

THERMODYNAMICS AND FLOW-FRAMES FOR DISSIPATIVE RELATIVISTIC FLUIDS

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ABSTRACT. A general thermodynamic treatment of dissipative relativistic fluids is introduced, where the temperature four vector is not parallel to the velocity field of the fluid. Generic stability and kinetic equilibrium points out a particular thermodynamics, where the temperature vector is parallel to the enthalpy flow vector and the choice of the flow fixes the constitutive functions for viscous stress and heat. The linear stability of the homogeneous equilibrium is proved in a mixed particle-energy flow-frame.

1. INTRODUCTION

It is commonly accepted that in relativistic fluids there is a freedom in the definition of the velocity field u^a , in the sense that one may fix it to different physical four vectors. The well known common choices are when the flow is defined by the particles (this is the Eckart flow-frame [1]), or by the energy (this is the Landau-Lifshitz flow-frame [2]). Therefore the velocity field of a one component fluid is a somewhat vaguely defined physical property, belonging to the flow of volatile quantities, once the energy, once the conserved charge. That attitude assumes that the physics is invariant when changing the flow-frame.

Other evidently flow-frame dependent properties of a relativistic dissipative fluid theory are related to the thermodynamic framework. Gibbs relation is classically given in the following form [1, 3]:

$$ds = \beta de - \alpha dn, \quad (1)$$

where $s = u_a S^a$ is the entropy density, $e = u_a u_b T^{ab}$ is the energy density, $n = u_a N^a$ is the particle number density, β is the reciprocal temperature and α is the chemical potential divided by the temperature. This non-relativistically motivated formula is defined by the parts of the covariant S^a and N^a entropy and particle number density vectors and the T^{ab} energy-momentum tensor, that are parallel to the u^a velocity field of the fluid. It is not evident that this particular form, or any generalization of that is invariant when changing the flow-frame.

The flow-frame dependence of the different relativistic fluid theories is more apparent when focusing on generic stability, on one of the fundamental problems in relativistic fluids. Thermodynamics is related to stability [4]. One expects in any reasonable theory of continua that the homogeneous equilibrium of an insulated system is asymptotically stable and the conditions of this stability are ensured by thermodynamics. In particular for one component fluids the concave entropy – thermodynamic stability – and nonnegative entropy production – nonnegative transport coefficients – should be the only conditions. A weaker requirement is the linear

stability of homogeneous equilibrium, that we call generic stability. However, the first order Eckart theory is violently unstable in the Eckart flow-frame and stable in the Landau-Lifshitz one. Therefore, according to the usual explanations, we must consider a higher order theory. For example the Israel-Stewart theory is believed to be a stable one.

However, the generic stability is proved only in the Eckart flow-frame and the conditions given by Hiscock and Lindblom are very complicated [5, 6] and lacking a simple physical interpretation. Other higher order theories and other flow-frames are not analysed from this point of view.

Our final example of flow-frame dependence comes from kinetic theory. There one assumes a temperature four vector, which is parallel to the flow. Then an ideal fluid is defined by the following form of particle number density and energy-momentum:

$$N_0^a = nu^a, \quad \text{and} \quad T_0^{ab} = eu^a u^b + p\Delta^{ab}. \quad (2)$$

here p is the static pressure. However, a simple flow-frame transformation leads to

$$N_0^a = n_0 u^a + j_0^a, \quad \text{and} \quad T_0^{ab} = e_0 u^a u^b + q_0^a u^b + u^a q_0^b + P_0^{ab}. \quad (3)$$

Here $j_0^a = n_0 w^a$, $q_0^a = h_0 w^a$ and $P_0^{ab} = p\Delta^{ab} + hw^a w^b$ and w^a is related to the u-orthogonal part of the temperature vector:

$$\beta^a = \frac{u^a + w^a}{T}. \quad (4)$$

Therefore the above formal concept of ideal fluid is flow-frame dependent.

