &\quad + \int_0^a \left(\frac{\partial^2 q_1}{\partial y^2} - \mu \frac{\partial^2 q_1}{\partial x^2} \right) dx + \int_0^b \left(\frac{\partial^2 q_2}{\partial x^2} - \mu \frac{\partial^2 q_2}{\partial y^2} \right) dy, \end{aligned} \quad (2)$$

where q is the intensity of the external load; D is the cylindrical stiffness; μ is the Poisson's ratio of the plate

material. $\delta''(x - x_j)$ is the second derivative of the Dirac delta. The Dirac special function can be defined as follows:

$$\delta(x) = \begin{cases} +\infty, & x = 0, \\ 0, & x \neq 0. \end{cases} \quad (3)$$

In practical applications, the generalization of the Dirac delta is used as a distribution. It has the following properties related to its derivatives,

$$\begin{aligned} \int f(x) \delta'(x - x_i) dx &= - \int f'(x) \delta(x - x_i) dx, \\ \int f(x) \delta'(x - x_i) dx &= -f'(x) H(x - x_i), \\ \int f(x) \delta''(x - x_i) dx &= - \int f'(x) \delta'(x - x_i) dx = - \left[\int f''(x) \delta(x - x_i) dx \right], \\ \int f(x) \delta''(x - x_i) dx &= - \left[\int f''(x) \delta(x - x_i) dx \right], \\ \int f(x) \delta''(x - x_i) dx &= -f''(x) H(x - x_i), \\ \int f(x) \delta^n(x - x_i) dx &= -(1)^n \int f^n(x) \delta(x - x_i) dx, \\ \int f(x) \delta^n(x - x_i) dx &= -(1)^n f^n(x) H(x - x_i), \end{aligned} \quad (4)$$

where $H(x - x_i)$ is the Heaviside's function, $H(x - x_i) = \begin{cases} 0, & x < x_i, \\ 1, & x \geq x_i. \end{cases}$

The second equation for the functions ω and F are obtained from the strain continuity relations. For the class of problems under consideration, it takes the form:

$$\frac{\partial^2 \epsilon_1}{\partial y^2} + \frac{\partial^2 \epsilon_2}{\partial x^2} - \frac{\partial^2 \omega}{\partial x \partial y} = 0. \quad (5)$$

Since the equations of the system have the same operators on the left side, it is convenient to reduce it to a single equation for the complex function $\varphi = \omega + \frac{in}{Eh} F$, where, $n = \sqrt{12(1 - \mu^2)}/h$ of the form:

$$L\varphi = \frac{q}{D} - \sum_{j=1}^N [A_{kj} \delta(x - x_j) - B_{kj} \delta''(x - x_j)] \quad (6)$$

where

$$\begin{aligned} A_{kj} &= \frac{E_j F_j}{D} \beta_k^2 \operatorname{Re} \varphi_k(x_j) - \frac{H b_j}{2D} \beta_k^2 t_j - \frac{b_j \beta_k^2 m_j}{D} \\ &\quad - \frac{\mu in}{Eh} \left[\frac{E_j F_j \beta_k^2}{n} \operatorname{Im} \left(\frac{d^2 \varphi_k}{dx^2} + \mu \beta_k^2 \varphi_k \right) + t_j b_j \right], \\ \beta_{kj} &= -\frac{in}{Eh} \left[\frac{E_j F_j}{n} \operatorname{Im} \left(\frac{d^2 \varphi_k}{dx^2} + \mu \beta_k^2 \varphi_k \right) + t_j b_j \right]. \end{aligned}$$

The desired function is represented in the form:

$$\varphi = \sum_{j=1}^N \varphi_k \sin(\beta_{kj} y), \quad (7)$$

where $\beta_k = k\pi/b$; a , b are the plate dimensions in plan.

