MTA SZTAKI Közlemények 37/1987 pp. 63-96

SCALING OF RANDOM VARIABLES AND ARRANGEMENT PROBLEMS IN LAYOUT DESIGN

B. GOETZE and *W. NEHRLICH*

Dept. of Appl. Discrete Math.
Karl-Weierstrass-Inst. of Math.
Acad. Sci. of the GDR
Mohrenstr. 39, Berlin, DDR-1086

1 Introduction

A key problem in Design Automation is the arrangement of components of a large and complicated system. As an important example consider the placement problem, where the modules of a circuit have to be assigned to favourable locations on a placement media. At the strongly idealized level of topological design large systems can be represented by graph- or hypergraph models. Moreover, in many cases it is also adequate to represent the one-, two- or multidimensional placement media by special graphs. (An important example is a grid graph.) Using such models placement problems can be considered as discrete optimization problems, in particular as embedding problems for graphs or hypergraphs. Some examples of embedding problems of this kind are the following NP-hard problems (see /GaJo/, /Jo/):

OPTIMAL LINEAR ARRANGEMENT OF GRAPHS,
MIN CUT LINEAR ARRANGEMENT OF GRAPHS,
MINIMUM BANDWITH, CROSSING NUMBER,
EDGE-EMBEDDING ON A GRID,
MINIMUM AREA EMBEDDING OF PLANAR GRAPHS,
WEIGHTED GRAPH EMBEDDABILITY.

Stimulated by the practical importance of such problems many attempts and different approaches have been made in order to find approximative solutions or solutions for subproblems. Among the numerous heuristic solution procedures for the various kinds of problems one can find greedy strategies, iterative and Monte-Carlo-procedures and even such kinds of solution

methods which are founded on non-discrete mathematical methods. Different from those discrete strategies which proceed stepwise on the basis of local decision, the non-discrete models represent global optimization criteria. Examples of non-discrete approaches and strategies concerning one- and two-dimensional arrangement problems can be found in /QuiBr/, /Otten1/, /Otten2/, /Fuku1/, /May/. In our paper we deal with a non-discrete heuristic approach which is based on a model of mathematical statistics - the scaling of random variables by optimization of the correlation coefficient.

Starting from an idea of /Fuku/ we shall apply this model to various arrangement problems, where the two-dimensional embedding of hypergraphs is in the centre of our attention. The mathematical treatment of this approach is explained in detail. Furthermore, the embedding algorithm is generalized to the case of additional placement constraints by incorporating these constraints into the model from the beginning. A natural and important application of this approach is the layout design for electronic circuits.

2 Scaling by Optimization of Correlation Coefficient

Here we consider the following problem: Let two discrete random variables be given, then we search for a scaling of these random variables such that the correlation coefficient is maximum. This problem leads to the determination of extremal points of quadratic forms, which is treated in detail in standard literature (cf. /Ga/, /Co/). In subsequent chapters we will apply scaling theory to the solution of certain arrangement problems. Though the mathematical solution of the scaling problem appears in full detail in statistics literature, we shall present a short derivation of this solution here, so that we can refer to this derivation in § 5.

Let two random variables $\tilde{x}$ and $\tilde{y}$ be given, which are varying within the sets $\tilde{X} = \{\tilde{x}_1, \ldots, \tilde{x}_m\}$ and $\tilde{Y} = \{y_1, \ldots, y_n\}$, respectively. Here $\tilde{x}_i$ and $\tilde{y}_j$ are arbitrary elementary events. Let the probability distribution on $\tilde{X} \times \tilde{Y}$ be given by the matrix $P = (p_{ij})_{m,n}$. This means that p_{ij} is the probability of the event "$\tilde{x} = \tilde{x}_i$ and $\tilde{y} = \tilde{y}_j$". For short, we will write $rp_i = \sum_{j=1}^{n} p_{ij}$ and $cp_j = \sum_{i=1}^{m} p_{ij}$ for row- and column-sums, respectively. These values represent the probability of "$\tilde{x} = \tilde{x}_i$" and "$\tilde{y} = \tilde{y}_j$", respectively.

If we are scaling the $\tilde{x}_i$ and $\tilde{y}_j$ to positions on the x-axis and the y-axis, then from $\tilde{x}$ and $\tilde{y}$ we obtain real random variables x and y, respectively. This permits to define the expectation values:

$$Ex = \sum_{i=1}^{m} rp_i \cdot x_i \quad , \quad Ex^2 = \sum_{i=1}^{m} rp_i \cdot x_i^2 \quad \text{(the definition for y is analogous)},$$

$$E(x \cdot y) = \sum_{i=1}^{m} \sum_{j=1}^{n} x_i \cdot p_{ij} \cdot y_j \ .$$

From this definition we may also define the variances and the covariance:

$\mathrm{Var}(x) = Ex^2 - (Ex)^2$ (analogous for y) ,

$\mathrm{Cov}(x,y) = E(x \cdot y) - Ex \cdot Ey$.

The correlation coefficient is defined as follows:

$$\varrho_{x,y} = \frac{\mathrm{Cov}(x,y)}{(\mathrm{Var}(x) \cdot \mathrm{Var}(y))^{1/2}}$$

The correlation coefficient ranges in size between -1 and +1 . If $|\varrho_{x,y}| = 1$, then the variables x and y are deterministically correlated, and the (x_i, y_j) with $p_{ij} \neq 0$ are all located on a straight line. If $\varrho = +1$, then this straight line has a positive slope, for $\varrho = -1$ the slope is negative.

The value of $|\varrho_{x,y}|$ is a measure for the concentration of the probability density function P around a straight line. Notice, that the correlation coefficient has the property of being invariant with respect to linear transformations, i.e.

$$\varrho_{(ax+b,cy+d)} = \varrho_{x,y} \ .$$

Our aim is a scaling of the random variables $\tilde{x}$ and $\tilde{y}$ (i.e. to find values x_i and y_j) such that we obtain a maximum value of ϱ .

First, we can assume without loss of generality, that the matrix P possesses neither zero-rows nor zero-columns. This is because an elementary event with zero-probability could be scaled on any place without influencing the value ϱ . Since ϱ is invariant with respect to linear transformations, we can assume the values of x and y to be normalized such that

$$Ex = Ey = 0 \qquad (2.1.)$$

$$\mathrm{Var}(x) = \mathrm{Var}(y) = 1 \ . \qquad (2.2.)$$

From (2.1.) we obtain

$\mathrm{Var}(x) = Ex^2$, $\mathrm{Var}(y) = Ey^2$ and $\mathrm{Cov}(x,y) = E(x \cdot y)$.

Thus, from (2.1.) and (2.2.) we conclude the simplified formula

$$\varrho_{x,y} = E(x \cdot y) \ . \qquad (2.3.)$$

We use the following notations:

$$x = (x_1,\ldots,x_m)^T \ , \ y = (y_1,\ldots,y_n)^T, \ e_m = (\underbrace{1,\ldots,1}_{m})^T \ ,$$

and

$$R_P = \begin{pmatrix} rp_1 & & 0 \\ & \ddots & \\ 0 & & rp_m \end{pmatrix} , \quad C_P = \begin{pmatrix} cp_1 & & 0 \\ & \ddots & \\ 0 & & cp_n \end{pmatrix} .$$

Thus, we can write

$$Ex = x^T \cdot R_P \cdot e_m \ , \quad Ey = y^T \cdot C_P \cdot e_n \ ,$$

$$Ex^2 = x^T \cdot R_P \cdot x \quad \text{and} \quad Ey^2 = y^T \cdot C_P \cdot y \ .$$

The normalization conditions (2.1.) and (2.2.) then read as follows:

$$x^T \cdot R_P \cdot e_m = y^T \cdot C_P \cdot e_n = 0 \qquad (2.1.a)$$

$$x^T \cdot R_P \cdot x = y^T \cdot C_P \cdot y = 1 \ . \qquad (2.2.a)$$

Finally, expression (2.3.) has the form

$$\varrho = x^T \cdot P \cdot y \ . \qquad (2.3.a)$$

Now, the problem is to determine extremal points of $\varrho = x^T \cdot P \cdot y$ under the conditions (2.1.a) and (2.2.a). By the use of Lagrangian multipliers we obtain necessary conditions for local extremal points under certain conditions.

