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The effect of teaching conceptual knowledge on students' achievement, anxiety about, and attitude toward mathematics

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Abstract
This study investigates how teaching for conceptual understanding alongside procedural knowledge affects students' achievement in, anxiety about, and attitude toward mathematics. Purposive sampling was used to select 200 secondary school students from Erbil-Iraq. An experimental approach was applied to evaluate the sample study. In the experimental group, conceptual teaching was the focus. In the control group, conventional teaching was used. Pre- and post-tests for an achievement test, mathematics attitude scale, and abbreviated math anxiety scale were applied to both groups to reveal the effect of conceptual knowledge on students' achievement, attitudes, and anxiety, respectively. Repeated measure ANOVA was used to analysis the data. The results show that there is a statistically significant difference in mathematics achievement between the two groups ($p<.001$). Students' attitudes toward mathematics in treatment group developed positively. Nevertheless, teaching mathematics conceptually reduced anxiety among female students more effectively than it did among male ones.

Keywords: conceptual knowledge, procedural knowledge, mathematics achievement, mathematics anxiety, attitudes toward mathematics

INTRODUCTION

In the last few decades, mathematics researchers have focused on the importance of teaching mathematics conceptually (see, for example, Crooks & Alibali, 2014). In conceptual teaching, mathematics teachers explain why particular mathematics operations work in specific ways. Students with conceptual knowledge acquire the ability to transfer their knowledge into new mathematical situations (National Council of Teachers of Mathematics [NCTM], 2000). Accordingly, learners must be taught the basics of mathematics (Generalao, 2012).

A lack of mathematics competence does not refer to a shortage of intelligence or an inability to learn mathematics. Instead, it indicates an inappropriate teaching method that leads students to lack mathematics skills, including conceptual understanding (Andamon & Tan, 2018). Understanding mathematics conceptually helps students diminish their mathematics anxiety, and it increases their confidence, thereby encouraging them to confront more challenging mathematical tasks (Mariquit & Luna, 2017).

Because anxiety about and attitude toward mathematics are serious issues among students, educators and researchers must consider them (Christiansen, 2021; Webb, 2017). The concept of attitude is used and understood in the way Pehkonen and Pietilä (2003) defined as a psychological construct, which belongs to the affective-emotional side of human personality. Anxiety also belongs to the emotional sphere of personality, but definitely to the negative side (Hembree, 1990). There is a negative relationship between students' anxiety and their performance on mathematics exams (Chernoff & Stone, 2012). A study conducted by Price (2015) to investigate the characteristics of students' anxiety and its impacts on their understanding of arithmetic showed that students with lower anxiety about mathematics better understood the concept of arithmetic compared to students with higher anxiety. Furthermore, mathematics anxiety plays an important role in students' choice of

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Contribution to the literature

- Teaching mathematics conceptually is necessary for students to absorb mathematical subjects (Andamon & Tan, 2018). In addition, teaching conceptual mathematics decrease students' mathematics anxiety, which leads to an increase in their performance (Khoule et al., 2017). Accordingly, there is a positive association between conceptual knowledge and academic achievement in mathematics (Zakaria et al., 2010).
- The lack of conceptual knowledge leads to a variety of challenges for students (Tekin-Sitrava, 2017). Hussein and Csíkos (2021) revealed that secondary school students in Erbil city had many problems with the concept of function.
- The study showed that students had difficulty defining the concept of function, and they could not recognize different representations of functions. Therefore, this study will investigate how conceptual teaching impacts students' achievement in, anxiety about, and attitude toward mathematics in secondary school students in Erbil-Iraq.

their future careers. University students with mathematics anxiety avoid majors that contain high mathematics requirements (Lefevre et al., 1992; Widmer & Chavez, 1982). A fear of mathematics hinders many people from pursuing certain professional opportunities (Tobias, 1993).

Students often have mathematics anxiety, and they think mathematics is a dreaded subject (Capinding, 2022). This anxiety impedes students' development and improvement in their mathematics competence (Cribbs et al., 2021; James et al., 2013). A study conducted by Capinding (2022) on students in high school, the results found that there was a negative relationship between anxiety and mathematical performance. The students who had mathematics anxiety performed worse than those who had not. In addition, Gunderson et al. (2018) revealed in their study that anxiety about mathematics had a negative impact on mathematics achievement, and low mathematics achievement indicated high mathematics anxiety for students. Jamieson et al. (2021) stated that students' exam performance was affected negatively by high mathematics anxiety. Ilhan et al.'s (2022) research among lower secondary students indicated that those who had higher mathematics anxiety were less successful in performance than those who had lower anxiety. Mathematics anxiety had a negative association with mathematics achievement and solving mathematical problems (Ilhan et al., 2022). Accordingly, both mathematics anxiety and performance affect each other negatively (Carey et al., 2016). Therefore, a holistic approach is needed to support the education process that concentrates on both emotion and skills toward mathematics (Sorvo et al., 2019). And it is vital to reduce students' mathematics anxiety because it is a barrier to improving their mathematics performance (Capinding, 2022).

Students' attitudes toward mathematics has a direct impact on their mathematics achievement. Students with a positive attitude toward mathematics are more academically prepared to develop in mathematics (Capinding, 2022). In their study, Zhang et al. (2020) found that students' attitudes had a positive relationship with their mathematics performance. Capinding (2022) indicated that there was a positive association between students' positive attitude toward and their achievement in mathematics. In the study conducted by Ilhan et al. (2022), the results revealed that the relationship between mathematics achievement and negative attitude was significantly negative. Simultaneously, according to Capinding's (2022) findings, there was a negative relation between students' anxiety about and their attitude toward mathematics. Students, who were anxious about mathematics, had a negative attitude toward it. And students, who had a negative attitude toward mathematics, had more anxiety. Consequently, a positive attitude toward mathematics overcome mathematics anxiety (Saha et al., 2020).

Gender differences in mathematics performance have long been in the focus of international comparative studies (see, for example, Hyde & Mertz, 2009). The issue is studied primarily with the intention to reveal a possible factor of inequity in an educational system independently of the form of education (coeducation vs segregated education). In the region where the current study took place, one preliminary study has been conducted on the topic by Hasan (2021) finding no significant difference in math performance on a university sample. Different results have been found in studies about gender differences among students with mathematics anxiety, namely, girls tend to possess a higher level of anxiety resulting in poorer math achievement, and from this respect it is very important to acknowledge and study the gender issue in mathematics education.

It is possible to improve students' understanding of mathematics and their confidence in their abilities by providing them with interactive teaching methods in a relaxing environment that can help to improve their attitudes toward mathematics (Jennison & Beswick, 2010). According to Turner et al. (2002), mathematics teachers should use a holistic teaching approach to involve learners in mathematics and to develop their attitudes toward this subject. Accordingly, in the present study, mathematics teachers in the experimental group focus on teaching for conceptual understanding. The control group follows traditional teaching methods. The goal of this study is to investigate how secondary school students' conceptual knowledge impacts their achievement in, anxiety about, and attitude toward mathematics in the Kurdistan region of Iraq.

THEORETICAL BACKGROUND

Conceptual and Procedural Knowledge

Conceptual knowledge is defined as an "understanding of the underlying structure of mathematics–the relationships and interconnections of ideas that explain and give meaning to mathematical procedures" (Faulkenberry, 2003, p. 13). Whereas, to the contrary, procedural knowledge is defined as "Mastery of computational skills and knowledge of procedures for identifying mathematical components, algorithms, and definitions" (Faulkenberry, 2003, p. 13). Students have conceptual knowledge if they can provide logical relationships between concepts (Mariquit & Luna, 2017). For example, after students' number sense has been developed, they can effectively solve mathematical tasks. The task 18*3 might be changed to double 9*3. The ability to make this change is called the flexibility of thought (Mariquit & Luna, 2017).

Mathematics education studies support the perspective that understanding mathematics requires students to make connections between procedures, facts, concepts, relationships, and mathematical ideas (Hiebert & Carpenter, 1992; Moschkovich et al., 1993; Skemp, 1976, 1989). According to Siregar and Siagian (2019), mathematics makes sense for students if they know the connections between the concepts. The ability to connect mathematical conception meaningfully is essential for students in every level of education (Siregar & Siagian, 2019), since it helps them to use the mathematical concepts that have been learned as basic knowledge to understand new concepts (NCTM, 2000). In addition, mathematical connections can be counted as a consequence of constructivist theory in mathematics learning and is a building of a mental network organized such as a spider's web that nodes can represent the pieces of information, and threads as the connections between them (Hiebert & Carpenter, 1992). Then for learning and thinking about connections between mathematics concepts, it is very important to look at mathematics as a whole (Siregar & Siagian, 2019). Consequently, understanding connections is fundamental in teaching mathematics (Eli et al., 2011). In this respect, if mathematics teachers focus on mathematical connections, then students acquire an interconnected understanding of mathematics (Evitts, 2005).