Substituting (7) into (6) and using the Bubnov-Galerkin procedure, the problem is reduced to a system of ordinary differential equations of the form:

$$L_k \varphi_k = \mathbf{p}_k - \sum_{j=1}^N \left[\mathbf{A}_{kj} \delta(x - x_j) - \mathbf{B}_{kj} \delta''(x - x_j) \right], \quad (8)$$

According to [2], the solution of Eq. (8) is represented as:

$$\varphi_k = \varphi_k^0 - \sum_{j=1}^N \mathbf{A}_{kj} \psi_{kj} - \sum_{i=1}^N \mathbf{B}_{kj} \psi_{kj}'', \quad (9)$$

where φ_k^0 is the solution of the equation $L_k \varphi_k^0 = \mathbf{p}_k$; ψ_{kj} is the solution of the equation $L_k \psi_{kj} = \delta(x - x_j)$; ψ_{kj}'' is the solution of the equation $L_k \psi_{kj}'' = \delta''(x - x_j)$.

The construction of the function φ_k^0 , ψ_{kj} , ψ_{kj}'' was carried out according to [2].

The second ribbed plate perceives only loads $\mathbf{t}_j$, $\mathbf{m}_j$, so the solution for it takes the form:

$$\varphi_k^{\text{II}} = - \sum_{j=1}^N \mathbf{A}_{kj}^{\text{II}} \psi_{kj} - \sum_{i=1}^N \mathbf{B}_{kj}^{\text{II}} \psi_{kj}'', \quad (10)$$

where upper index II means the second ribbed plate.

Unknown odds $\text{Re} \varphi_k(x_j)$, $\mathbf{t}_j(x_j)$, $\mathbf{m}_j(x_j)$, $\text{Im} \varphi_k^{\text{II}}(x_j) + \mu \beta_k^2 \text{Im} \varphi_k(x_j)$ incoming (9) and (10) is determined from the condition of compatibility of the movements of the ribs and the skin along the separation line. Compiling these conditions, a system of 4N algebraic equations has been obtain.

Solutions (9) and (10) include functions ψ_{kj} , ψ_{kj}'' .

The graphs of these functions are smooth, and their derivatives have jumps and kinks corresponding to the force and moment effects distributed along the lines. Due to this, solutions in the form (6) using (8) and (9) have fast convergence, both in the local vicinity of the edges and in the continuum zones. For practical calculations, you can limit yourself to holding 1–3 members of the series.

3. STABILITY OF THREE-LAYER PANELS WITH DISCRETE CORE IN THE FORM OF RIBS

Under the action of vertical loads, in addition to strength, it is also necessary to investigate the issues of stability of the panels under consideration.

Replacing the load function by the sum of the projection of external forces acting in the plane tangent to the deformable surface, as a result of its buckling, onto the normal

$$\mathbf{q}_3 = - \left(T_1 \frac{\delta w^*}{\delta x^2} + 2S \frac{\delta^2 w^*}{\delta x \delta y} + T_2 \frac{\delta w^*}{\delta y^2} \right), \quad (11)$$

in the general case the stability equation or the critical state equation have been obtain. Then there are for each plate

$$\begin{aligned} \Delta^2 \varphi = & -\frac{1}{D} \sum_{j=1}^N \left[E_j I_j \text{Re} \left(\frac{\delta^4 \varphi}{\delta y^4} \right) + \frac{H b_j}{2} \frac{\delta^2 \mathbf{t}_j}{\delta y^2} + b_j \frac{\delta^2 \mathbf{m}_j}{\delta y^2} \right] \delta(x - x_j) \\ & + \sum_{j=1}^N \frac{i n}{E h} \left[\frac{E_j F_j}{n} \text{Im} \left(\frac{\delta^2 \varphi}{\delta x^2} - \mu \frac{\delta^2 \varphi}{\delta y^2} \right) + \mathbf{t}_j b_j \right] \delta''(x - x_j) \\ & - \mu \sum_{j=1}^N \frac{i n}{E h} \left[\frac{E_j F_j}{n} \text{Im} \left(\frac{\delta^4 \varphi}{\delta x^2 \delta y^2} - \mu \frac{\delta^4 \varphi}{\delta y^4} \right) + b_j \frac{\delta^2 \mathbf{t}_j}{\delta y^2} \right] \delta(x - x_j) \\ & - \frac{1}{D} \left[T_1 \text{Re} \frac{\delta^2 \varphi}{\delta x^2} + 2S \text{Re} \frac{\delta^2 \varphi}{\delta x \delta y} + T_2 \text{Re} \frac{\delta^2 \varphi}{\delta y^2} \right] \end{aligned} \quad (12)$$