First we will consider only the second restriction from (2.2.a), i.e.

$$y^T \cdot C_P \cdot y = 1 \ .$$

Thus we have to define a function

$$H(x_1,\ldots,x_m,y_1,\ldots,y_n) = x^T \cdot P \cdot y - \varrho' \cdot y^T \cdot C_P \cdot y \ .$$

Necessary conditions for extremal points are:

$$\frac{\partial H}{\partial y_j} = 0 \qquad \text{for} \quad j = 1,\ldots,n$$

or, for short,

$$\frac{\partial H}{\partial y} = 0 \ .$$

Hence we obtain

$$\frac{\partial}{\partial y} (x^T \cdot P \cdot y) = \varrho' \cdot \frac{\partial}{\partial y} (y^T \cdot C_P \cdot y)$$

$$P^T \cdot x = 2 \varrho' \cdot C_P \cdot y \ . \qquad (2.4)$$

In order to clarify the meaning of the factor $2\varrho'$ we multiply equation (2.4.) by y^T and obtain

$$y^T \cdot P^T \cdot x = 2\varrho' \cdot y^T \cdot C_P \cdot y \ .$$

Under the above condition this leads to

$$x^T \cdot P \cdot y = 2\varrho' \quad , \text{ i.e.}$$

$$\varrho = 2\varrho' \ .$$

Thus, from (2.4.) we obtain

$$P^T \cdot x = \varrho \cdot C_P \cdot y \qquad (2.4.a)$$

Since the matrix P does not contain any zero-rows or zero-columns, R_P and C_P are regular matrices.

Thus, from (2.4.a) we obtain

$$\varrho \cdot y = C_P^{-1} \cdot P^T \cdot x \ . \qquad (2.4.b)$$

On the other hand, multiplying (2.3.a) by ϱ , we have

$$\varrho^2 = \varrho \cdot x^T \cdot P \cdot y = x^T \cdot P \cdot (\varrho \cdot y) \ .$$

Substituting (2.4.b) we conclude

$$\varrho^2 = x^T \cdot (P \cdot C_P^{-1} \cdot P^T) \cdot x \ . \qquad (2.5.)$$

Equation (2.5.) describes ϱ^2 as a quadratic form in x . Our aim is to find maximum values of ϱ^2 under the conditions (2.1.a) and (2.2.a). Thus, we seek for

$$\max_{\substack{x^T \cdot R_P \cdot x = 1 \\ x^T \cdot R_P \cdot e_m = 0}} x^T \cdot (P \cdot C_P^{-1} \cdot P^T) \cdot x \qquad (2.6.)$$

For this purpose, the well-known theorem about extremal points of quadratic forms (cf. /Co/, /Ga/) can be used.

Theorem

Let S and D be $(m \times m)$-matrices, S being symmetric and D positive definite. Let the (real) eigenvalues of

$$S \cdot x = \lambda \cdot D \cdot x$$

be $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_m$, and let $x^{(i)}$ be the eigenvector corresponding to λ_i. Then

$\lambda_i = \max \{ x^T \cdot S \cdot x \mid x^T \cdot D \cdot x = 1$ and $x^T \cdot D \cdot x^{(j)} = 0$ for all $j = 1, \ldots, i-1 \}$,
where the maximum value is assumed for $x = x^{(i)}$.

Hence, for the special case $i = 2$ the theorem yields

$$\lambda_2 = \max_{\substack{x^T \cdot D \cdot x = 1 \\ x^T \cdot D \cdot x^{(1)} = 0}} x^T \cdot S \cdot x$$

where the maximum value is assumed for $x = x^{(2)}$. For our purpose the theorem can be applied to (2.6.) with $S = (P \cdot C_P^{-1} \cdot P^T)$ and $D = R_P$. The maximum value (2.6.) equals to the second-largest eigenvalue of

$$(P \cdot C_P^{-1} \cdot P^T) \cdot x = \lambda \cdot R_P \cdot x \quad \text{or}$$

$$\boxed{(R_P^{-1} \cdot P \cdot C_P^{-1} \cdot P^T) \cdot x = \lambda \cdot x} \qquad (2.7.)$$

For the proof of this assumption it remains to show, that $x^{(1)} = e_m$ holds.

Lemma

The matrix $R_P^{-1} \cdot P \cdot C_P^{-1} \cdot P^T$ is a stochastic matrix.

Proof. Both the matrices $N_1 = R_P^{-1} \cdot P$ and $N_2 = C_P^{-1} \cdot P^T$ are clearly stochastic, i.e. all items are nonnegative and all row sums equal to one. Thus, $N_1 \cdot N_2$ is again a stochastic matrix.

According to the lemma, (2.7.) has the largest eigenvalue $\lambda_1 = 1$, with e_m as a corresponding eigenvector.

We discuss some more conclusions from the theorem.

Notice, that $Ex^{(2)} = x^{(2)T} \cdot R_P \cdot e_m = x^{(2)T} \cdot R_P \cdot x^{(1)} = 0$.
The same holds true for the other eigenvectors $x^{(3)}, \ldots, x^{(m)}$.
The eigenvalues $\lambda_3, \lambda_4, \ldots$ are also extremal points of φ^2 under the conditions (2.1.a) and (2.2.a), where the additional condition of orthogonality of the vectors (with respect to the matrix R_P) holds.

Hence it is evident, that all eigenvalues of (2.7.) are real and positive: $1 = \lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_m \geqslant 0$. Notice that these properties can also be seen directly from the structure of (2.7.). First note that (2.7.) is equivalent to

$$(R_P^{-1/2} \cdot P \cdot C_P^{-1} \cdot P^T \cdot R_P^{-1/2}) \cdot z = \lambda \cdot z \quad ,$$

which is a symmetric problem, and therefore all eigenvalues are real ones. Furthermore, let $F = R_P^{-1/2} \cdot P \cdot C_P^{-1/2}$, then the problem has the form

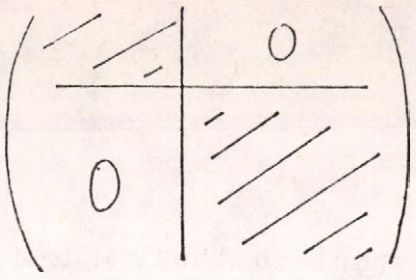
$$(F \cdot F^T) \cdot z = \lambda \cdot z \quad .$$

Hence $\lambda \geqslant 0$ follows from the fact, that this matrix is positive definite.
Now we consider a further fact concerning the spectrum of (2.7.)

Lemma

For the eigenvalue problem (2.7.) the property $1 > \lambda_2$ holds if and only if the matrix P is connected.

A matrix P is called unconnected iff there are permutations of the rows and columns transforming P into



Given a $(m \times n)$ matrix P we can define a corresponding graph $G_P = (V,E)$ in a canonical way as follows:

$V=\{1,\ldots,m\}$ and $(i,k) \in E \leftrightarrow \exists j \in \{1,\ldots,n\}: p_{ij} \neq 0 \wedge p_{kj} \neq 0$.

The graph defined in this way is connected (in the usual sense) iff the matrix P is connected.

Proof of the lemma

Let G_P be the canonical graph corresponding to P. Consider the matrix $B = (R_P^{-1} \cdot P \cdot C_P^{-1} \cdot P^T)$ of the eigenvalue problem (2.7.). Clearly, sgn (B) is the adjacency matrix of G_P. Then P is connected iff sgn (B) is connected. Now the theorem of PERRON/FROBENIUS (cf./Ga/) can be applied to the stochastic matrix B. This yields the assumption of the lemma.

The claim of P being connected will be natural for our applications (cf. subsequent chapters). Thus, for the spectrum of (2.7.) we can presume

$$1 = \lambda_1 > \lambda_2 \geqslant \lambda_3 \geqslant \ldots \geqslant \lambda_n \geqslant 0 .$$

Hence, the eigenvalue λ_2 together with the vector $x = x^{(2)}$ is a nontrivial solution for the problem (2.6.).