According to Skemp (1976), there are forms of understanding, namely, relational understanding and instrumental understanding of mathematics. Relational understanding helps the students to use an appropriate procedure to solve mathematics problems and logical reasoning (Patkin & Plaksin, 2018; Utomo, 2020). It also provides the students capability to deduce appropriate procedures in mathematics problem solving (Minarni et al., 2016; Skemp, 2006). However, instrumental understanding is the student's ability to apply the mathematics rules to solve the problems regardless of knowing the reason for working strictly according to the rules (Skemp,1976). There are many advantages for relational understanding of mathematics, mainly, it is easier to memorize and comprehend concepts that help to solve complicated problems more efficiently, and it helps the students to generate an original idea, which is the achievement of the learning aim (Skemp, 2006). Therefore, relational understanding should be encouraged, and conceptual structures that contain relevant concepts should be developed to achieve this understanding (Star & Stylianides, 2013).

To develop conceptual knowledge, learners are trained to solve mathematical problems that contain new ideas. This technique helps them to think deeply and to apply previously learned information to solve the new tasks. Training students to solve challenging tasks helps to improve their critical thinking skills, thereby leading to better performance in mathematical problem solving (Mariquit & Luna, 2017). A study managed by Khoule et al. (2017) revealed how teaching mathematics conceptually affects students' performance in mathematics exams. An experimental approach was used. Two groups were formed. One was taught conceptually, while the other was taught procedurally. Two quizzes were developed: a conceptual quiz and a procedural quiz. The results showed that the conceptual group performed better than procedural group in both the conceptual and the procedural quizzes. Nonetheless, the procedural group practiced procedural mathematics problems more frequently than the other group. The test questions were designed to reveal the participants' knowledge about the subject matter. The results showed the procedural group had less understanding of the subject matter than the conceptual group. This suggests that the conceptual group had a greater ability to reason logically, to formulate solutions, and to understand mathematics flexibly. Compared to the procedural group, they better used their knowledge as a tool in problem solving.

Often, the ability to assess mathematics learning depends on the ability of students to manipulate knowledge procedurally (De Zeeuw et al., 2013). Students are taught and expected to produce correct answers on the exam. The tools used to assess students encourage educators to focus only on procedural knowledge, not conceptual knowledge (De Zeeuw et al., 2013). Therefore, the practice of teaching mathematics procedurally, which is the most common teaching method, raises concerns about students' performance in and anxiety about mathematics. A study by Khoule et al. (2017) indicated that teaching procedurally does not overcome students' mathematics anxiety. Because this method of pedagogy concentrates on mastering rules, it suggests that students should remember the material that they studied in the exam. In turn, this teaching approach increases students' anxiety.

Results from research on metacognition may provide a powerful and sound basis for enhancing students' knowledge at the same time as enriching their conceptual understanding and reducing their anxiety. Conceptual knowledge of mathematics assumes that students understand why, when, and how to use mathematical procedures. This kind of knowledge belongs to what the literature usually labels as metaknowledge or metacognition. To simplify the distinction between metacognition and cognition, metacognition is the process of monitoring and controlling students' needs and thinking about the problem-solving process. Cognition, in contrast, comprises more or less automatized processes. Metacognition is thinking about the thinking process; it also defined as awareness and management of the cognitive process (Kuhn, 2000).

Among educators and researchers, metacognition-based educational approaches have become effective for teaching and learning mathematics. This is because these approaches have positive effects on the construction of new knowledge and because they develop students' mathematical skills (Du Toit & Kotze, 2009). The metacognitive components of mathematical thinking may fulfill several different roles in the process of human development; the case of arithmetic performance has been analyzed by Csíkos (2022). To summarize, both metacognitive and non-metacognitive components have important functions in the context of human development. These functions vary according to the differences between individuals, task demands, and the context of mathematical problems. Consequently, it is very important for mathematics teachers to balance between conceptual and procedural approaches, both of which are necessary to improve students' performances (Capraro & Joffrion, 2006). Nevertheless, according to NCTM (2000), mathematics teachers must explain mathematics subjects conceptually before they explain them procedurally. It is believed that educators who wish to provide more meaningful mathematics knowledge to learners should start with visual models and end with a symbolic model (Ketterlin-Geller, 2007; NCTM, 2000).

Anxiety

Fifty years ago, Richardson and Suinn (1972) conducted research on mathematics anxiety when they examined its psychometric properties. Mathematics anxiety is common among students (Curtain-Phillips, 1999). According to Rossnan (2006), children often struggle with mathematics anxiety that hinders their ability to understand mathematics as a part of their daily lives. Mathematics anxiety is a negative feeling of distress that arises when confronting mathematics problems (Jansen et al., 2013). It has been defined as "feelings of tension and anxiety that interfere with the manipulation of numbers and solving mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551).

Students' mathematics anxiety generally develops in response to previous bad experiences. First, mathematics teachers with anxiety could transfer it to their students (Lau et al., 2022; Vinson, 2001). Secondly, parents can pass anxiety to their children just by talking about the difficulty of mathematics (Lau et al., 2022; Wang et al., 2014). Nevertheless, the majority of researchers believe that mathematics anxiety is created in the classroom (Finlayson, 2014; Lerner & Friesema, 2013). Students' mathematics anxiety originates with low levels at the very beginning of school. After students are unable to do certain mathematics problems, their anxiety increases step by step (Shore, 2005). Finally, gaps in students' mathematics development appear. These gaps significantly increase mathematics anxiety, which will remain a permanent block until the learners confronted it (Shore, 2005).

The strong negative relationship between mathematics anxiety and mathematics achievement has been recorded in many studies (see, for example, Ashcraft, 2002; Ashcraft & Moore, 2009; Hembree, 1990; Ma, 1999). Mathematics anxiety poses an obstacle to students' performance in mathematics courses (Ashcraft & Moore, 2009; Vinson, 2001). For example, students can forget information and lack self-confidence as a result of their mathematics anxiety (Tobias, 1993). A study by Ashcraft and Kirk (2001) revealed that it was difficult for students with mathematics anxiety to concentrate their attention on the tasks at hand. Their distracted thoughts prevented them from developing their mathematics competence. According to Meece et al. (1990), students with mathematics anxiety in grades 7-9 had poor self-confidence toward mathematics, and they failed to enroll in advanced mathematics education courses.

Teaching mathematics conceptually is important to decrease students' mathematics anxiety. A study conducted by Khoule et al. (2017) revealed the relationship between methods of teaching mathematics and students' mathematics anxiety. The anxiety of students who had been taught conceptually was compared to that of those who had been taught procedurally.

The results showed a statistically significant relationship between students' anxiety and teaching for conceptual understanding. Teaching for conceptual understanding reduced students' mathematics anxiety to a greater degree than procedural teaching could do. Teaching mathematics procedurally not only failed to reduce mathematics anxiety; it actually increased it (Khoule et al., 2017; Skemp, 1971).

Price (2015) found that female students experience higher levels of mathematics anxiety than male ones. Female students experience less enjoyment of mathematics and less confidence in mathematics class; they also report their mathematics anxiety more often. It can be said that female students have more mathematics anxiety than male ones (Beesdo et al., 2009; Geist, 2010; Hembree, 1990). Nonetheless, some studies have revealed that both men and women experience the same level of mathematics anxiety (Jameson, 2014; Wood, 1988). A study conducted by Jameson (2014) on mathematics anxiety among students from kindergarten to grade 6 found no differences between genders. Therefore, it can be said that there are diverse findings about gender differences and their relation to mathematics anxiety.

Attitude

Research on the role of students' attitudes toward mathematics, along with challenges to that role, have attracted the attention of mathematics educators and educational researchers (Chen et al., 2018). Attitude is a psychological propensity that is expressed by evaluating a specific entity with approval or disapproval (Fisher & Rickard, 2008). In addition, in line with the definition by Pehkonen and Pietilä (2003), "attitude is a relatively stable psychological tendency toward a particular idea, object, or entity with a certain degree of positivity or negativity" (Sunghwan & Taekwon, 2021, p. 2).

