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Investigating teeth root stresses of various spur gears using stress function and FEM

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ABSTRACT

The gear is the most efficient and reliable portion of the transmission system. In machine tools, gears are utilized to handle heavy loads. These gears are used indefinitely if the prerequisites are met. Gear tooth bending stress for numerous spur gears has been examined in this research. The influence of changing the pressure angles on the spur gear tooth root stress has been investigated. The purpose of this study is to look for a new way of finding the stresses generated in the roots of the gear teeth as well as the effect of changing pressure angles on these stresses. Solidworks 2021 uses parametric formulation to model gears, analyzed for deformation and maximum bending stress in ANSYS. The airy stress function is used to conduct the nominal root tooth stress. The enhancement percentages of modified cases are 5.92%, 1.70%, 10.70%, 17.22%, 10.86%, 24.32%, 20.91%, and 26.15% for the cases 2, 3, 4, 5, 6, 7, 8, and 9, respectively according to the stress function findings.

KEYWORDS

gear, bending stress, pressure angle, deformation

1. INTRODUCTION

Gears are one of the essential components of machines with circular and rotating motions or the beating heart of these machines. Therefore, designers, manufacturers, and developers are also interested in improving and developing gears to suit a variety of tasks, like increasing their capacity to withstand greater loads with higher torques and speeds and reducing the noise caused by contact and friction. Because it is the weakest area of the gear, the stress generated in the roots of the gear teeth is one of the primary causes of tooth failure. This study examines the stresses generated at the roots of the spur gear tooth with varying pressure angles and variable combination ratios.

2. RELATED WORKS

Here some important topics are summarized that can be described as previous results of researchers.

A paper by Bodzas [1] detailing the computer-aided design method for spur gear pairs with conventional and modified straight teeth. The determination of geometrical parameters is required for design. With the use of computer programs, gear pairs with regular and modified teeth and any arbitrary characteristics may be analyzed, constructed, and modeled. With these programs, gear pairs with certain geometry have been created, and Computer-Aided Designs (CAD) models with further meshing, geometrical, and Tooth Contact Area (TCA) analysis have also been Bodzas. Publication [2] provided a finite element analysis study of the impact of the addendum modification coefficient on the normal stress and the normal deformation. Five different kinds of helical gears were modeled. Only their addendum modification coefficients vary from one another. The other initialization variables

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are all fixed. After the result is acquired from the mechanical parameters and the addendum modification coefficients, the inferences have been drawn based on the diagram's findings. Wable et al. [3] have investigated gear pitting failure. In CATIA V5, gears are modeled using parametric formulation. In ANSYS, the gear's deformation and maximum contact stress, which produces pitting, are studied. On photo-elastic apparatus, experimental analysis is conducted using the photo-elastic methodology. Pedersen [4] presents an investigation on how to directly optimize the design of the final gear in order to reduce the bending stress of involute teeth. The cutting tool (the gear envelope) that can produce this optimum gear shape is then found using the optimized shape. Emphasis is placed on the significance of separating the form parameterization from the finite element analysis of stresses, and a simple but sufficiently flexible root parameterization is used. Significant reductions in stress are seen. A set of spur gear teeth employed by Jangid et al. [5], is often exposed to bending stresses and contact stress. This project aims to evaluate and validate the bending stress appropriation over involute spur gears using the modern commercial Finite Element Method (FEM) program ANSYS paired with Solidworks. Gear profiles would be created in Solidworks using a parametric mathematical definition of involute bend in addition to a complete model, based on certain magic properties in the number of teeth, diametric pitch, and mass point. These models must afterwards be examined for 3D and 2D bending stresses using ANSYS simulation. Musa et al. [6] used the theory of elasticity represented by the stress function approach in a case study to examine the impact of utilizing gears with an asymmetric tooth profile on stress concentrations in gear keyways.

3. METHODS

The analytical model was constructed using the Airy stress function [6] since its objective is to determine the stress in a specific surface area using any 2D coordinate system, like the x-y plane. This equation aims to determine the possibility of extracting stress in a specific area or any other area of the spur gear. Therefore, the equation derived from reference [6] was used, as it had previously been used in the spur gear key-way area. However, in this work, it will be used in the spur gear root area and compared with the old traditional form bending stress equation, as well as the stress concentration factor extracted in order to look at how different spur gear pressure angles and types affect or have an impact on gear tooth root stresses.

3.1. Stress function formulation

In 1862 George Airy was the first to construct the stress function, thus the name "Airy's stress function", which follows the basic concept of "plane strain and plane stress" [7]. Scientists acknowledged that the tooth of the gear should be viewed as a "beam fixed from the section" of the rim, exposed to the greatest stresses at the filet (fixed region) [8].