Similarly to (2.7.), one can derive an eigenvalue problem for the vector y:

$$(C_P^{-1} \cdot P^T \cdot R_P^{-1} \cdot P) \cdot y = \lambda \cdot y \qquad (2.8.)$$

Again, for the eigenvalues the equation $\lambda = \varrho^2$ holds. If we take

$$N_1 = R_P^{-1} \cdot P \quad \text{and} \quad N_2 = C_P^{-1} \cdot P^T \ , \qquad (2.9.)$$

then the problems (2.7.) and (2.8.) have the form

$$(N_1 \cdot N_2) \cdot x = \lambda \cdot x \quad \text{and}$$
$$(N_2 \cdot N_1) \cdot y = \lambda \cdot y \quad .$$

Such problems are equivalent with respect to their non-zero eigenvalues (including the multiplicities). The relations between equivalent eigenvectors are given by

$$x = N_1 \cdot y \quad \text{and} \quad y = N_2 \cdot x \quad .$$

In our case we obtain the following formulas:

$$x = (R_P^{-1} \cdot P) \cdot y \quad \text{and}$$
$$y = (C_P^{-1} \cdot P^T) \cdot x \quad . \qquad (2.10.)$$

3 Bandshape Optimization for Rectangular Matrices

There are numerous practical applications, where bandshape optimization for rectangular matrices is a key problem. Bandshape optimization means to permute rows and columns of a given matrix in such a way that the non-zero elements (and especially the "heavy" elements) are located as tight as possible around a diagonal line.

Example

$$A = \begin{pmatrix} 1&0&0&0&5 \\ 0&0&1&5&0 \\ 1&1&0&0&0 \\ 0&1&0&1&0 \\ 0&1&0&0&0 \\ 0&0&6&0&0 \\ 6&0&0&0&1 \end{pmatrix} \qquad A' = \begin{pmatrix} 6&0&0&0&0 \\ 1&5&0&0&0 \\ 0&1&1&0&0 \\ 0&0&1&0&0 \\ 0&0&1&1&0 \\ 0&0&0&6&1 \\ 0&0&0&1&5 \end{pmatrix}$$

If the row-sequence of A is permuted according to $\pi_0 = (6,2,4,5,3,7,1)$, and the column-sequence of A is permuted according to $\sigma_0 = (3,4,2,1,5)$, then we obtain the matrix A' which possesses a very good bandshape.

The fuzzy notion of bandshape can be made precise with the help of the correlation coefficient, regarding the given matrix as a probability density function.

Let $A = (a_{ij})_{m,n}$ be a real nonnegative rectangular matrix with $A \neq 0$. Then we turn to the the matrix

$$P = \frac{1}{a} \cdot A \quad , \quad \text{where} \quad a = \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \, . \qquad (3.1.)$$

Then P is a probability distribution on $\{x_1,\ldots,x_m\} \times \{y_1,\ldots,y_n\}$. The correlation coefficient ρ for the distribution P under the special assignment $x_i = i$ and $y_j = j$ represents one possible measure of the bandshape of A (or P respectively). The closer ρ is to +1 , the better is the bandshape of A. For our example, we obtain $P = \frac{1}{30} \cdot A$ and $\rho \approx -0.5931$. The permutation $\pi_0 = (6,2,4,5,3,7,1)$ corresponds to the new assignment $x_1 = 7$, $x_2 = 2$, $x_3 = 5$, $x_4 = 3$, $x_5 = 4$, $x_6 = 1$, $x_7 = 6$.

The permutation $\sigma_o = (3,4,2,1,5)$ corresponds to $y_1=4$, $y_2=3$, $y_3=1$, $y_4=2$, $y_5=5$. For the new assignment we obtain a correlation coefficient $\varrho' \approx +0.9712$, which expresses the bandshape quality of A'.

Let us now assume the given matrix A to be connected. Hence, the matrix A does not have any zero-rows or -columns. Clearly, P is also connected. The re-arrangement of rows and columns of A can be determined by scaling of the variables x_i and y_j. The eigenvalue-problem (2.7.) can be reformulated in the following form:

$$\boxed{(R^{-1} \cdot A \cdot C^{-1} \cdot A^T) \cdot x = \lambda \cdot x} \quad , \qquad (3.2.)$$

where $R = R_A$ and $C = C_A$. Then the eigenvector corresponding to the largest nontrivial eigenvalue λ_2 of (3.2.) yields the optimal real values for the x_i. According to (2.4.b) or (2.10.) the corresponding scaling of the y_j is derived from the transformation

$$y = (C^{-1} \cdot A^T) \cdot x \quad .$$

Notice, that x and y are of course real vectors and thus in general do not represent permutations of the rows and columns of A. The real components of these vectors have to be transformed to discrete ones. This can be performed simply by sorting the components.

For our example, the largest nontrivial solution is

$\lambda \approx 0.9671$ with

$$x = \begin{pmatrix} 0.0888 \\ -0.0807 \\ 0.0411 \\ -0.0368 \\ 0.0022 \\ -0.1048 \\ 0.0815 \end{pmatrix} \qquad y = \begin{pmatrix} 0.0773 \\ 0.0022 \\ -0.1014 \\ -0.0734 \\ 0.0875 \end{pmatrix}$$

The sorting of the components leads also to the permutations π_o and σ_o, which produce the above matrix A'.

The correlation coefficient is $\varrho = \sqrt{\lambda} \approx 0.9834$, whereas the matrix A' has only a correlation coefficient $\varrho' \approx 0.9712$. The loss of bandshape quality is connected with the transformation of the optimal eigensolution to the (discrete) sorted components. This is the reason why the method described here does not necessarily produce an optimal permutation of the given matrix. It is an open problem to evaluate the quality of approximation of this non-discrete method.

The method described here was presented in various forms and independently in /Fuku1/, /Otten1/ and /May/ (see also /Fuku2/, /MaMe/). Note that the derivation of this heuristic method with the help of the correlation coefficient - as described above - is due to /Fuku1/. In /Otten1/ the problem MIN CUT LINEAR ARRANGEMENT of hypergraphs was considered.[1] This problem reads as follows:

Let a finite hypergraph $H = (V,E)$ be given. Determine an embedding (i.e. a one-one-mapping) $\pi : V \to \{1,2,\ldots,|V|\}$ such that the out-width (or track number)

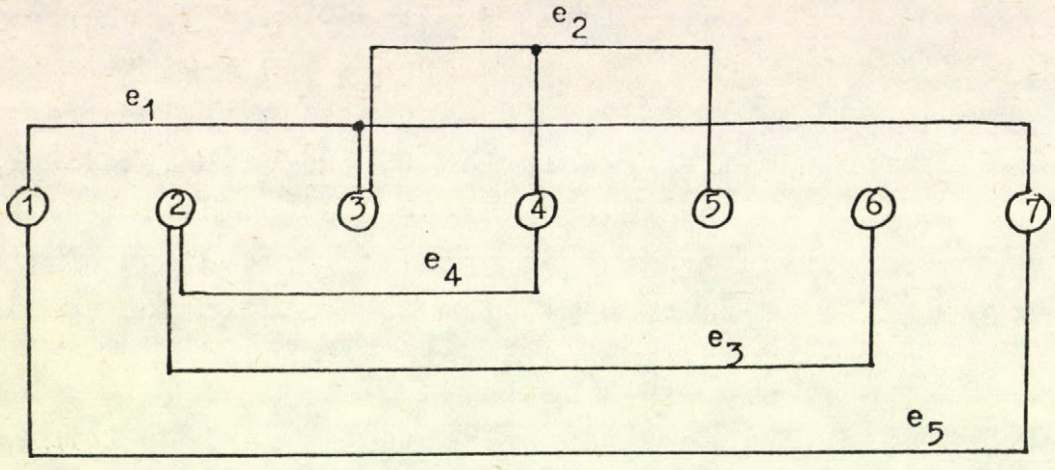
$$\max_{1 \le i \le |V|-1} \left| \{ e : e \in E \text{ and } \min_{u \in e} \pi(u) \le i < \max_{v \in e} \pi(v) \} \right| \text{ is minimum.}$$

This problem is NP-hard. (In 1974 STOCKMEYER proved that the MINCUT LINEAR ARRANGEMENT problem is NP-hard for graphs, cf. /GaJo/. This problem is NP-hard even when restricted to graphs with degree ≤ 3, whereas it becomes solvable in time $O(n \log n)$ for arbitrary trees, see /CMST/.)