Students' different experiences with mathematics form their attitudes toward it. They develop a positive or negative attitude because of their accumulated experiences with a subject affect their psychological state (Sunghwan & Taekwon, 2021). For instance, according to Mullis et al. (2020), students with a positive attitude toward mathematics were interested in participating in mathematics courses and spent more time studying mathematics than students with a negative attitude. In contrast, students with a negative attitude toward mathematics perceived it as an unnecessary subject and felt afraid to participate in courses dedicated to it (Guo et al., 2015; Wigfield et al., 2016).

A study conducted by Esuong and Edoho (2018) revealed that primary school students with positive attitudes toward learning mathematics performed well in mathematical tasks. Students with negative feelings about mathematics performed poorly. Students with positive attitudes enjoy studying and practicing mathematics, which increases their competence (Aiken, 1970; Andamon & Tan, 2018; Ashcraft & Kirk, 2001). Students with negative attitudes, however, have inappropriate feelings about the subject (Jennison & Beswick, 2010). Consequently, students with positive attitudes toward mathematics perform much better than those with negative attitudes (Papanastasiou, 2000).

Mathematics teachers should consider students' cognitive and emotional needs (McLeod, 1992). Students' mathematics anxiety inhibits their cognitive development by creating negative attitudes that affect their long-term futures (Wu et al., 2012). Student with positive attitudes toward mathematics perform well. Thus, to succeed in mathematics, it is important to maintain a positive attitude toward it (Dowker et al., 2012).

Academic Achievement

Academic achievement is "specified level of attainment or proficiency in academic work as evaluated by the teachers, by standardized tests or by a combination of both" (Bhat & Bhardwaj, 2014). Better academic achievement means that students tend to excel (Robiah, 1994). In the present study, students' achievement refers to their success in an exam that based on an eighth-grade curriculum (Zulnaidi & Zamri, 2017). In this paper, the term achievement and performance have been used interchangeably. Distinct definitions may be found in the literature, such as achievement often refers to a kind of summative assessment while performance is a more neutral term in this respect.

Hypothesis

The hypotheses are formed as null hypotheses. This was done for the purpose of a straightforward testing of them, and based on the literature the researchers' real expectations can be formed as the alternative hypotheses of the following.

1. In terms of students' achievement, there will be no statistically significant difference between the control group and the experimental group.
2. In terms of decreasing students' anxiety, there will be no statistically significant difference between the control group and the experimental group.
3. In terms of improving students' positive attitudes toward mathematics, there will be no statistically significant difference between the control group and the experimental group.

Research Questions

1. Does teaching mathematics conceptually affect students' achievement?

2. Does teaching mathematics conceptually affect students' anxiety?
3. Does teaching mathematics conceptually affect students' attitude?

METHODOLOGY

Participants

Two hundred students (110 female and 90 male) in grade 8 participated in the present study. They were 14 years old. Purposive sampling was used to select three public secondary schools in Erbil, in the Kurdistan region of Iraq. According to the previous studies, on average, taking 200 students as a sample study is sufficient for this kind of study (see, for example, Andamon & Tan, 2018; Krejcie & Morgan,1970). Each class contains approximately 30 to 35 students. Accordingly, six classes were taken, three of them were chosen randomly as experimental groups and the rest of them were control groups. The three schools out of a total 130 schools were chosen based on the similarity of some important characteristics: geographical location, socioeconomic background, and students' previous aptitude in mathematics and science. These three aspects were considered in sample selecting to get an accurate result. In Erbil, people with different socioeconomic background live in different parts of geographical locations. For example, there is a Golden area where most of the people who live there have a rich economy. In these high socioeconomic status families, most of their children study in the top private schools, or their children have a private teacher for each subject. The aptitude of those students cannot equalize and combine with students who are from lower socioeconomic backgrounds. In the present study, the three schools were chosen in medium socioeconomic area. Another aspect that was considered in choosing the sample study was, students' previous aptitude in mathematics and science, they were checked by looking at their last year's grades to equalize the groups. The three school administrators were asked to provide us the students grades for mathematics and science subjects. The students who had on average less than 60% were sorted to a low level, an average of 60% to 80% sorted to a medium level, and an average over 80% sorted to a high level. Then the groups were redistributed based on the students' level where each group contains an approximately equivalent amount of low, medium, and high level students.

There were two groups of students in each of the three sample schools. We divided the students at each school into two numerically equivalent groups based on their average mathematics grades from the previous year. Therefore, the participants in all the three schools represented two large groups whose average mathematics grades resembled one another. Each group contained low, average, and high achievers. Their ages were the same, and they read the same mathematics subjects. One of the groups was designated the control group. The other was designated an experimental group; it was chosen randomly. Each group in each of the three schools was called a subgroup (**Table 1**).

Table 1. Sample study

Group	Sub-groups		
Experimental	A1	A2	A3
Control	B1	B2	B3

We chose students in eighth grade due to the substantial nature of this period. Grade 8 is the main period when students develop their understanding of mathematics (Hembree, 1990). Another reason for our choice was that the majority of studies on mathematics anxiety and mathematics attitude have focused on either primary school or adult students; few of them have focused on students in the middle grades (Ashcraft & Kirk, 2001). Therefore, the present study took students in eighth grade as a sample.

Instrumentation

In this study, an experimental approach was used to reveal how teaching mathematics conceptually affects students' achievement in, anxiety about, and attitude toward mathematics. The study tries to prove or disprove its hypotheses statistically (Ross et al., 2005). The experimental method is a research method that attempts to reveal the impact of an independent (experimental) variable that the researcher purposely manipulates while holding all other conditions constant. Comparing the experimental group to the control group reveals the effect of the independent variable (Jonassen et al., 2008).

To reveal how teaching mathematics conceptually affects students' achievement, pre- and post-tests were conducted on both groups of students (**Appendix A**). Pre- and post-tests were designed to compare groups and to measure changes resulting from a certain treatment (Dimitrov & Rumrill, 2003). The same achievement test was applied to the experimental and control groups. The pre-test was conducted at the beginning of the experiment to evaluate students' academic performances. After five weeks of teaching, the post-test was conducted to reveal how conceptual knowledge impacted students' achievement.

Conceptual knowledge can be measured by scores on problem-solving exams (Mariquit & Luna, 2017). All of the questions on the achievement tests were developed from an eighth-grade curriculum. The pre-test covered the topics namely, meaning of fractions, simplifying fractions, addition and subtraction of fractions, multiplication and division of fractions, absolute value, inequalities, power, decimal, and square roots. The post-test covered curriculum topics that students were taught during the experimental period, namely, equations, solving multi steps inequality, comparing fractions, probability, statistics (mean, median, and mode), and geometry (parallel lines, congruent triangles, isosceles triangles, equilateral triangles, and parallelogram). We chose these topics that were taught during the experimental period, based on the schools' principle that we had to follow the curriculum topics' order. The tests were prepared and applied by means of collaboration between us and the mathematics teachers. There were 25 multiple-choice questions for each of the pre- and post-test. Students received 1 point for each correct answer and 0 points for each incorrect answer. Therefore, a perfect mark was 25 points. For analysis purposes, the marks were converted to a scale of 100.

Mathematics attitude scale (MAS) developed by Aiken and Dreger (1961) and abbreviated math anxiety scale (AMAS) developed by Hopko et al. (2003) were adopted to determine how conceptual teaching affects students' attitudes toward and anxiety about mathematics, respectively. AMAS and MAS were constructed of nine questions and 20 questions respectively, all of the questions were multiple-choice questions and have five alternatives (**Appendix B** & **Appendix C**). At the beginning and the end of the experiment, students in the experimental and control groups were asked to complete MAS and AMAS evaluations. Analyzing the answers that students gave during the pre- and post-experiment periods revealed the effect of conceptual knowledge on students' attitudes and anxiety.