The stresses may be summed up as follows [6]:

$$\sigma_x = \frac{\partial^2 \varphi}{\partial y^2}\,;\ \sigma_y = \frac{\partial^2 \varphi}{\partial x^2}\,;\ \tau_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y}. \tag{1}$$

For Airy's equilibrium and compatibility equations in Cartesian coordinates, the function $\varphi(x, y)$ is the stress function. Therefore, the total y-direction stress may be calculated as follows:

$$\sigma_y = \frac{12 W_t \cdot x \cdot y}{bh^3} + \frac{W_r}{bh}, \tag{2}$$

where σ_y is the stress in the y-direction; W_t is the tangential force; W_r is the radial force; x, y are the coordinates of any point in spur gear surface; b is the face width of the spur gear; and h is the width of the required weakest section of the tooth.

3.2. Spur gears modeling

Modeling programs as Pro-Engineer, Solidworks, Catia and many more would be the greatest alternatives to create unexpected geometry for analysis. The gear shape created in this study using Solidworks 2021 [9], among other things, may be necessary for the FEM investigation conducted on the ANSYS workbench 2021. There are several methods for creating an involute spur gear profile. The spur gear modeling method was taken based on [6]. The parameters required to sketch the spur gear are available in Table 1.

These three characteristics stated in Table 1 are fundamental and essential factors in determining the full depth involute profile spur gears. All other parameters may be determined from these parameters: circular pitch, diametric pitch, module, addendum, and dedendum for complete depth standard involute profile tooth system. Table 2 shows the cases studied in this paper, where symmetric

Table 1. Spur gear modeling parameters

Symbol	Description	Value
N	No. of Teeth	14
m	Module	7 mm
b	Face width	10 mm
Ø	Pressure angle	Depending on each case

Table 2. Studied cases

Case	Pressure Angle ° Ø$_{\text{loading}}$–Ø$_{\text{unloading}}$	Loading Angle °
1*	14.5–14.5	29.19
2	20.0–20.0	32.38
3	25.0–25.0	35.87
4	14.5–20.0	29.19
5	14.5–25.0	29.19
6	20.0–14.5	32.38
7	20.0–25.0	32.38
8	25.0–14.5	35.87
9	25.0–20.0	35.87

Table 3. Terms required for Eq. (2)

Case	y (mm)	x_i (mm)	b (mm)	h_i (mm)	W_{ti} (N)	W_{ri} (N)
1*	13.99	4.97	10	9.94	436.4866	243.8841
2	15.00	4.90	10	10.25	422.2668	267.7513
3	14.13	5.99	10	10.30	405.1794	292.9670
4	13.99	5.17	10	10.35	436.4866	243.8841
5	13.99	5.48	10	10.96	436.4866	243.8841
6	14.04	5.17	10	10.40	422.2668	267.7513
7	14.04	5.68	10	11.36	422.2668	267.7513
8	14.13	5.48	10	10.96	405.1794	292.9670
9	14.13	5.68	10	11.36	405.1794	292.9670

and asymmetric gears were modeled using Solidworks to investigate the effect of the asymmetry and the different pressure angles on the tooth root stresses, which is the weakest section.

Table 3 gives the specifics for each case in Table 2 that are required for Eq. (2) when 500 N is represented the normal-force where W_t and W_r. The data were taken from the sketched models in Solidworks.

3.3. Finite element method analysis

ANSYS workbench 2022 [10] was used to analyze the spur gear cases that were saved as (.IGS) files from Solidwork 2021. The boundary conditions and mesh generating were employed based on references [1, 11] as it is plotted in Fig. 1. A tangential force of 500 N was attached to the tip of tooth face A, and the fixed support was taken on face B (Fig. 1) in order to investigate the stresses induced at the weakest area. The generated mesh was taken as Solid 185 elements with finer meshing and tetrahedral faces, which are required for this analysis.

4. RESULTS

4.1. Stress function results

The results of the stress function are listed in Table 4 based on the data extracted from the Solidworks models as it is listed in Table 3 and by using Eq. (2).

The results show a joyful reduction in the stresses induced in the root area of the spur gear tooth for analytical analysis. The enhancement percentage reached 26.15% for the asymmetric spur gear with 25°–20° pressure angle as a result of the increase in terms y, x, and h and the effect of the asymmetry on the normal force.