As an example, consider the hypergraph $H = (\{1,\ldots,7\}, \{e_1,\ldots,e_5\})$, where $e_1 = \{1,3,7\}$, $e_2 = \{3,4,5\}$, $e_3 = \{2,6\}$, $e_4 = \{2,4\}$, $e_5 = \{1,7\}$ together with the embedding $\pi(i) = i$. We have the following figure:

---

[1] This problem is often called Board Permutation problem.



The cut-width for $\pi(i)=i$ equals 5. It is assumed for $i=3$.

Let us consider the incidence matrix of the hypergraph.

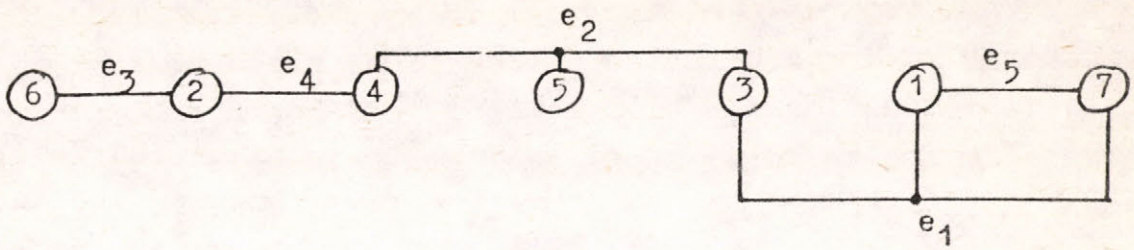
$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Here the rows correspond to the vertices, and the columns correspond to the edges of H.

The lines in the above figure (corresponding to the edges) appear as non-zero intervals, i.e. the regions between the first and the last non-zero element in the columns of A. Thus one can expect a reduction of the cut-width by reducing the lengths of these intervals. This goal clearly corresponds to the bandshape optimization for the incidence matrix. For our example, we obtain the above mentioned permutations π_0 and σ_0 for the rows and columns, respectively. The resulting matrix is

$$A' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

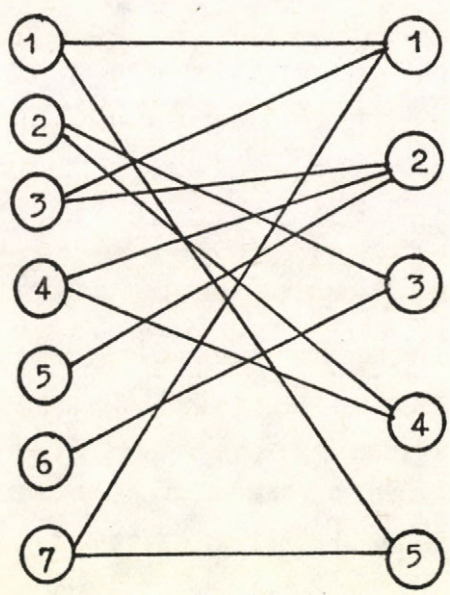
The corresponding re-arrangement of the hypergraph yields



The cut-width under this embedding equals 2.

Another arrangement problem is considered in /May/, /MaMe/ where bipartite graphs are investigated.

For an illustration of this problem consider the following example.

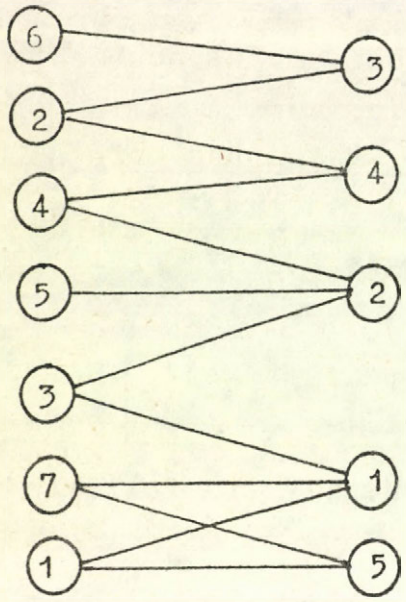


Roughly speaking, the problem consists of finding permutations of both columns such that we obtain a clear and readable representation of the graph. This fuzzy description was made precize in /May/ by formulating the following tasks:

a) Minimize the total sum of edge-lengths.
b) Minimize the number of edge crossings.

The heuristic procedure developed in /May/ is based on the algorithm described above.
This can be seen by describing the given bipartite graph by a rectangular 0/1-matrix, whose rows/columns correspond to the "left" / "right" vertices of the graph. For our example this matrix coincides with the incidence matrix A of the hypergraph presented above. (This is because hypergraph and bipartite graph are equivalent notions if we identify "right" vertices with edges and "left" vertices with the vertices of the hypergraph.) Thus we can expect, that a bandshape optimization will reduce both the total sum and the crossing number

for bipartite graphs. For our example, the permutations π_o and σ_o mentioned above produce the following representation of the bipartite graph.



Notice, that - similarly to the MINCUT problem - no results concerning the quality of the approximation are known for this optimization problem.

Now we shall discuss another aspect of the eigenvalue problems (2.7.) and (3.2.). It is connected with the power method (v. Mises method) which can be applied here.

Let $B \cdot x = \lambda \cdot x$ be an eigenvalue problem, and assume that all eigenvalues are real and nonnegative. The power method consists of the iteration of $x := B \cdot x$, where the vector x has to be normalized from time to time during the iteration process. In our problem we have

$B = N_1 \cdot N_2$, where

$N_1 = R^{-1} \cdot A$ and $N_2 = C^{-1} \cdot A^T$.

Since N_1 and N_2 represent the transformations between x and y (cf. 2.10.), we can divide the iteration step of the power method into:

$$y := N_2 \cdot x \qquad (3.3.a)$$

$$x := N_1 \cdot y \qquad (3.3.b)$$

Consider (3.3.a) :

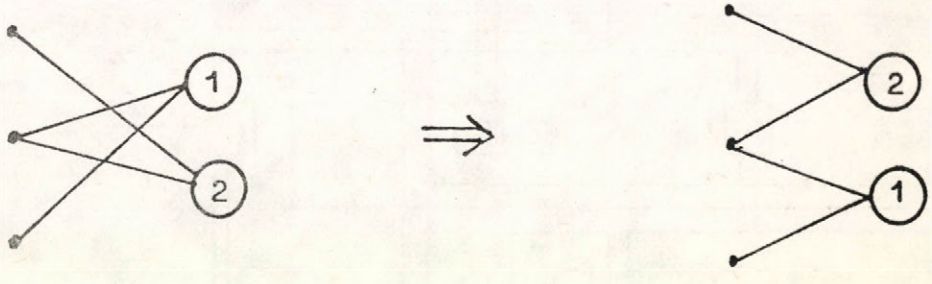
$y := C^{-1} \cdot A^T \cdot x$

$$y_j := \frac{1}{ca_j} \cdot \sum_{i=1}^{m} a_{ij} \cdot x_i \qquad j=1,\ldots,n \; . \qquad (3.4.a)$$

Analogously, from (3.3.b) we get:

$$x_i := \frac{1}{ra_i} \cdot \sum_{j=1}^{n} a_{ij} \cdot y_j \qquad i=1,\ldots,m \; . \qquad (3.4.b)$$