Procedure

This study's experimental approach was used to determine the impact of a manipulated (independent) variable: teaching for conceptual understanding (Sekaran, 1992). In the experimental (treatment) groups, mathematics instructors focused on teaching for conceptual understanding (**Appendix D**). In the control groups, they taught mathematics subjects conventionally (Gurbuz et al., 2010). Conventional teaching in the Kurdistan region of Iraq mostly depends on communicating procedural knowledge. In procedural teaching, the teachers teach how to use the mathematics rules to solve the problem regardless of explaining the relationships between the concepts, and answer about how and why questions in the steps in the problem solving (**Appendix E**). Teachers are prepared to teach both conceptual and procedural knowledge. Nonetheless, for many reasons, they focus on procedural understanding; it is more comfortable for them, the learning materials are available in textbooks, and it does not require them to change their teaching styles (Maryunis, 1989). Teachers in both groups had almost had the same amount of teaching experience, which ranged from 8-11 years per person. The duration of the experiment was five weeks, beginning on May 16 and ending on June 20, 2021. In each group 25 lessons were taught; the period of each lesson was 40 minutes. In the experimental group's classes, teachers combined conceptual and procedural teaching by focusing on three main aspects: using instructional language carefully and avoiding naked numbers, focusing more on concepts than algorithms and shortcuts, and building connections among concepts. In addition, these teachers concentrated on providing in-depth explanations of the relationships among the steps required to solve a mathematics problem. This teaching method also utilized thinking aloud, one of the most common strategies in metacognition (Moghadam & Fard, 2011). Teaching mathematics conceptually is not a simple process; a certain kind of knowledge is required (Putnam et al., 1992). Nevertheless, we coordinated with mathematics teachers to successfully investigate the experimental process. We worked with mathematics teachers to design and deliver lessons that fostered conceptual knowledge of mathematics. They also attended most of the classes in both the experimental and control groups to observe the teaching processes and to ensure that the experiment proceeded correctly.

The rules and regulations of research ethics were considered carefully in the present study. According to Robson (2011), ethical principles should be considered in studies that contain participants. All participation in the present study was voluntary, not physically or psychologically compelled. The names of participating schools and pupils have all been anonymized.

Evidence of Validity and Reliability

Both internal and external validity were considered in this study (Gall et al., 2003). The characteristics that threaten internal validity were taken in consideration, mainly, using different methods of teaching in the groups, different levels of students' aptitude, socioeconomic background, and the number of students in a classroom. The researchers controlled both procedural and conceptual teaching in the groups by making a meeting with the mathematics teachers before each class to exchange information on teaching methods, thereby, the researchers attended in each class to be sure that there is no bias in the teaching method process. Students who were taken as a sample study had almost the same level of aptitude, and they approximately had the same socioeconomic background. Finally, the number of students in the groups was equalized. In terms of external validity, the results of this study can only be generalized to students who have characteristics similar to those of the participants in this study (Dimitrov & Rumrill, 2003).

A group of five mathematics education experts validated the achievement test and assessed its content validity. Content validity, "refers to the steps taken to ensure that assessment items reflect the construct they are intended to measure" (Cook & Hatala, 2016, p. 3).

The questions were given to a judging panel of subject matter experts (SMEs). The expert panel made only a few suggestions. Otherwise, the items in the instrument were found to be commensurate with the research questions. Based on the experts' suggestions, the tests were revised to make them appropriate for use in the study. The measure of the content validity of the whole test called content validity index (CVI) is defined as the average of CVR score for all questions in the test (Nikolopoulou, 2022). The CVI for pre- and post-test were 0.98 and 1, respectively.

To test the reliability of the achievement test, a pilot study was applied to 30 students. A difficulty index, discriminant index, and Cronbach's alpha were used to measure the test's reliability. The difficulty indices of the pre- and post-test were moderate: 45% and 41%, respectively (Suherman & Sukjaya, 1990). The discriminant index for the pre-test was .77, and it was .72 for the post-test. These are good levels (Lim, 2007). Each test consisted of 25 questions, yielding good reliability (Cronbach's alpha=.85 and .84).

When studying mathematics anxiety, Hopko et al. (2003) reported excellent reliability values, strong convergent validity, and appropriate internal consistency: Cronbach's alpha=0.67. The test-retest reliability of AMAS was considered: r=0.66. In addition, according to Aiken and Dreger (1961), MAS has excellent internal consistency and temporal stability: positive attitude subscale Cronbach's alpha=0.911, and negative attitude subscale Cronbach's alpha=0.902.

Implementation of Teaching Conceptually

To teach mathematics conceptually, the teachers in the present study focused on the three main aspects: using instructional language carefully and avoiding naked numbers, focusing more on concepts than algorithms and shortcuts, and building connections among concepts (Molina, 2014).

Using instructional language carefully and avoiding naked numbers

When teachers use the correct language of mathematics, they protect students from confusion. Even a slight deviation in language can cause content errors because mathematics is one of the most accurate disciplines. Language mistakes might occur because of carelessness by mathematics teachers. For example, if a teacher writes the fraction 9/12 and asks students to reduce it, the expected solution is 3/4. Understanding the concept of reduction is different from understanding the concept of simplification, however. These terms have contradictory meanings in relation to the concept of equality between the fractions 9/12 and 3/4.

Teachers should avoid using naked numbers, or numerals without descriptors. It is necessary to connect the idea of measurement and the wider idea of representation. For example, take the problem 6÷1/2. If students are taught in careless language, this expression may be interpreted as "how many times does 1/2 go into 6?" What does this expression mean? If students get the answer, do they know what it represents? Do they know why the result is larger than the original instead of smaller? What if the teacher asks, "How many halves are there in 6"? in that case, students realize that the answer is 12 halves, not only the naked number 12.

Focusing more on concepts than algorithms and shortcuts

Algorithms and shortcuts are beneficial only when they help conceptual understanding rather than hindering it. Teachers must provide students with detailed steps about how to solve a mathematics problem. They can explain why those steps happen and connect the concepts with the process.

Understanding mathematics procedurally rather than conceptually makes it harder for students to absorb more complicated subjects. When using a shortcut method, the problem-solving notion remains fuzzy for students. Accordingly, each step in the problem-solving process must be included. This inclusion is fundamental for a deeper understanding of what exactly happens when an equation is solved.

Building connections among concepts

Finding connections among mathematical concepts and ideas can be used to develop mathematics pedagogy. Recognizing connections is the basis for a deeper understanding of mathematical concepts. For example, take the concept of average. The main point here is a profound understanding of the concept of multiplication. Generally, multiplication is defined as repeated addition. The crucial missing point in this definition is the repetition of groups of equal sizes. Therefore, multiplication can be defined as the repeated addition of groups of equal sizes. In 2×5, for example, the explanation could be 5+5. This equation represents two groups, each of which has five units. To achieve a deep understanding of the concept of average, it is necessary to relate it to the concept of division. Students will recognize that division means equal distribution and that the average is defined as an equal redistribution. This conceptual definition could not be obtained without making a connection to multiplication and division.

RESULTS

Different techniques were utilized to statistically analyze the data and thereby to answer the three research questions. Descriptive statistics–namely, standard deviation and mean–were used to describe students' performance in, anxiety about, and attitude toward mathematics (Andamon & Tan, 2018) (**Table 2**).

Table 2. Descriptive statistics for experimental & control group measures (SD: Standard deviation)

Measure	Group	n	Mean (%)	SD
Pre-grade	Experimental	100	69.0	15.89
	Control	100	69.4	14.51
Post-grade	Experimental	100	72.1	16.42
	Control	100	69.9	14.79
Pre-attitude	Experimental	100	55.1	16.83
	Control	100	59.0	16.55
Post-attitude	Experimental	100	69.6	12.90
	Control	100	62.6	13.17
Pre-anxiety	Experimental	100	31.6	3.55
	Control	100	31.8	3.77
Post-anxiety	Experimental	100	26.5	3.74
	Control	100	30.6	3.58

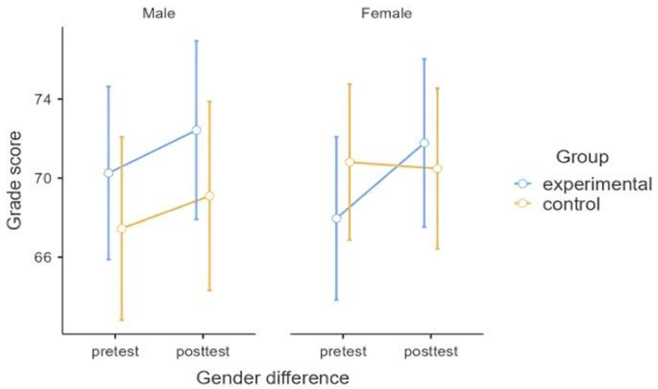

Figure 1. Difference between grade score for male & female students in experimental & control groups (Source: Authors' own elaboration, using Jamovi software)

To determine whether the developmental sessions produced any improvement in mathematical achievement, grade scores were analyzed using mixed analyses of variance (2 groups × 2 genders × 2 measurements) with Tukey multiple comparisons. The difference between the mathematical achievement pre-test and post-test scores was significant: $F(1, 196)=18.48$, $p<.001$, partial eta squared=0.086. Students achieved higher scores in the post-test than the pre-test. The difference between genders was not significant: $F(1, 196)=0.04$, $p=0.83$, partial eta squared=0.000. Likewise, the interaction between group and gender was not significant: $F(1, 196)=0.79$ $p=0.375$, partial eta squared=0.004. In contrast, the interaction of group × gender × measurement was significant: $F(1, 196)=4.52$, $p=0.035$, partial eta squared=0.023. Based on Tukey multiple comparisons, the experimental group's mathematical abilities improved significantly more for girls than boys. There was no change in the control group throughout the five weeks (**Figure 1**).