According to our investigations the generalization of thermodynamics, where the temperature vector is not parallel to the four velocity of the fluid improves the stability properties [7, 8, 9, 10]. The analysis of the problematics of flow-frames resulted in the following form of the entropy production [11]:

$$\begin{aligned} \Sigma &= \partial_a S^a = (nw^a - j^a)\partial_a \alpha + (q^a - hw^a)(\partial_a \beta + \beta \dot{u}_a) \\ &+ (\Pi^{ab} - q^{(a} w^{b)})\partial_a \beta_b + q^{[b} w^{a]}\partial_a (\beta(u_b - w_b)) \geq 0. \end{aligned} \quad (5)$$

Here the background Gibbs relation was assumed in a form that corresponds to the general form of the temperature (4):

$$ds = \beta de + \beta w_a dq^a - \alpha dn. \quad (6)$$

We may have an important observation here. The classical assumption $w^a = 0$, which may be considered as an equation of state specifying a flow-frame fixed to the thermometer, leads to the Eckart form of the entropy production. The corresponding fluid theory is unstable [12]. There are two particular choices that lead to generic stable relativistic hydrodynamics. The first one introduces an entropy density that depends on the "length" of the energy-momentum vector: $s = s(E, n)$, where $E = \sqrt{u_b T^{ab} T_{ac} u^c}$. In this case $w^a = q^a/e$ [13]. However, this result is not quite compatible with the concept of kinetic equilibrium. The second one introduces a different thermodynamics, where the Gibbs relation is defined according to kinetic equilibrium: $w^a = q^a/h$, with $h = e + p$ being the enthalpy density [14]. The hydrodynamics based on this suggestion is proved to be generic stable in an Eckart frame [10].

In these mentioned investigations we did not consider the compatibility of thermodynamic requirements and frame fixing. In the present paper we develop a general thermodynamic analysis based on the kinetic compatible Gibbs relation. In

the derived expression of entropy production one can identify two thermodynamic force-current pairs for the three constitutive quantities: the diffusion current density, heat current and viscous stress. In this way a flow-frame fixes the constitutive framework, but does not introduce any restriction on the physics.

In this work we do not investigate the question of causality, which is traditionally considered the most important problem in relativistic context and the main motivation of the development of second order fluid theories [15]. There is also a general belief that causality implies stability [16, 17]. One may have two comments in this respect. First of all the continuum limit reduces the propagation speed of real signals [18, 19, 20] and, on the other hand, several second order theories are not proved to be symmetric hyperbolic or divergence type [21, 22, 23, 24]. An important benchmark is the comparison of the numerical solutions of different fluid theories and the Boltzmann equation in various cases [25, 26].

This paper is organized as follows. In the next section we introduce the kinetic compatible relativistic thermodynamics. In the third section the entropy production is calculated and analysed. In the fourth section the generic stability of the dissipative hydrodynamics is proved in a mixed particle-energy flow-frame.

2. THERMODYNAMICS OF MOTION

In this paper we use the Lorentz form with the sign convention $(+, -, -, -)$, the indexes a, b, c, \dots run over $0, 1, 2, 3$. We use natural units, $c = 1$.

For isotropic media the entropy density S^a is connected to the particle number density N^a and energy-momentum density T^{ab} by the isotropic pressure:

$$p\beta^a = S^a + \alpha N^a - \beta_b T^{ab}. \quad (7)$$

Standard kinetic theory definitions and calculations for ideal substances satisfy the above expressions in thermodynamic equilibrium [27, 14]. Eq. (7) is our starting point in the following investigations, we propose its validity for nonideal substances and also out of hydrodynamic equilibrium. An arbitrary flow, u^a , decomposes the quantities of the above expression as follows:

$$S^a = su^a + J^a, \quad (8)$$

$$N^a = nu^a + j^a, \quad (9)$$

$$T^{ab} = eu^a u^b + q^a u^b + u^a q^b + P^{ab}, \quad (10)$$

$$\beta^a = \beta(u^a + w^a). \quad (11)$$

Here n is the flow-frame particle number density, j^a is the diffusion current, the non-convective particle number current density in this frame, e is the energy density, q^a is the momentum density and the heat current and P^{ab} is the pressure tensor. These components are flow-frame dependent, in particular $j^a u_a = 0$, $q^a u_a = 0$ and $P^{ab} u_b = 0$. Then the u-timelike and u-spacelike parts of (7) deliver

$$s = \beta h + \beta w_b q^b - \alpha n, \quad (12)$$

$$J^a = \beta(q^a + w_b \Pi^{ab} - \mu j^a). \quad (13)$$

Here we have introduced the viscous stress, as $\Pi^{ab} = P^{ab} + \Delta^{ab} p$.