This equation contains delta functions and their derivatives as coefficients and can be integrated using the

methods described in [7], i.e., by representing the desired solution as an expansion in terms of regular and specially selected discontinuous functions with some desired coefficients. This method makes it possible to obtain practically the same convergence for moments and shear forces, both in the regular region and near the edges, and, consequently, to determine the concentration of internal forces even when keeping a small number of expansion terms in series.

However, when determining the critical load, there is no need to look for the concentration of forces in small areas, since the critical load in the event of buckling of the entire plate or the entire structure is some integral characteristic. Those are little sensitive to local perturbations of the stress-strain state and are determined mainly by the nature of the distribution of displacements, which are rather smooth functions. In this regard, one should expect quite satisfactory convergence of the process in determining the critical load even when the desired values are approximated by functions that are completely regular, for example, trigonometric functions.

Depending on the load application scheme, it can be transmitted through one plate or through two plates.

Consider the case of compression by a uniformly distributed load applied to two plates. In this case, you should accept $T_2 = S = 0$; $T_1 = 0.5q$. Let's represent the function φ as a double series

$$\varphi = \sum_k \sum_l \varphi_{lk} \sin \beta_l y \sin \alpha_k x, \quad (13)$$

where $\beta_l = l\pi/b$; $\alpha_k = k\pi/a$.

Substituting (13) into (12) and using Bubnov-Galerkin for the expansion term in the external load series, has been obtained

$$\begin{aligned} \frac{a}{2} (\alpha_k^2 + \beta_l^2)^2 \varphi_{lk} = & - \sum_{j=1}^N \sum_i \left\{ -\frac{1}{D} E_j I_j \beta_l^4 \text{Re} \varphi_{lk} - \beta_l^2 \frac{H b_j}{2D} \mathbf{t}_{lk} - \frac{b_j \beta_l^2 \mathbf{m}_{lk}}{D} \right. \\ & \left. + \frac{i n}{E h} \left[\frac{E_j F_j}{n} (\alpha_k^4 - \mu \beta_l^2 \alpha_k^2) \text{Im} \varphi_{lk} - \alpha_k^4 b_j \mathbf{t}_{lk} \right] \right\} \sin \alpha_k x_j \sin \alpha_i x_j \\ & - \frac{1}{D} \frac{a}{2} (0.5 q \alpha_k^2 \text{Re} \varphi_{lk}) \end{aligned} \quad (14)$$

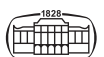
where $\varphi_{kl} = \varphi_{kl}(x_l)$.

The coefficients and included in this equation are found from the conditions of compatibility of displacements along the line of separation of the ribs. Substituting these conditions into (14), a system of homogeneous equations is obtained. This system has a nontrivial solution if its determinant is equal to zero. Solving system (13) has been found.

The influence of various design factors on the magnitude of the critical force has been studied.

Figure 2 shows the dependence of the critical load on the number of reinforcing elements at constant dimensions.

Figure 3 shows the dependence of the critical force on the thickness of the carrier layer. In this case, the total thickness of the panel was kept constant.