Repeated ANOVA measurements showed a statistically significant difference between the control group and the experimental group in terms of pre- and post-test scores: $p=0.007$. The participants in the treatment group achieved higher mathematics scores than control group (**Figure 2**).

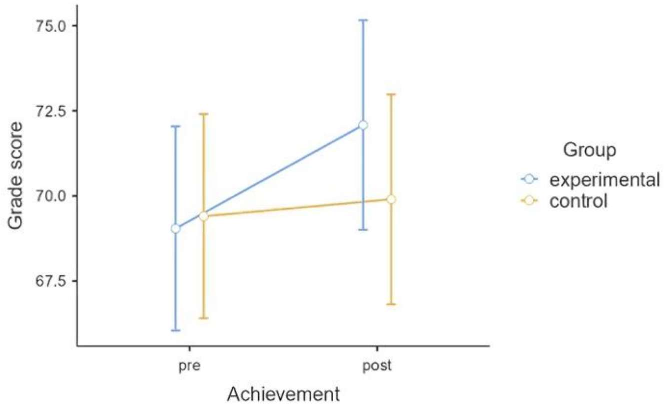

Figure 2. Difference between grade score for students in experimental & control groups (Source: Authors' own elaboration, using Jamovi software)

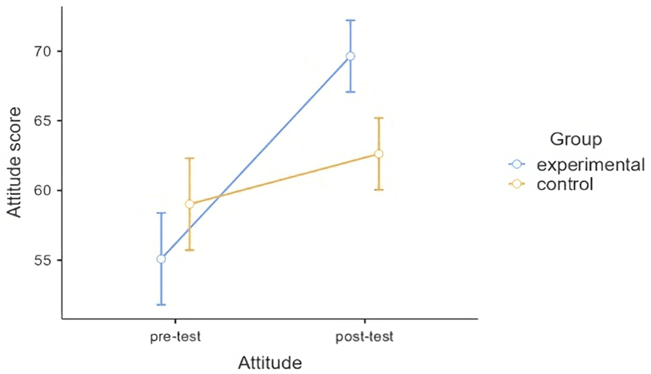

Figure 3. Estimated marginal means for positivity of students' attitude toward mathematics (Source: Authors' own elaboration, using Jamovi software)

According to the repeated ANOVA measurements, there was statistically significant difference between the attitudes of the experimental and control groups: $F(1, 198)=149$, $p<0.001$, partial eta squared=0.429 (**Figure 3**). Likewise, there was a statistically significant difference between the anxiety levels of the experimental and control groups: $F(1, 198)=117$, $p<0.001$, partial eta squared=0.372 (**Figure 4**). Girls had higher anxiety than boys: $F(1, 196)=4.33$ $p<0.001$ partial eta squared=0.022.

From that perspective, the three null hypotheses in the present study are rejected. The following substitutes replace them: "There is a statistically significant difference between the control group and the experimental group in terms of students' achievement," "There is a statistically significant difference between the control group and the experimental group in terms of the degree to which students' anxiety decreased," and "There is a statistically significant difference between the control group and the experimental group in terms of the degree to which students' positive attitude toward mathematics improved."

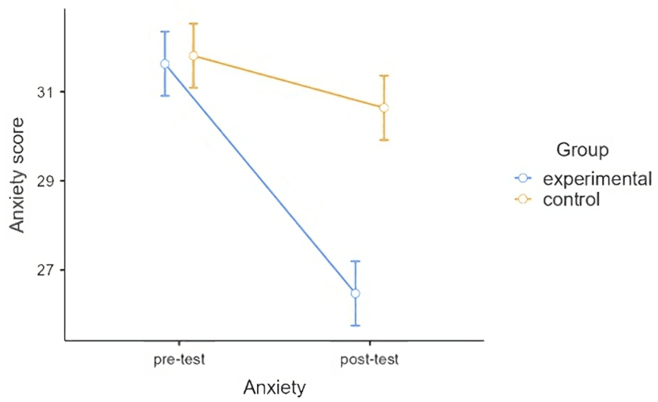

Figure 4. Estimated marginal means for students' anxiety (Source: Authors' own elaboration, using Jamovi software)

DISCUSSION

This study investigated how teaching mathematics conceptually impacts students' achievement in, anxiety about, and attitude toward mathematics.

The results indicate that, compared to procedural teaching, conceptual teaching has a more positive effect on achievement, anxiety, and attitudes.

This study shows that, in terms of mathematics achievement, there is a statistically significant difference between the experimental and control groups. The teachers in the treatment group followed lesson plans focused on conceptual understanding. Teachers in the control group followed the principles of conventional teaching. Teaching mathematics conceptually led students in the experimental group to achieve higher mathematics exam scores than students in the control group. The experimental group's mean achievement post-test score was much higher than its mean pre-test score.

In the control group, only a slight difference was recorded between pre- and post-test scores. This finding indicates that conceptual teaching affected mathematics achievement in the experimental group. This finding is consistent with Khoule et al.'s (2017) study, which showed that the students in a conceptual group performed better on conceptual and procedural quizzes than those in a procedural group. Furthermore, students in the conceptual group were more able to reason logically, to formulate solutions, and to understand mathematics flexibly.

Teaching mathematics conceptually helped students achieve higher scores on mathematics exams because these students learned mathematics based on relations and concepts rather than procedures. Relational understanding helped the students remember mathematics rules more easily, and it provided them with the ability to adapt their knowledge to solve new mathematics problems. Students who possess relational understanding will be active in finding new areas in which to apply mathematics–for instance, they might apply it to the roots of trees that extend in all directions (Skemp, 1976). In contrast, it was difficult for students in the control group to apply what they learned in the classroom on the post-test. In particular, they failed to answer questions with different contexts from those that they solved during class. This finding is consistent with Zakaria et al.'s (2010) study in which a significant relationship between conceptual knowledge and mathematics achievement was recorded. Conceptual teaching helps students to better achieve the learning process goal (Hurrell, 2021).

This study also found that teaching mathematics conceptually had a different impact on different genders (**Figure 1**). Female students in the experimental group achieved higher exam scores than male students. This result contradicts the findings of Hyde et al. (1990), who found no statistically significant difference in mathematics achievement between male and female students. This might suggest that female students have more mathematics anxiety than male ones (Beesdo et al., 2009). We found that teaching for conceptual understanding reduces female students' anxiety more than it does male students'. As a result, female students achieved higher scores than male ones on the mathematics test. Students with lower levels of mathematics anxiety tend to have a stronger understanding of arithmetic than students with higher levels of mathematics anxiety (Price, 2015).

Students in the experimental group learned to self-monitor by asking themselves many questions during the problem-solving process. For instance, they asked, "How can I start to solve this problem?" "What is the relationship among these steps?" and "What is another method to solve this problem?" These kinds of questions help learners improve their metacognitive ability, which helps them to understand mathematics in a deeper and more meaningful way (Ilyas & Basir, 2019; Kramarski et al., 2002; Salehi, 2002). This finding is consistent with the findings of previous studies. For example, Amin and Sukestiyarno's (2015) study found that metacognitive knowledge plays an active role in improving students' achievement. According to Grant (2014), students' ability to monitor themselves during the learning process can increase their ability to solve problems.

Reducing mathematics anxiety helps students achieve better results in mathematics exams (Furner, 2019). The correlation between students' mathematics anxiety and their performance in mathematics courses was negative and very high (Hembree, 1990). To increase students' mathematics achievement, educators must work on reducing their anxiety. Lau et al. (2022) found in their study that to reduce students' anxiety, teachers have to use an appropriate teaching method and have confidence in teaching mathematics. According to Ashcraft and Moore (2009), "math anxiety is a significant impediment to math achievement" (p. 197). Additionally, Jameson (2014) found that mathematics anxiety negatively impacts mathematics achievement in both children and adults. The findings in the present study support Ashcraft and Kirk's (2001) study, which revealed that reducing students' mathematics anxiety allows them to concentrate their attention more on mathematical tasks that develop their competence.