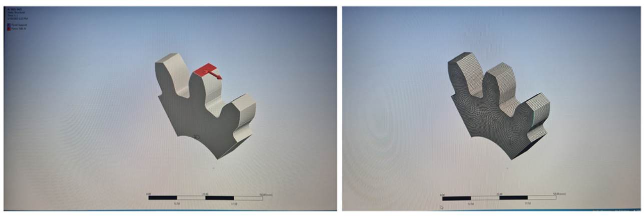


Fig. 1. FEM analysis boundary conditions and mesh

Table 4. Analytical stress results

Case	σ_y (MPa)	Enhancement percentage reference to case 1 (%)
1*	39.54	0.00%
2	37.20	5.92%
3	40.51	1.70%
4	36.49	10.70%
5	32.73	17.22%
6	35.24	10.86%
7	29.92	24.32%
8	31.27	20.91%
9	29.20	26.15%

4.2. Numerical stress results

The numerical results were obtained using ANSYS workbench 2022, and Table 5 lists the numerical stress in the y-direction.

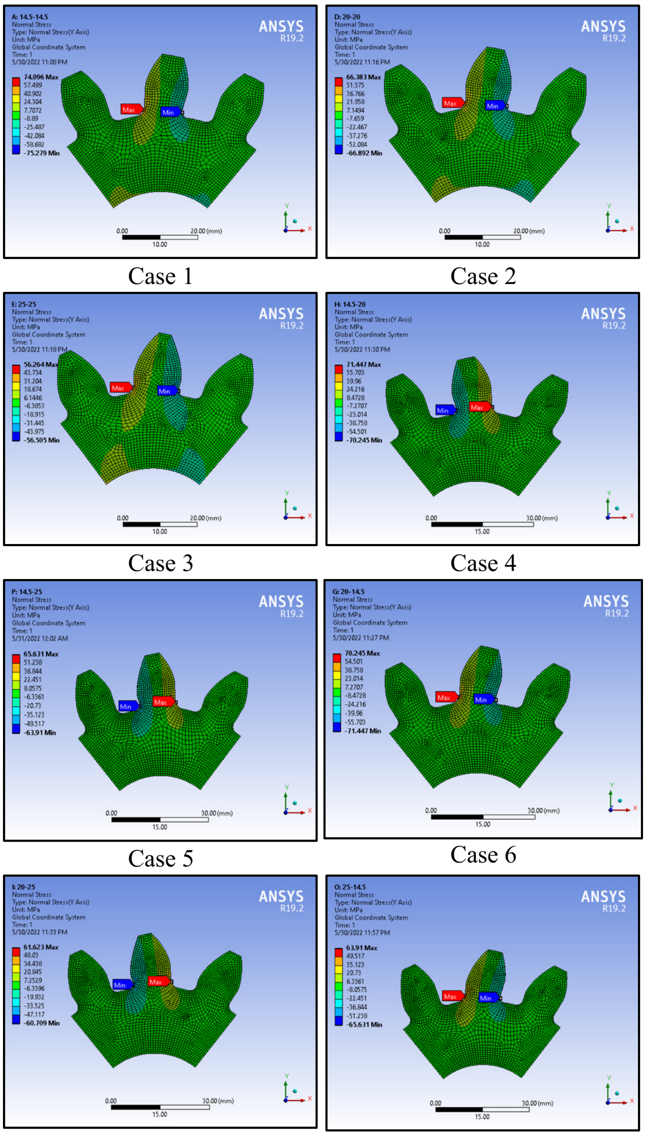
Figure 2 illustrates the numerical results, where the maximum stresses appeared at the spur gear tooth root area. The maximum stresses are available in Table 5 also. It is clear that the numerical stress results confirm the analytical results, where the influence of the asymmetry is clearly shown as the enhancement percentage reach 24.02%.

4.3. Stress concentration factor K_t

A change in the shape of a body or component indicates a region of stress concentration or a weak area. Thus, designers pay special attention to these places since they are the most susceptible to failure.

Table 5. Numerical stress results

Case	σ_n (MPa)	Enhancement percentage reference to case 1 (%)
1*	74.10	0.00
2	66.40	10.39
3	56.30	24.02
4	71.45	3.58
5	65.63	11.43
6	70.25	5.20
7	61.62	16.84
8	63.91	13.75
9	60.71	18.07



Case 1 Case 2

Case 3 Case 4

Case 5 Case 6

Case 7 Case 8

Fig. 2. Numerical stress results

The stress concentration factor K_t is a term that designers rely on for defining the area of stress concentration:

$$K_t = \frac{\sigma_{\max}}{\sigma_{nominal}}, \tag{3}$$

where K_t is the stress concentration factor; $\sigma_{\max}$ is the maximum stress from experimental or numerical results; and σ_{nominal} is the analytical stress. The stress concentration factor results are shown in Table 6,

$$K_{t\ \text{Relative}} = \frac{\sigma_{\max}\ of\ any\ case}{\sigma_{nominal}\ of\ referace\ case\ 1}. \tag{4}$$

4.4. Validation of stress concentration factor result

Table 7 shows the comparison between the value of $K_{t\ \text{Relative}}$ of the present work with the same value of reference [12], where the discrepancy reach 0.81%, which a great and joyful results. According to the result it is clear that there is a good