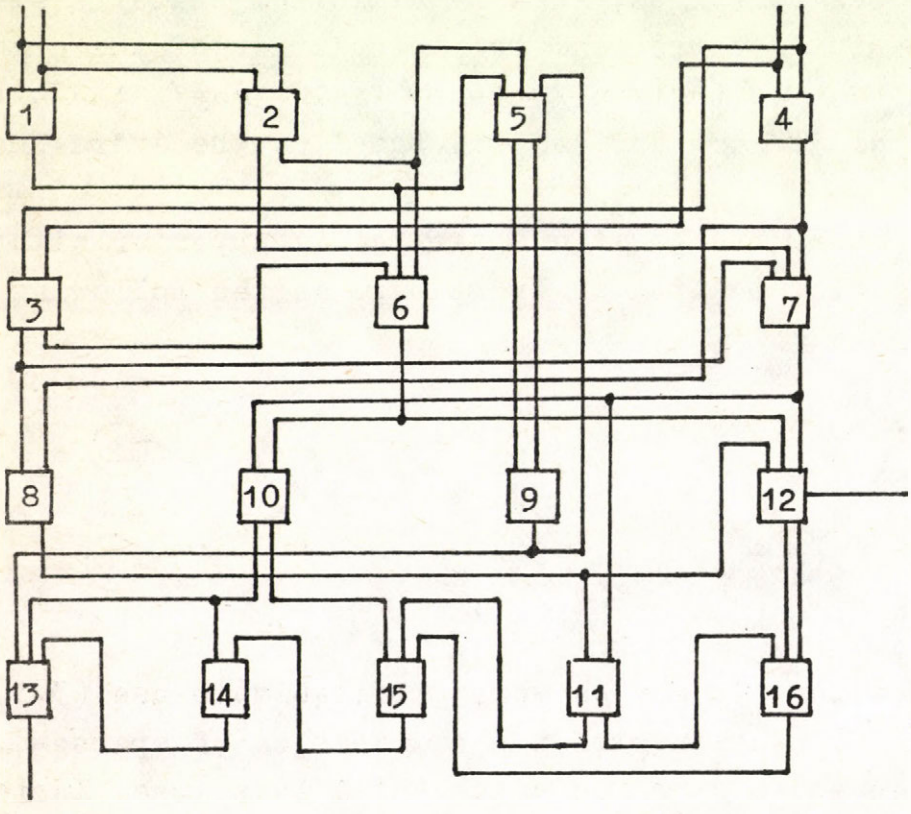
Now we turn back to the arrangement problem for bipartite graphs. Let x_i be the (real) locations of the m "left" vertices. Then applying step (3.4.a) means, that the "right" vertices are shifted into the center of gravity of its "left" neighbours. Similarly, applying (3.4.b) means to shift the "left" vertices into the center of gravity of its "right" neighbours. Therefore, the successive iteration process, alternating (3.4.a) and (3.4.b), is called "averaging". The averaging process converges, if an equilibrium state is attained. Those states correspond to the solution of the eigenvalue problem. Notice that in /Otten1/ and /May/ the heuristic procedures are derived and based on the averaging model.
Note that the averaging positions are really superior with respect to the minimization of crossings, as the following example demonstrates.



Finally, notice that it may be very profitable to use (3.3.a) and (3.3.b) in connection with the application of sparse-matrix techniques, even in the case that A is sparse. Instead of $B = N_1 \cdot N_2 = R^{-1} \cdot A \cdot C^{-1} \cdot A^T$, (which may be very dense in comparison with A) one has only to handle the sparse matrix A and the diagonal matrices R^{-1} and C^{-1} .

4 Placement of Circuits in the Plane

Design-automation for electronic circuits includes searching for sophisticated methods for the solution of placement and routing problems on the level of topological design. This level is characterized by a more or less rigorous idealization inasmuch as the geometrical shape and size of the modules and of the connecting wires are disregarded. Thus, in topological design, usually the representation of circuits by hypergraphs is a sufficient model. The modules correspond to the vertices, and the nets (signal sets) are represented by the edges of the hypergraph. Consider the following example of an electronic circuit.



The corresponding hypergraph has 16 vertices and 25 edges. The following table defines an enumeration $e_1,\ldots,e_{25}$ of the edges.

The table shows the incidence matrix of the given hypergraph.

i \ j	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	1	1		1																					
2	1	1	1		1																				
3						1	1	1	1																
4							1	1		1															
5			1	1							1	1											1		
6			1	1		1							1												
7					1				1	1				1											
8									1	1					1										
9											1	1											1		
10													1	1		1						1			
11														1	1					1	1				
12													1	1	1									1	1
13																1	1						1		
14																1	1	1							
15																		1	1	1		1			
16																			1		1			1	1

Notice that the model of an edge-weighted hypergraph may be more efficient in practical computation. In our example, we could join the edges e_1 and e_2, e_7 and e_8, e_{11} and e_{12}, e_{24} and e_{25}, respectively, to constitute new edges with weight = 2. For the subsequent discussion, however, it is more convenient to handle the simpler model of unweighted hypergraphs.

We consider the topological placement problem, i.e. the problem to locate the modules (vertices) in the plane in such a way that the subsequent routing process is able to find a "good" wiring. We shall assume that a wiring is good if it has minimum (or at least near-optimal) total wire-length. Since - in the placement step - the real final wiring has not yet been performed, the total wire-length has to be approximated in a certain way. In spite of the concrete model used for this estimation, a general goal for any placement procedure should be that strongly connected modules are located close together, i.e. that proximity reflects connectivity.

Notice that the connectivity of vertices intuitively determines distances d_{ij} between vertices. The distances derived from connectivity (and possibly from additional data) may be defined in various ways (cf. /Fuku/, /Otten2/) . In any case, a good placement result will be an embedding of the given hypergraph in Euclidean plane such that the distances are preserved "as well as possible".[1] In mathematical statistics, this problem is called the multidimensional scaling problem (MDS). The formulation of the placement problem as MDS-problem is an idea due to OTTEN, and it was pointed out in /Otten1/ and /Otten2/. However, we shall not use here the classical MDS-solution (due to SCHOENBERG, cf. /MaKeBi/) which was applied in /Otten2/ . We consider a slightly different solution, proposed in /Fuku/. The investigation of this approach leads to a generalization which will be presented in § 5.

The problem of "scaling of a hypergraph" can be treated by solving the eigenvalue problem (3.2.), maximizing the correlation coefficient. The eigenvector corresponding to the largest nontrivial eigenvalue yields a favourable embedding of the vertices of the hypergraph on the real axis. According to MDS-theory it is proposed in /Fuku/ to use the eigenvector according to the second-largest nontrivial eigenvalue for the location of the vertices in the second dimension of the plane. Since the eigenvectors are orthogonal, this leads to a favourable distribution of the vertices with zero-correlation. Let λ_2 and λ_3 be the largest nontrivial eigenvalues, and let $u = x^{(2)}$ and $v = x^{(3)}$ be the corresponding eigenvectors. The correlation coefficients are $\varrho_2 = \sqrt{\lambda_2}$ and $\varrho_3 = \sqrt{\lambda_3}$, respectively. In order to take into consideration the different weights of the two solutions, the eigenvectors are normalized to $\|u\| = \varrho_2$ and $\|v\| = \varrho_3$.

---

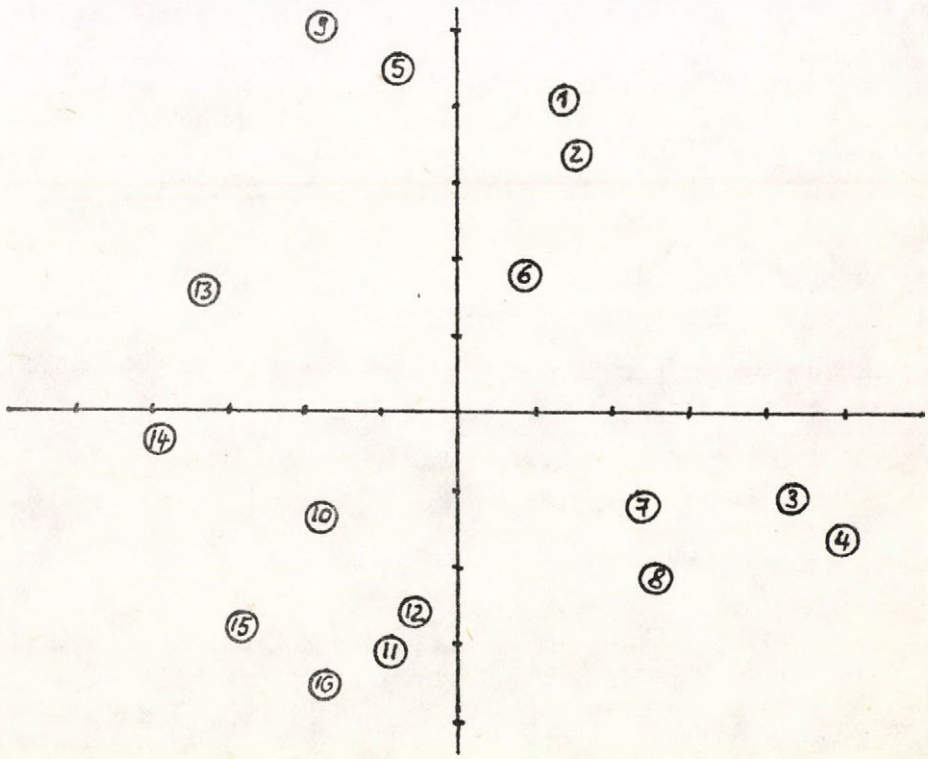
1) Notice that this problem is connected with the WEIGHTED GRAPH EMBEDDABILITY problem (cf. /Jo/) .