The present study is consistent with the finding of a study conducted by Casty et al. (2021) on mathematics anxiety, attitude, and performance among secondary school students, that both students' anxiety about and attitude toward mathematics affected their performance and achievement in mathematics. Students with high anxiety about and negative attitude toward mathematics had less confidence to carry out mathematics tasks. Accordingly, for students' better achievement in mathematics, students' level of anxiety and attitude toward mathematics should be considered and moderated by using an appropriate teaching method (Casty et al., 2021; Richland et al., 2020).

Students' mathematics anxiety increases unless teachers teach conceptually. According to a theory by Skemp (1971), procedural teaching increases students' mathematics anxiety. The present study found a negative relationship between conceptual knowledge and participants' mathematics anxiety. After learning mathematics conceptually for five weeks, the mathematics anxiety of students in the experimental group was reduced. In terms of anxiety reduction, the treatment group outperformed the control group. The experimental group's mean score on the anxiety post-test dropped remarkably compared to its pre-test results. In the control group, however, the mean score in the pre-test was 31.8, a number that only dropped to 30.6 on the post-test. These results indicate that teaching mathematics conceptually helps students to reduce their anxiety, which leads to higher achievement.

Students in school are often taught how to memorize mathematics rules and how to use those rules to solve mathematics problems, but they do not work on comprehending the explanations of the skills they learn (Rossnan, 2006). In a study conducted by Khoule et al. (2017), algebra teachers in community colleges mostly depended on procedural methods. Focusing on memorization techniques rather than conceptual understanding encourages students' mathematics anxiety (Newstead, 1998). According to a study by Curtain-Phillips (1999), students' anxiety has increased due to the fact that teachers do not cater to various learning styles. Mathematics anxiety can develop in response to unsupportive teaching styles (Webb, 2017). Therefore, to overcome students' mathematics anxiety, educators must find different methods of teaching mathematics (Rossnan, 2006). For example, they can teach for conceptual understanding in addition to teaching conventionally. Teachers can start with visual models and finalize the lesson with a symbolic model that expresses abstract concepts in symbols (Ketterlin-Geller, 2007).

Compared to students in the experimental group, students in the control group found it harder to remember mathematical rules during the post-test. Many students in the control group asked, "Teacher, could you explain to me which rule I have to use to answer question number x?"

The present study statistically revealed that the conceptual method of teaching can overcome students' anxiety about mathematics. The traditional method, also known as the procedural method, cannot reduce students' anxiety because it depends on memorizing rules. The students had high levels of anxiety because they were taught mathematics procedurally, which increased their anxiety rather than overcoming it (Khoule et al., 2017). Moreover, high levels of anxiety make it difficult for them to remember mathematical rules (Rayner et al., 2009).

For students to be successful in learning mathematics, teachers must take both cognitive and affective aspects into consideration (McLeod, 1994). In the present study, the effect of a conceptual teaching method on students' attitudes toward mathematics revealed that the attitudes of participants in the experimental group improved more than attitudes of those in control group. The experimental group's mean post-test score for positive attitude was higher than its mean pre-test score, but this was not the case for the control group. According to Ashcraft and Kirk (2001), maintaining a positive attitude toward mathematics helps to increase students' competence in mathematics courses. This finding is consistent with a study conducted by Jennison and Beswick (2010) on the effect of an interactive teaching method on students' attitudes toward and performance in mathematics. That study's results revealed that the majority of participants increased their confidence and their positive attitudes toward mathematics by the end of the study. In addition, a study conducted by Zamir et al. (2022) on determining students' attitudes and achievements through problem based learning, revealed that mathematics attitude is considered as a critical element in the process of mathematics learning. Confidence in learning mathematics and mathematics motivation had a significant role in the students' attitudes toward problem based learning. Therefore, there is a strong association between students' attitudes toward mathematics and their performance in this subject. Students with a positive attitude toward mathematics have higher performance than whom have negative attitudes toward it (Naungayan, 2022; Segarra & Julià, 2021).

A high F value represents an effect variance that exceeds the error variance by a large amount. In the present analysis, the F value was high, which aligns with prior educational research. For example, in a study on the consistency and variability of learning strategies in different university courses, four university courses were taken. That study's authors found that "Lack of regulation was reported less frequently in the private law and criminal law courses compared to the other courses in both studies ($F[1, 84]=86.19$, $p<0.001$, and $F[1, 62]=56.06$, $p<0.001$)" (Vermetten et al., 1999, p. 13).

The results of the present study show that students will earn higher score in mathematics if they participate in thinking and exploring rather than merely learning mathematics rules mechanically (Khoule et al., 2017). Teaching mathematics conceptually built confidence among this study's participants. It decreased their mathematics anxiety, and it helped them create the confidence needed to absorb new mathematical knowledge. To summarize, teaching mathematics conceptually not only improves students' achievement in mathematics but also reduces their mathematics anxiety and improves their positive attitudes toward this subject.

CONCLUSION & RECOMMENDATIONS

The present study investigated how teaching mathematics conceptually impacts students' achievement in, anxiety about, and attitudes toward mathematics. The results revealed that there are statistically significant relationships between teaching for conceptual understanding and the three aforementioned variables. Compared to the control group, the experimental group performed better in a mathematics exam. This result indicates that participants in the experimental group outperformed the control group in mathematics achievement, reduced their mathematics anxiety, and improved their attitudes toward mathematics. One limitation of this study is that it focused only on students in state secondary schools in Erbil. Furthermore, only a limited number of students was used as a sample.

The conventional teaching method should be revised to match the skills that students need to be productive (Rossnan, 2006). Students need practical mathematics classes, and they should be involved in thinking and analyzing rather than merely learning the rules and applying them (Curtain-Phillips, 1999). When learning mathematics, metacognition must be emphasized as much as cognition because it is one of the main elements in students' achievement.

Educators should encourage students to acquire the ability to confront mathematics problems (Furner & Berman, 2003). Students are often afraid of mathematics, which generates anxiety more than other disciplines (Shore, 2005). According to one study, almost two-thirds of adults in the United States had a deep fear of mathematics (Burns, 1998). However, anxiety can be controlled and reduced (Tobias, 1993). To reduce mathematics anxiety, educators must focus less on speed tests (Reys et al., 1995). Instead, they have to focus more on adopting new teaching methods that are conformable to mathematics (Zamir et al., 2022). Additionally, mathematics teachers must make mathematics an enjoyable subject by using a meaningful method of teaching, and they must also explain the importance of mathematics to everyday life and to students' future careers (Cruikshank & Sheffield, 1992).

We have the same belief as Taylor and Brooks (1986): both basic concepts and correct procedures are important when solving mathematics problems. Therefore, we recommend that mathematics teachers should use multiple teaching methods to build connections between abstract thought and conceptual learning (Hurrell, 2021).

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Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX A

Pre-test mathematics, Kurdistan Region of Iraq, Erbil City, 2021, State School, Grade 8th

Q1) Choose the correct answer (25 degree for all, only one degree for each question).

1. The simplest form of this fraction $\frac{16}{24}$ is

A) $\frac{8}{12}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) $\frac{4}{6}$

2. The simplest fraction form of this decimal 1.125 is.

A) $\frac{9}{8}$ B) $\frac{45}{40}$ C) $\frac{8}{9}$ D) $1\frac{5}{40}$

3. The decimal form of this fraction $-\frac{37}{20}$ is.

A) 1.85 B) -0.85 C) -1.58 D) -1.85

4. The result of $|7| + |-14|$ is.

A) 7 B) 21 C) -7 D) -21

Q2) Compare. Use $<$ or $>$ or $=$.

1. $|-4| \cdots 3$
2. $|3-5| \cdots |5| - |13|$
3. $\left|-1\frac{2}{3}\right| \cdots \frac{3}{2}$

Q3) If $y = -\frac{3}{9}$, which is NOT equal to y?

A) $\frac{-1}{3}$ B) $-\frac{1}{3}$ C) $-\left(\frac{-1}{3}\right)$ D) $-\left|\frac{-1}{-3}\right|$

Q4) What is the correct answer of the following.

1. $-8.01 - 9.02=$

A) 17.03 B) -1.01 C) 1.01 D) -17.03

2. $\frac{4}{9} + \frac{7}{15} =$

A) $\frac{41}{45}$ B) $\frac{11}{24}$ C) $\frac{11}{15}$ D) $\frac{42}{45}$

3. $3\frac{1}{2} + \left(-7\frac{4}{5}\right)=$

A) $4\frac{3}{10}$ B) $-4\frac{3}{10}$ C) $-10\frac{13}{10}$ D) $4\frac{3}{3}$

4. $8.25 - \frac{5}{16} =$

A) 8.5625 B) -7.9375 C) 7.9375 D) 3.25

5. $10.71 \div (-0.7) =$

A) 15.3 B) -15.3 C) -7.497 D) -15.5

6. $6\frac{3}{7}\left(\frac{7}{8}\right)=$

A) $6\frac{35}{56}$ B) $6\frac{21}{56}$ C) $5\frac{35}{56}$ D) $6\frac{10}{15}$

Q5) The value of $\left(\frac{1}{2}\right)^4 - 3^2$ is.