Table 6. Stress concentration factor results

Case	σ_y (MPa)	σ_n (MPa)	K_t	$K_{t\ \text{Relative}}$
1*	39.54	74.10	1.87	1.87
2	37.20	66.40	1.79	1.68
3	38.87	56.30	1.45	1.42
4	35.31	71.45	2.02	1.81
5	32.73	65.63	2.01	1.66
6	35.24	70.25	1.99	1.78
7	29.92	61.62	2.06	1.56
8	31.27	63.91	2.04	1.62
9	29.20	60.71	2.08	1.54

Table 7. Validation of result with [12]

$K_{t\ \text{Relative}}$ for Present Work case 1	$K_{t\ \text{Relative}}$ for [12]	Discrepancy %
1.87	1.859	0.81%

agreement between the results of the present work and the results obtained by reference [12] that give this work reliability, validity and broad acceptance [12].

4. CONCLUSIONS

The most important conclusions obtained from the present work can be outlined as follows:

1. According to the results, it is clear that using different pressure angles and asymmetric teeth significantly affects the stresses induced in the root or the weakest section of the spur gear tooth;
2. The enhancement percentages of modified cases are 5.92%, 1.70%, 10.70%, 17.22%, 10.86%, 24.32%, 20.91%, and 26.15% for cases 2, 3, 4, 5, 6, 7, 8, and 9, respectively according to the stress function findings;
3. The enhancement percentages of the modified cases are 10.39%, 24.02%, 3.58%,11.43%, 5.2%, 16.84%, 13.75%, and 18.07% for cases 2, 3, 4, 5, 6, 7, 8 and 9, respectively according to the FEM numerical results;
4. FEM and stress function results proved that case 9 shows joyful outcomes, which is the 25°–20° asymmetric spur gear, and it is the best combination since the stresses induced are the smallest due to wider cross suction;
5. It is clear that there is a good agreement between the results of the present work and the results obtained by references, where the discrepancy reaches 0.81% a great and joyful result.

REFERENCES

[1] S. Bodzás, "Computer aided designing and modeling of spur gear pairs having normal and modified straight teeth," *Int. Rev. Appl. Sci. Eng.*, vol. 10, no. 2, pp. 157–163, 2019.

[2] S. Bodzás, "Tooth contact analysis of the effect of the addendum modification coefficient of helical gears having modified teeth," *Int. Rev. Appl. Sci. Eng.*, vol. 12, no. 1, pp. 1–9, 2021.

[3] V. Wable and D. M. Mate, "An investigative study of spur gear failure by FEA and photo-elastic method," *Int. J. Sci. Res.*, vol. 5, no. 7, pp. 544–549, 2013.

[4] N. L. Pedersen, "Minimizing tooth bending stress in spur gears with simplified shapes of fillet and tool shape determination," *Eng. Optim.*, vol. 47, no. 6, pp. 805–824, 2015.

[5] A. Jangid and S. Kumar, "Modeling and simulation analyses for bending stresses in involute spur gears by finite element method," *Int. J. Appl. Eng. Res.*, vol. 13, no. 12, pp. 10914–10923, 2018.

[6] H. A. Musa and M. Q. Abdullah, "Influence of using asymmetric teeth profile on stress concentrations in gear key-way," in *IOP Conference Series: Materials Science and Engineering*, vol. 1094, 2020, Paper no. 012067.

[7] S. Timoshenko and J. Goodier, , *Theory of Elasticity*. Mc Graw-Hill, 1951.

[8] W. Lewis, "Investigation of the strength of gear teeth," *Proc. Eng. Club*, Philadelphia, vol. 2, no. 3, pp. 241–243, 1892.

[9] M. S. Al-Din Tahir, S. S. Hassan, and J. S. Chiad, "The mathematical model for lateral stiffness of variable length conical spring," *Pollack Period.*, vol. 17, no. 2, pp. 31–35, 2022.

[10] A. Benbouaza and A. Benbouaza, "Detection of unbalance defect by the vibration analysis technique," *Pollack Period.*, vol. 17, no. 3, pp. 36–40, 2022.

[11] S. Bodzás, "Geometric design, modeling and finite element analysis of spur gear pairs having normal teeth based on the modification of the pressure angle," *Int. Rev. Appl. Sci. Eng.*, vol. 13, no. 3, pp. 285–297, 2022.

[12] M. Q. Abdullah and M. J. Jweeg, "Analytical solution of bending stress equation for symmetric and asymmetric involute gear teeth shapes with and without profile correction," *Innovative Syst. Des. Eng.*, vol. 3, no. 6, pp. 19–33, 2012.