The set of points (u_i, v_i) - the scatter diagram - is called graphspace /Fuku/. Consider again our example. The solution of the eigenvalue problem (3.2.) yields

$\lambda_2 = 0.884$ and $\lambda_3 = 0.858$, where

$x^{(2)} = (1.29, 1.57, 4.39, 4.89, -0.85, 0.92, 2.38, 2.64,$
$-1.81, -1.79, -0.93, -0.68, -3.36, -3.90, -2.93, -1.81)$

$x^{(3)} = (+4.13, +3.32, -1.15, -1.72, +4.51, +1.76, -1.21, -2.23,$
$+5.00, -1.42, -3.15, -2.69, +1.64, -0.35, -2.77, -3.66)$



Thus we have an embedding of the hypergraph in Euclidean space, preserving in some sense the distances, i.e. reflecting the connectivity. In many applications (e.g. in gate array techniques), the set of possible module locations (slots) form a discrete and regular set of points.

An equidistant grid is a simple model of a discrete placement media which is often used. In this case the points of the scatter diagram produced by the non-discrete method have to be shifted to the grid points.

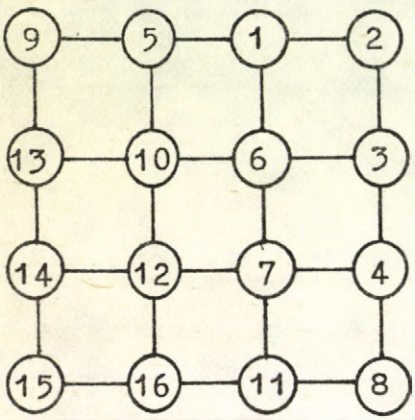
This is performed in two steps.

1. The set of points derived from the eigensolution is normalized in such a way that the first and second moments are

equal to the first and second moments of the grid, respectively.

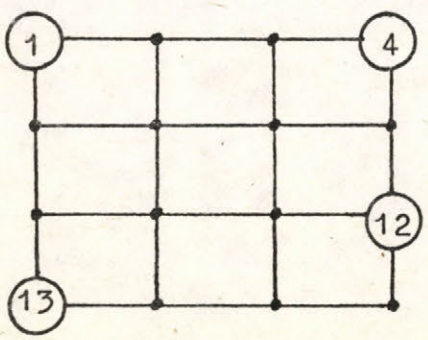
2. The points are translated to the grid points (see also /Fuku/). This step is performed with the help of a linear-assignment procedure (see e.g. /BuDe/).

For our example we obtain the following assignment to a regular (4 x 4)-grid:



5 The Placement Problem under Constraints

In most real-world placement problems we have to satisfy additional constraints as far as certain modules have a-priori fixed places on the placement media (grid). In consideration of such constraints in /Fuku/ a solution is proposed which yet disregards the connections between the fixed and the free modules. However, there is a possibility to incorporate the constraints (from the very beginning) into the eigenvalue problem. We shall explain the idea for this approach in terms of the x-direction of the grid. According to the relations between the components corresponding to the fixed modules, not all vectors $x = (x_1,\ldots,x_m)^T$ are admissable now. The admissable vectors form a subspace, which can be described as the range of a linear transformation. Without loss of generality assume that the variables $x_1,\ldots,x_q$ correspond to the a-priori fixed, and the remaining variables $x_{q+1},\ldots,x_m$ correspond to the free modules. Consider the example presented in § 4 under the following placement constraints.

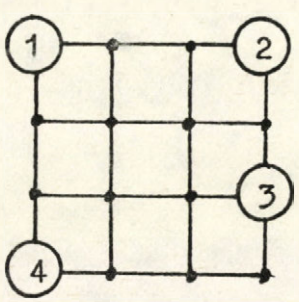


First we have to perform a re-numeration of the modules:

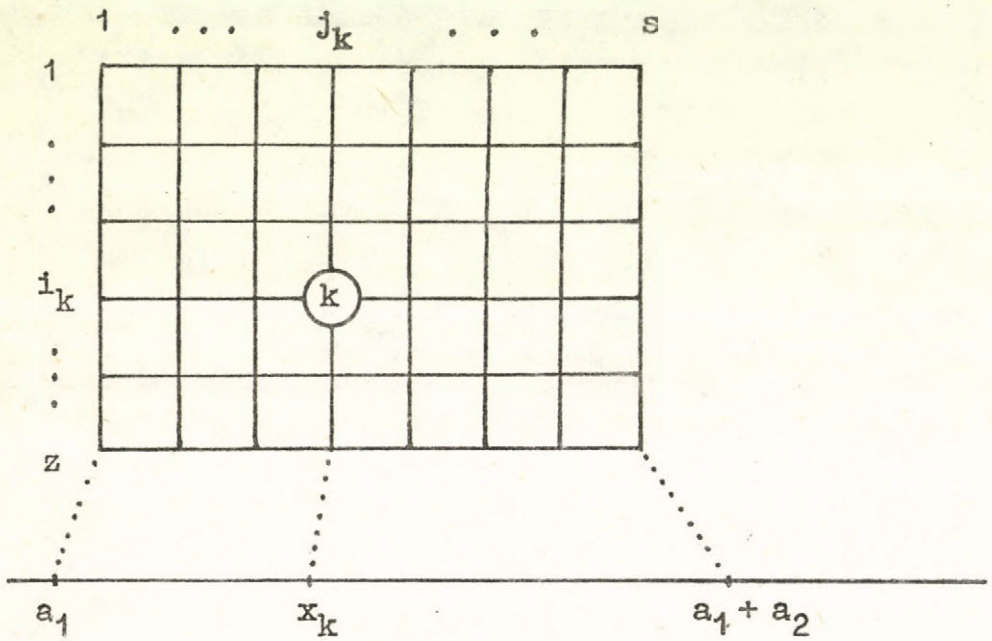
old index : 1 4 12 13 : 2 3 5 ... 11 14 15 16

new index : 1 2 3 4 : 5 6 7 ... 13 14 15 16

According to the new indices we have the following placement constraints.



With respect to the solution method using scaling theory it would not be adequate to formulate the constraints (related to the x-axis) in the form $x_1 = x_4 = 1$, $x_2 = x_3 = 4$. What we need in this model (because of the conditions $Ex = 0$ and $Var(x) = 1$) are two degrees of freedom: translation and expansion. They serve as parameters for $x_1,\ldots,x_q$. Therefore, we introduce two variables a_1 and a_2 instead of $x_1,\ldots,x_q$. Here the variable a_1 corresponds to the left border of the placement media (grid), whereas the variable a_2 corresponds to the expansion of the media.



Assume that module k is pre-assigned to the j_k-th column of the grid, and assume that the grid has s columns. Then we define

$$\sigma_k = \frac{j_k - 1}{s - 1} \qquad (k=1,\ldots,q) \,. \qquad (5.1.)$$

For our example, we have $\sigma_1 = \sigma_4 = 0$ and $\sigma_2 = \sigma_3 = 1$. Then for each x_k $(k \leq q)$ we have the condition

$$x_k = a_1 + \sigma_k \cdot a_2 \quad , \quad \text{i.e.} \qquad (5.2.)$$

$$\begin{pmatrix} x_1 \\ \vdots \\ x_q \end{pmatrix} = \begin{pmatrix} 1 & \sigma_1 \\ \vdots & \vdots \\ 1 & \sigma_q \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad (5.3.)$$

If we define $x_q^1 = \begin{pmatrix} x_1 \\ \vdots \\ x_q \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & \sigma_1 \\ \vdots & \vdots \\ 1 & \sigma_k \end{pmatrix}$, $a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$,

then from (5.3.) we get

$$x_q^1 = \Sigma \cdot a \quad . \qquad (5.4.)$$

If we assume, that not all fixed modules occupy the same column, then the rank of Σ equals 2. Otherwise, this rank equals 1. In the latter case we could turn to the reduced form

$$x_q^1 = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \cdot (a_1) \quad . \qquad (5.5.)$$