A) $-\frac{142}{16}$ B) $\frac{143}{16}$ C) $-\frac{144}{16}$ D) $-\frac{143}{16}$

Q6) Multiply or Divide. Write the product as one power.

7. $\frac{4^7}{4^3}$
8. $y^8 \times y^{-8}$
9. $(7^4)^3$

Q7) The standard notation of 4.05×10^{-6} is.

A) 0.00000405 B) 0.00000045 C) 4050000 D) -0.00000405

Q8) The scientific notation of 0.000000615 is.

A) 615×10^{-6} B) 615×10^{9} C) 615×10^{-9} D) 615×10^{6}

Q9) The result of $5(\sqrt{225}-10)$ is.

A) 125 B) 5 C) -25 D) 25

Q10) Which is NOT equivalent to $3\times3\times3\times3\times3\times3$?

A) 729 B) 18^2 C) 3^6 D) 9^3

Q11) Awaz has a pot containing $\frac{3}{4}$ liters of liquid, he needs to put them in cups of capacity $\frac{1}{8}$ liters. How many cups he need?

A) 6 cups B) $\frac{2}{4}$ C) 8 cups D) $\frac{2}{32}$

Q12) India's population is approximately 1.08×10^9. How do you write this in standard form?

A) 1080000000 B) 1080000 C) 108000 D) 180000000

Q13) In Lana's refrigerator 5 grape juice cans, she drinks $\frac{1}{4}$ can per day. For how many days are these cans enough for?

A) 20 days B) 80 days C) 5 days D) 4 days

Post-test mathematics, Kurdistan Region of Iraq, Erbil City, 2021, State School, Grade 8th

Q1) Choose the correct answer (25 degree for all, only one degree for each question).

1. The solution of $3x-6>18$ is?

A) $x>21$ B) $x>8$ C) $x<6$ D) $x<8$

2. On her last three science test, Mariam got 84,96, and 88. What grade she need to get on her next test to get average of 90 on these 4 tests?

A) 90 B) 95 C) 92 D) 100

3. Ahmed and Dara together have 36 posters. Ahmed has double what Dara has, how many posters does each one has?

A) Ahmed 24, Dara 12 B) Ahmed 12, Dara 24 C) Ahmed 18, Dara 18 D) Ahmed 15, Dara 20

4. What is the value of $(5-3)^{-3}+(845-3)^0$?

A) $\frac{1}{8}$ B) 1 C) $1\frac{1}{8}$ D) $\frac{7}{8}$

5. Which number is not a solution for the inequality $n-7<1$?

A) 2 B) 4 C) 6 D) 8

6. In order to have the 600000 D he needs for a bike, Dlir plans to save an amount of money each week for the next 15 weeks. What is the minimum amount that Dlir has to save each week in order to reach his goal?

A) 60000 B) 66000 C) 30000 D) 40000

7. The solution of $\frac{5}{6}x+\frac{1}{2}<\frac{2}{3}+\frac{1}{6}x$.

A) $x<\frac{1}{4}$ B) $x>\frac{1}{4}$ C) $x<4$ D) $x>4$

8. Which one is not the same as this fraction $\frac{13}{15}$?

A) $\frac{39}{50}$ B) $\frac{65}{75}$ C) $\frac{91}{105}$ D) $\frac{26}{30}$

9. The solution of $\frac{2}{3}+\frac{5}{7}$ is?

A) $\frac{7}{10}$ B) $\frac{29}{21}$ C) $\frac{7}{21}$ D) $\frac{10}{21}$

10. The solution of $\frac{41}{48}-\frac{5}{6}$ is?

A) $\frac{1}{48}$ B) $\frac{36}{48}$ C) $\frac{36}{42}$ D) $\frac{1}{6}$

11. If $9+3x=2y$, which of the following is a solution of this equation?

A) $\frac{9+3y}{2}=x$ B) $x=\frac{2}{3}$y-9 C) $x=\frac{2}{3}y-3$ D) $x=2y-3$

12. Naveen spins the spinner at right. What is the probability that the spinner will land on the number 4?

A) 4 B) $\frac{1}{4}$ C) 2 D) $\frac{1}{2}$

13. Which percent best shows the probability that Amir will randomly draw a non-odd number from five cards numbered 2,4,6,8, and 10?

A) 75% B) 25% C) 50% D) 100%

14. Azad made 26 of the 32 free throws he attempted. Which percent is closest to the experimental probability that he will make his next free throw?

A) 50% B) 60% C) 70% D) 80%

15. What is the mean of data set: 272, 276, 281, 279, 276?

A) 276.8 B) 267.8 C) 282.1 D) 285

16. In which data set the mean, median, and mode all the same number?

A) 6, 2, 5, 4, 3, 4, 1 B) 4, 2, 2, 1, 3, 2, 3 C) 2, 3, 7, 3, 8, 3, 2 D) 4, 3, 4, 3, 4, 6, 4

17. The median of this set 92, 88, 65, 68, 76, 90, 84, 88, 93, 89 is.

A) 83 B) 90 C) 88 D) 93

18. A bag contains 25 blue balls, 22 brown balls and 68 red balls. What is the probability of randomly selecting a blue ball from the bag?

A) $\frac{115}{25}$ B) $\frac{22}{115}$ C) $\frac{5}{23}$ D) Other

19. A bag contains 13 yellow balls, 5 black balls and 17 red balls. What is the percentage probability of randomly selecting a yellow ball from the bag?

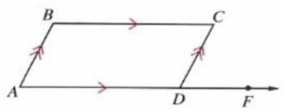
A) 20% B) 52% C) 68% D) 13%

20. Which expression is true for this data set 15, 18, 13, 15, 16, 14?

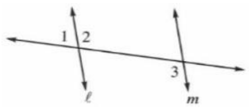
A) Mean<mode B) Median>mean C) Median=mean D) Median=mode

21. $m\,\widehat{BCD} = (x+50)^\circ$; $m\,\widehat{CDA} = (3x+20)^\circ$ Find $m\,\widehat{BAD}$.

A) 15° B) 27.5° C) 65° D) 77.5°



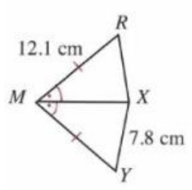
22. What is the relation between 1 and 3?



A) supplementary B) alt-int C) same-side int D) given vertically opposite angles

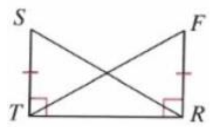
23. What is the perimeter of the polygon MRXY?

A) 29.9 cm B) 39.8 cm C) 49.8 cm D) 59.8 cm



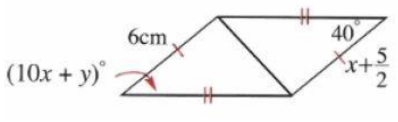
24. What postulate you can use to prove $\Delta STR \cong \Delta FRT$?

A) ASA B) SSS C) HS D) SAS



25. What is the value of y in the adjacent figure?

A) 5 B) 20 C) 35 D) 10



APPENDIX B

Abbreviated Math Anxiety Scale (Amas) Developed by Hopko et al. (2003)

Please write your name in the upper right-hand corner. Read the following statements carefully and decide how anxious you would be (how anxious you would feel) in the following situations? Please cross the correct number with an X.

1 - Not at all 2 - A little 3 - Medium 4 - Quite 5 - Very

For example, if you feel that you are not bothered at all when you have to answer, mark 1, and if you really do, mark 5.

There is no right or wrong solution, we want to know your feelings in each situation.

Situations	Scale				
1. Use the tables at the back of the mathematics textbook.	1	2	3	4	5
2. To think about the mathematics test due in 1 day.	1	2	3	4	5
3. Watch the teacher solve an algebraic equation on the board.	1	2	3	4	5
4. Take an exam in a mathematics course.	1	2	3	4	5
5. Get homework with complicated tasks, which should be solved by the next hour.	1	2	3	4	5
6. Listen to a lecture in mathematics class.	1	2	3	4	5
7. Listen to another student explain a mathematical formula.	1	2	3	4	5
8. Write a pamphlet in math class.	1	2	3	4	5
9. Start a new chapter in a mathematics book.	1	2	3	4	5

APPENDIX C

Mathematics Attitudes Scale (Mas) Developed by Aiken and Dreger (1961)

Please write your name in the upper right-hand corner. Each of the statements on this opinionnaire expresses a feeling which a particular person has toward mathematics. You are to show, on a five-point scale, the extent of agreement between the feeling expressed in each statement and your personal feeling. The five points are: Strongly Disagree (1), Disagree (2), Undecided (3), Agree (4), Strongly Agree (5). You are to encircle the number which best indicates how closely you agree or disagree with the feeling expressed in each statement AS IT CONCERNS YOU.