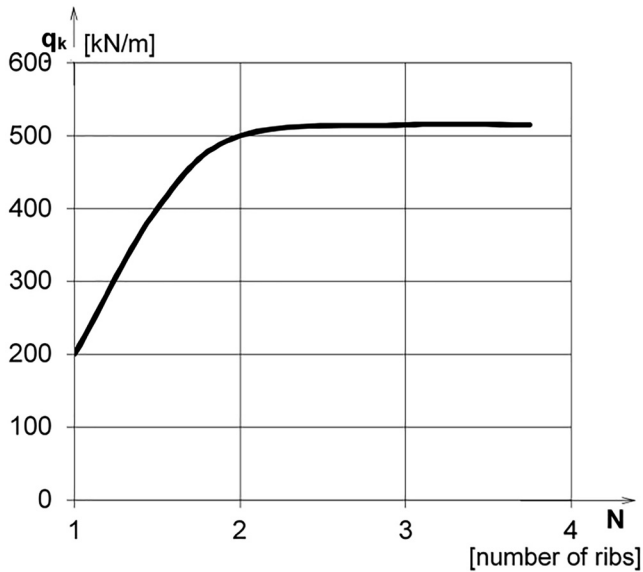


Fig. 2. The dependence of the critical load q_{kr} from the number of reinforcing elements–ribs N for a reinforced concrete panel

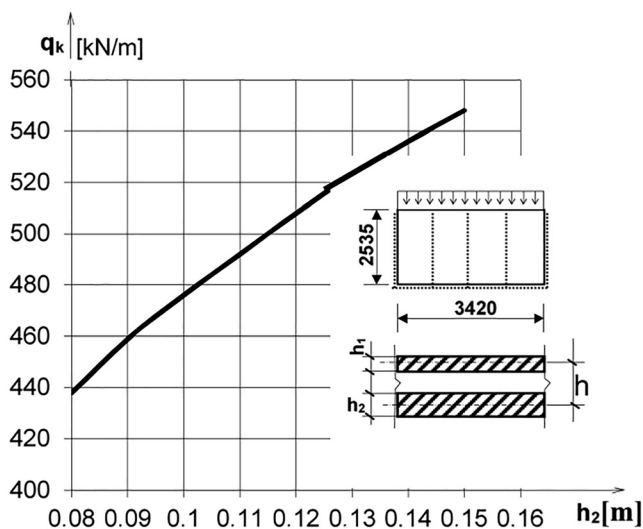


Fig. 3. The dependence of the critical load q_{kr} from the thickness h_2 of the carrier layer for a reinforced concrete panel

Figure 4 shows the dependence of the critical load on the value of the total thickness of the concrete panel - package is given. The thicknesses of the bearing and structural layers do not change.

4. CONCLUSION

- Theoretical foundations and practical methods for calculating the stress-strain state, stiffness and stability of multilayer concrete panels have been developed;
- The general structure of a comprehensive study is presented to determine the stress-strain state of multilayer concrete panels, taking into account their design features and materials used;

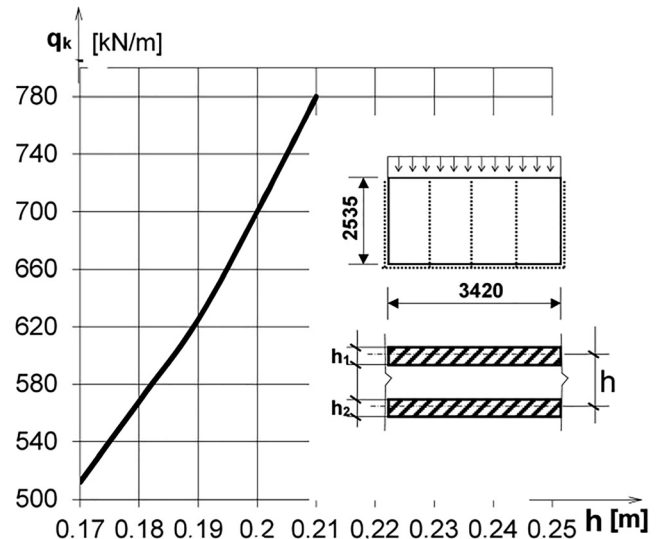


Fig. 4. The dependence of the critical load q_{kr} from the total thickness of the panel-package h for reinforced concrete panel

- Theoretically, the stress-strain state of multi-layer concrete panels made of composite concrete with rigid connections was determined;
- The results of the analysis of factors that improve the quality of calculations for the considered load schemes are presented, allowing for an effective selection of the design model;
- Development of methodological and theoretical foundations for designing and creating a concept for the production of multilayer enclosing structures, ensuring an increase in their technical level and reliability, as well as reducing manufacturing and operation costs.

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