In order to unify both cases, we can assume in (5.4.), that

$$a = \begin{pmatrix} a_1 \\ \vdots \\ a_t \end{pmatrix} \text{ and } \Sigma = (\sigma_{ij})_{q,t} \text{ with rank } (\Sigma) = t \quad . \qquad (5.6.)$$

Now we define

$$u = \begin{pmatrix} a_1 \\ \vdots \\ a_t \\ x_{q+1} \\ \vdots \\ x_m \end{pmatrix} \quad \text{and} \quad \Phi = \left(\begin{array}{c|c} \Sigma & 0 \\ \hline 0 & \mathfrak{E}_{m-q} \end{array}\right)$$

Instead of (5.4.) we obtain

$$x = \Phi \cdot u \quad , \quad \text{where rank } (\Phi) = m - q + t \quad . \qquad (5.7.)$$

Furthermore, we can assume (cf. (5.3.) and (5.5.)) that there exists $\tilde{e} \in \mathbb{R}_{m-q+t}$ such that

$$e_m = \Phi \cdot \tilde{e} \quad . \qquad (5.8.)$$

Now we can apply (2.6.) using the formulas (5.7.) and (5.8.) derived from the constraints. Doing this, we obtain the following optimization problem:

$$\max_{\substack{u^T(\Phi^T \cdot R_P \cdot \Phi)\cdot u=1 \\ u^T(\Phi^T \cdot R_P \cdot \Phi)\cdot \tilde{e}=0}} [u^T (\Phi^T \cdot P \cdot C_P^{-1} \cdot P^T \cdot \Phi)\cdot u] \qquad (5.9.)$$

Here the matrix R_P is a quadratic nonnegative diagonal matrix. Hence $\sqrt{R_P}$ also exists. Thus

$$\Phi^T \cdot R_P \cdot \Phi = (\sqrt{R_P}\cdot\Phi)^T \cdot (\sqrt{R_P}\cdot\Phi) \qquad (5.10.)$$

is a Gramian and obviously a positive semidefinit matrix. Moreover, since the columns of Φ are linear undependent, this is also true for $\sqrt{R_P}\cdot\Phi$. Therefore, the matrix $\Phi^T \cdot R_P \cdot \Phi$ is even a regular and in particular a positiv definit matrix.

Hence, similarly to the derivation shown in § 3, we can use the theorem about extremal points of quadratic forms. Doing this, we obtain the eigenvalue problem

$$(\Phi^T \cdot P \cdot C_P^{-1} \cdot P^T \cdot \Phi)\cdot u = (\Phi^T \cdot R_P \cdot \Phi)\cdot \lambda \cdot u \ .$$

Considering this eigenvalue problem we first remark, that - similarly to § 3 - we can write this problem in the following form:

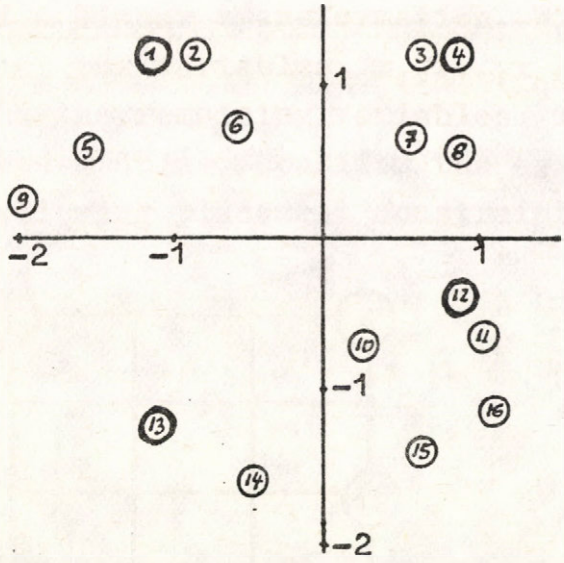
$$(\Phi^T \cdot A \cdot C^{-1} \cdot A^T \cdot \Phi)\cdot u = (\Phi^T \cdot R \cdot \Phi)\cdot \lambda \cdot u \ . \qquad (5.11.)$$

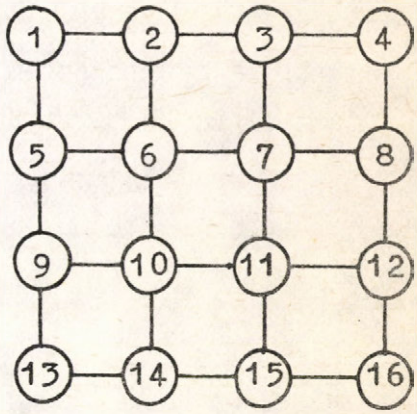
Because of the relation $e_m = \Phi \cdot \tilde{e}$, the vector $\tilde{e}$ is clearly the trivial solution corresponding to $\lambda_1 = 1$. Hence we are interested in the second-largest eigenvalue λ_2 and the corresponding eigenvector $u^{(2)}$.

From this solution, using the transformation (5.7.), we obtain the vector x containing the x-coordinates of the m modules. (Note that we have to turn back to the original enumeration of the modules.)

For our example we have $\sigma_1 = \sigma_4 = 0$ and $\sigma_2 = \sigma_3 = 1$ (cf. 5.3.). The eigenvalue problem (5.11.) then yields the solution $\lambda_2 = 0.881$ with

x = (-1.082,-0.892, 0.671, 0.882,-1.648,-0.510, 0.588, 0.936,
-1.954, 0.285, 1.004, 0.882,-1.082,-0.358, 0.676, 1.130)
(Here the order of components already corresponds to the original enumeration of modules.)
The y-coordinates of the desired grid placement can be computed in a completely analogous manner. For our example we have $\sigma_1' = \sigma_2' = 1$, $\sigma_3' = 1/3$, $\sigma_4' = 0$. The solution of the eigenvalue problem (5.11.) then yields $\lambda_2' = 0.860$ with
y = (1.247, 1.225, 1.241, 1.247, 0.500, 0.698, 0.644, 0.552,
0.205,-0.865,-0.752,-0.407,-1.235,-1.661,-1.508,-1.067).
The following figure shows the graph space together with the final solution derived from the application of a linear assignment procedure to the "free points".





We add some remarks concerning the placement procedure described in this chapter.

First notice that certain problems may arise using the procedure for special instances. As an example, assume $\sigma_i = \sigma_i'$ for $i = 1,\ldots,q$. Then both eigenvalue problems (5.11.) derived from Σ and Σ', respectively, would coinside. Hence all modules would be located on a straight line. This is also the case, if $1 - \sigma_i = \sigma_i'$ for $i=1,\ldots,q$. Moreover, if there exists an automorphism α of the hypergraph such that

$\{\alpha(1),\ldots,\alpha(q)\} = \{1,\ldots,q\}$ and

$\sigma_i = \sigma_{\alpha(i)}'$ $(i=1,\ldots,q)$ or $1 - \sigma_i = \sigma_{\alpha(i)}'$ $(i=1,\ldots,q)$,

then all free modules would be located on a straight line. Notice that we have indicated only sufficient conditions yielding pathological solutions. For practical applications we recommend the following procedure. First recognize by considering the graph space, whether the free modules possess a good distribution. This should be done by determining the correlation coefficient ρ_0 of the free points in the graph space. In the case that $\rho_0 \notin [-1+\varepsilon, 1-\varepsilon]$ we would recommend to take the second-largest nontrivial solution of (5.11.) (instead of the largest one) to be the locations in one of the dimensions. If then the distribution of the graph space of the free modules is not improved, then take the third-largest solution etc.