Situations	Scale				
1. I am always under a terrible strain in a math class.	1	2	3	4	5
2. I do not like mathematics, and it scares me to have to take it.	1	2	3	4	5
3. Mathematics is very interesting to me, and I enjoy math courses.	1	2	3	4	5
4. Mathematics is fascinating and fun.	1	2	3	4	5
5. Mathematics makes me feel secure, and at the same it is stimulating.	1	2	3	4	5
6. My mind goes blank, and I am unable to think clearly when working math.	1	2	3	4	5
7. I feel a sense of insecurity when attempting mathematics.	1	2	3	4	5
8. Mathematics makes me feel uncomfortable, restless, irritable, and impatient.	1	2	3	4	5
9. The feeling that I have toward mathematics is a good feeling.	1	2	3	4	5
10. Mathematics makes me feel as though I'm lost in a jungle of numbers and cannot find my way out.	1	2	3	4	5
11. Mathematics is something which I enjoy a great deal.	1	2	3	4	5
12. When I hear the word math, I have a feeling of dislike.	1	2	3	4	5
13. I approach math with a feeling of hesitation, resulting from a fear of not being able to do math.	1	2	3	4	5
14. I really like mathematics.	1	2	3	4	5
15. Mathematics is a course in school which I have always enjoyed studying.	1	2	3	4	5
16. It makes me nervous to even think about having to do a math problem.	1	2	3	4	5
17. I have never liked math, and it is my most dreaded subject.	1	2	3	4	5
18. I am happier in a math class than in any other class.	1	2	3	4	5
19. I feel at ease in mathematics, and I like it very much.	1	2	3	4	5
20. I feel a definite positive reaction to mathematics; it's enjoyable.	1	2	3	4	5

APPENDIX D

A Lesson Plan Sample for Teaching Conceptual Understanding

Subject: Fraction

Tools: Whiteboard, color marker, learning cards

Time: 40 minutes

Objective: The objective of this lesson was to help the students to investigate a deep understanding of the concept of fraction by providing examples in real life and explaining the relation of fraction with division and ratio, also explaining the meaning of simplifying fractions.

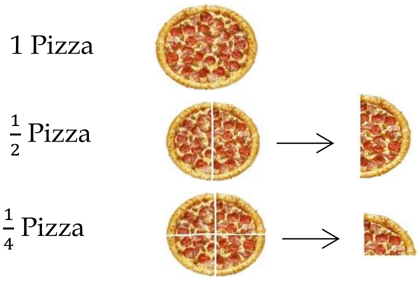
Methodology: Conceptual teaching was utilized, that focus on understanding the concepts, and relations.

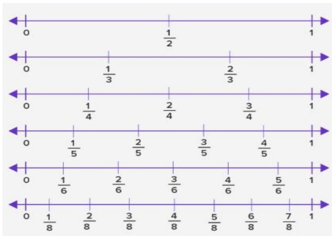
Stages: There were six stages in this lesson plan.

Stage 1 (4 minutes): In this stage, the meaning of fraction was provided, "Is a numerical quantity that represents a part of the whole". Also, writing of fraction and the name of each part, numerator, and denominator is explained.

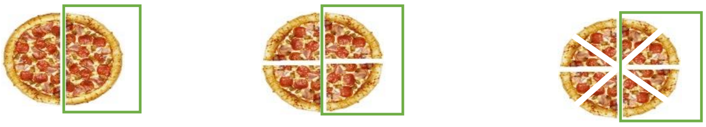
Stage 2 (10 minutes): Some real examples to explain the notion of fraction were provided in the classroom. In this stage, the teacher spends more time deepening in explanation and providing more examples, visually and orally.

For example:





The same technique was used to explain the notion of simplifying and equality of fractions.

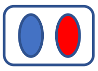


$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

Stage 3 (5 minutes): In this stage, the subjects that have a relation with fraction were explained. Here the concept of division and ratio were explained and compared with fraction.

For example:

In ratio $\frac{1}{2}$ means one of two, such as one ball is red in two balls.



In ratio $\frac{2}{3}$ means two of three, such as two are boys out of three students.

More examples were provided to them with graphs, $\frac{5}{10}, \frac{4}{5}, \frac{7}{9}, \ldots$

Also, the relation between fraction and division was explained by the teacher. Fraction is a single number, while division is an operation between two numbers. For example, $9 \div 3, 48 \div 12, 18 \div 9$.

Stage 4 (6 minutes): After explaining the notion of fraction, simplifying, and equality of fractions conceptually, the teacher talked about the fraction formulas and how to use them for solving a mathematical problem.

$$\text{Fraction} = \frac{x}{y}\ \forall\, x, y \in Z;\ \text{for example, } \frac{3}{5}, \frac{7}{12}, \frac{72}{115}, \ldots, \text{etc.}$$

In simplifying fraction, we have to find a number that is both of numerator and denominator divided by it, and we will continue until there is not any more division.

$$\frac{4 \div 2}{8 \div 2} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

$$\frac{36}{123}, \frac{225}{510}, \frac{50}{310}, \frac{28}{84}, \frac{121}{286}, \ldots, \text{etc.}$$

Stage 5 (10 minutes): Questions in the textbook were solved by the students with the teacher's help in this stage.

Stage 6 (5 minutes): Evaluate students' understanding of the concept of fraction by asking them some questions, such as, what is fraction? What is different among fraction, division, and ratios? Why does the value of fraction not change if we divide or multiply both the numerator and denominator by the same number? In addition, the participants were evaluated during the 5th stage.

APPENDIX E

A Lesson Plan Sample for Teaching Procedurally (Traditional Teaching)

Subject: Fraction

Tools: Whiteboard, color marker, learning cards

Time: 40 minutes

Objective: The objective of this lesson was to help the students to understand the meaning of fraction mathematically, know the formulas and how to use them in solving problems, and explain how to simplify a fraction.

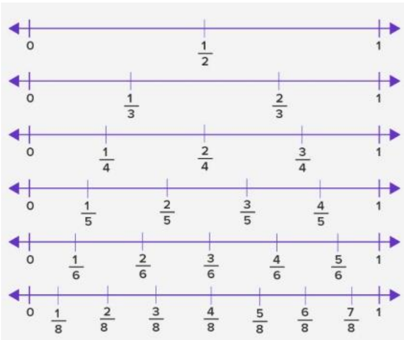
Methodology: Procedural teaching was utilized, that focus on how to use the rules to solve a fraction problem.

Stages: There were five stages in this lesson plan.

Stage 1 (5 minutes): In this stage, the meaning of fraction was provided, "Is a numerical quantity that represents a part of the whole". Also, writing of fraction and the name of each part, numerator, and denominator were explained.

Stage 2 (5 minutes): Some examples to explain the meaning of fraction was provided in the classroom. In this stage, the teacher spends less time on explanations, and providing some examples.

For example:



Stage 3 (10 minutes): The teacher talked about fraction formulas and how to use them for solving a mathematics problem.

$$\text{Fraction} = \frac{x}{y}\ \forall\, x, y\ \in Z;\ \text{for example, } \frac{3}{5}, \frac{7}{12}, \frac{72}{115}, \ldots, \text{etc.}$$

In simplifying fraction, we have to find a number that is both of numerator and denominator divided by it, and we will continue until there is not any more division.

$$\frac{4\div 2}{8\div 2} \quad = \quad \frac{2\div 2}{4\div 2} \quad = \quad \frac{1}{2}$$

$$\frac{36}{123}, \frac{225}{510}, \frac{50}{310}, \frac{28}{84}, \frac{121}{286}, \ldots, \text{etc.}$$

Stage 4 (15 minutes): The questions in the textbook were solved by the students with the teacher's help.

Stage 5 (5 minutes): Evaluate students' understanding of fractions by asking them some questions, such as, what is fraction? How do simplify fraction? Why does the value of fraction not change if we divide or multiply both the numerator and denominator by the same number? In addition, the participants are evaluated during the 4th stage.