The second remark concerns the numerical solution of (5.11.) and constitutes a relation to the averaging model discussed in § 3. This remark is directed to the application of sparse-matrix techniques.
Remember that we have the problem

$$B \cdot u = \lambda \cdot u \quad , \quad \text{where}$$

$$B = (\Phi^T \cdot R \cdot \Phi)^{-1} \cdot (\Phi^T \cdot A \cdot C^{-1} \cdot A^T \cdot \Phi) \; .$$

Define

$$N_1 = (\Phi^T \cdot R \cdot \Phi)^{-1} \cdot \Phi^T \cdot A$$

$$N_2 = \qquad C^{-1} \quad \cdot (\Phi^T \cdot A)^T \; .$$

Then we can write

$$B = N_1 \cdot N_2$$

Consider the special shape of these matrices:

$$\Phi = \begin{pmatrix} \Sigma & 0 \\ 0 & \mathfrak{E} \end{pmatrix} \quad , \quad R = \begin{pmatrix} R_0 & \\ & R_1 \end{pmatrix} \begin{matrix} \} q \\ \} m-q \end{matrix}$$

$$A = \begin{pmatrix} A_0 \\ A_1 \end{pmatrix} \begin{matrix} \} q \\ \} m-q \end{matrix} \; .$$

Then we have

$$\Phi^T \cdot A = \begin{pmatrix} \Sigma^T & 0 \\ 0 & \mathfrak{k} \end{pmatrix} \cdot \begin{pmatrix} A_0 \\ \cdots \\ A_1 \end{pmatrix} = \begin{pmatrix} \Sigma^T \cdot A_0 \\ \cdots \\ A_1 \end{pmatrix} \tag{5.13.}$$

and

$$(\Phi^T \cdot R \cdot \Phi)^{-1} = \left[\begin{pmatrix} \Sigma^T & 0 \\ 0 & \mathfrak{k} \end{pmatrix} \cdot \begin{pmatrix} R_0 & 0 \\ 0 & R_1 \end{pmatrix} \cdot \begin{pmatrix} \Sigma & 0 \\ 0 & \mathfrak{k} \end{pmatrix} \right]^{-1} = \tag{5.14.}$$

$$= \begin{pmatrix} \Sigma^T \cdot R_0 \cdot \Sigma & 0 \\ \cdots & \cdots \\ 0 & R_1 \end{pmatrix}^{-1} = \begin{pmatrix} \Sigma^T \cdot R_0 \cdot \Sigma)^{-1} & 0 \\ \cdots & \cdots \\ 0 & R_1^{-1} \end{pmatrix}.$$

Hence we obtain

$$N_1 = \begin{pmatrix} (\Sigma^T \cdot R_0 \cdot \Sigma)^{-1} \cdot (\Sigma^T \cdot A_0) & 0 \\ \cdots & \cdots \\ 0 & R_1^{-1} \cdot A_1 \end{pmatrix} \tag{5.15.}$$

$$N_2 = C^{-1} \cdot ((\Sigma^T \cdot A_0)^T \vdots A_1^T) = (C^{-1} \cdot (\Sigma^T \cdot A_0)^T \vdots C^{-1} \cdot A_1^T).$$

Considering (5.15.) we see that there is an evident similarity to the formulas (3.3.). Notice that the approach presented here can also be derived using the averaging model, modified by additional conditions. Then in the alternating averaging process because of the constraints arises the problem of approximating an overdetermined systems of linear equations. Using the least squares method the eigenvalue problem (5.11.) can be derived (see /NeGoe/).

Note that the matrices N_1 and N_2 can be determined more concretely when separating between the cases

$$\Sigma = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 1 & r_1 \\ \vdots & \vdots \\ 1 & r_q \end{pmatrix}$$

The derivation is omitted here. From the above consideration follows: In order to solve the eigenvalue problem with the help of the power method, it is necessary to store the matrices

$$\begin{pmatrix} \Sigma^T \cdot A_0 \\ \cdots\cdots \\ A_1 \end{pmatrix} \quad , \quad C, R_1 \quad \text{and} \quad (\Sigma^T \cdot R_0 \cdot \Sigma)^{-1} \; .$$

Here it may be efficient to apply sparse-matrix techniques for the matrix A_1 . Finally remark that for both the x-axis solution and the y-axis solution of the placement problem the majority of the necessary information is identical, in particular the matrices A_1 , C and R_1 . For a simultaneous solution of the eigenvalue problems this could be used in order to improve the efficiency of the whole procedure.

Acknowledgements

We are indebted to Dr. L. Telegdi and Dr. M. Simonowics (Budapest) for useful discussions contributing to this subject.

References

/Be/ Berge, C.: Graphs and hypergraphs, Amsterdam, 1973

/BuDe/ Burkard, R.E., U. Derigs: Assignment and Matching Problems: Solution Methods with FORTRAN programs, Lecture Notes in Economics and Mathem. Systems, vol. 184 (1980), Springer-Verlag, Berlin

/CMST/ Chung, M.J., F. Makedon, I.H. Sudborough, J. Turner: Polynomial time algorithms for the min cut problem on degree restricted trees, SIAM J. Comp., 14 No1 (1985)

/Co/ Collatz, L.: Eigenwertaufgaben. Mit technischen Anwendungen, Leipzig, 1949

/Fuku1/ Fukunaga, K., S. Yamada, H. Stone, T. Kasai: Placement of Circuit Modules Using a Graph Space Approach, IEEE Proc. 20th Design Automation Conference (1983)

/Fuku2/ Fukunaga, K., S. Yamada, H. Stone, T. Kasai: A Representation of Hypergraphs in the Euclidean Space, IEEE Transactions on Computers, vol. C-33 No4 (1984)

/Ga/ Gantmacher, F.R.: Matrizenrechnung I, Berlin, 1958

/GaJo/ Garey, M.R., D.S. Johnson: Computers and Intractability, San Francisco, 1979

/GoeNe/ Goetze, B., W. Nehrlich: Placement of Electronic Circuits under Module Constraints, Proc. 7-th European Conf. on Circuit Theory and Design, Prague 1985, 33-36

/Jo/ Johnson, D.S.: The NP-Completeness Column: An Ongoing Guide, Journal of Algorithms 3 (1982)

/KeSt/ Kendall, M., Stuart, A.: The advanced theory of statistics 2, London, 1961

/Ma Kc Bi/ Mardia, K.V., Kent, J.T., Bibby, J.M.: Multivariate Analysis, London, 1979

/May/ May, M.: Zwei Graphenalgorithmen und ihre Anwendung in der Strukturoptimierung und in der Theorie linearer Gleichungssysteme. ZKI-Inf. (1982) 4, 119-136

/Ma Me/ May,M., P. Mennecke: Layout of Schematic Drawings. Syst. Anal. Model. Simul. 1 (1984) 4, 307-338

/Ne Goe/ Nehrlich, W., B. Goetze: Äquivalenz verschiedener Ansätze zur nichtdiskreten Bearbeitung schwerer diskreter Optimierungsaufgaben, unpublished manuscript, 1985

/Otten1/ Otten, R.H.J.M.: Eigensolutions in top-down layout design, Proc. International Symposium on Circuits and Systems ISCAS (1982)

/Otten2/ Otten, R.H.J.M.: Automatic Floorplan Design, IEEE Proc. 19th Design Automation Conference (1982)

/Qui Br/ Quinn, N., M.A. Breuer: A Force Directed Component Placement Procedure for Printed Circuit Boards, IEEE Transactions on Circuits and Systems, vol. CAS-26 (1979)

/Rao/ Rao, C.R.: Linear statistical inference and its applications, New York, 1965

Scaling of random variables and arrangement problems in layout design

B. Goetze and W. Nehrlich

Summary

An optimization model from mathematical statistics - scaling of random variables by optimization of correlation coefficients - is applied to several arrangement problems for graphs and hypergraphs. The two-dimensional embedding of hypergraphs is in the centre of our attention because of its importance for the topological design of electronic circuits. For this case, the optimization model is generalized for an important case of additional constraints. The mathematical derivation of this non-discrete optimization strategy is presented in detail. Moreover, relations to similar non-discrete methods as well as some computational aspects of this approach are discussed.

Valószinüségi változók skálázása és elrendezési problémák az ültetési tervben

B. Goetze, W. Nehrlich

Összefoglaló

A szerzők a valószinüségi változók skálázására a matematikai statisztikában használt optimalizációs modellt /korrelációs együtthatók optimalizálása/ alkalmazzák gráfok és hipergráfok elrendezési problémáira. Figyelmüket leginkább a hipergráfok két-dimenziós elhelyezésére összpontositják, mert ez egy fontos feladat az elektromos áramkörök "topológiai" tervezésénél. Ebben az esetben az optimalizációs modellt ugy általánositják, hogy az korlátozó mellékfeltételek jelenléte esetében is müködjön. Az igy felmerülő nem-diszkrét optimalizálási stratégiát a szerzők részletesen is ismertetik. Ezen felül a módszert más nem-diszkrét módszerekkel is összevetik és a módszer számitástechnikai aspektusait is tárgyalják.