The covariant analysis of kinetic equilibrium requirements resulted in the following Gibbs relation [10]:

$$ds = \beta_a dE^a + \beta p w_a du^a - \alpha dn. \quad (14)$$

Then one can obtain the corresponding Gibb-Duhem relation with the help of (12):

$$\beta dp + h d\beta + q^a d\beta_a - \beta w^a h du_a - n d\alpha = 0. \quad (15)$$

3. FLOW-FRAME INDEPENDENT ENTROPY PRODUCTION

Introducing the substantial time derivative $\frac{d}{d\tau} := u^a \partial_a$, abbreviated by and overdot, the balances for the particle current density and energy-momentum tensor are expressed in terms of the local rest frame quantities:

$$\partial_a N^a = \dot{n} + n \partial_a u^a + \partial_a j^a = 0, \quad (16)$$

$$\begin{aligned} \partial_b T^{ab} &= \dot{e} u^a + e \dot{u}^a + e u^a \partial_b u^b + \dot{q}^a + q^a \partial_b u^b + \\ &u^a \partial_b q^b + q^b \partial_b u^a + \partial_b P^{ab} = 0^a. \end{aligned} \quad (17)$$

The energy and momentum balances are the time and spacelike components of the energy-momentum tensor balance projected onto the flow-frame:

$$u_a \partial_b T^{ab} = \dot{e} + e \partial_b u^b + u_a \dot{q}^a + \partial_b q^b - P^{ab} \partial_b u_a = 0, \quad (18)$$

$$\begin{aligned} \Delta_c^a \partial_b T^{cb} &= e \dot{u}^a + \Delta_b^a \dot{q}^b + q^a \partial_b u^b + q^b \partial_b u^a + \Delta_c^a \partial_b P^{cb} \\ &= h \dot{u}^a + \Delta_b^a \dot{q}^b + q^a \partial_b u^b + q^b \partial_b u^a + \Delta_c^a \partial_b \Pi^{cb} - \Delta^{ab} \partial_b p = 0^a. \end{aligned} \quad (19)$$

The frame related quantities are important in the separation of the ideal and dissipative parts of the basic fields. This separation is best performed by analyzing the thermodynamical relations. In order to achieve this one postulates the existence of an additional vector field, the entropy current as a function of the basic fields $S^a(N^a, T^{ab})$. It must not decrease by fulfilling the condition of the balances (16) and (17). That conditional inequality can be best represented by introducing the Lagrange-Farkas multipliers¹ α and β^a , respectively:

$$\Sigma := \partial_a S^a + \alpha \partial_a N^a - \beta_b \partial_a T^{ba} \geq 0. \quad (20)$$

The left hand side of this inequality shows, that the definition of the entropy production precedes the specification of the flow-frame.

The separation of ideal and dissipative parts of basic physical quantities is a consequence to the choice of that flow-frame. Citing the authors of [31], when arguing about the uniqueness of the Landau-Lifshitz frame “The uniqueness of the energy frame comes from ... the physical assumption that the dissipative effect comes from only the spatial inhomogeneity.”. However, the thermodynamic choice is primary, the Landau-Lifshitz frame is an additional condition which may or may not be compatible with the preferred thermodynamics.

Entropy is not an independent physical quantity: both the entropy density and the current density of the entropy are functions of the primary fields, as it is reflected by the Gibbs relation (14) and by (13). Moreover, the entropy inequality is conditional, the conservation of particle number, energy and momentum being the constraints. The best way to introduce these restrictions is by substituting the

¹Lagrange multipliers are introduced for conditional extrema. For conditional inequalities Gyula Farkas suggested analogous quantities and proved the corresponding theorem of linear algebra, called Farkas’ lemma [28, 29, 30].

corresponding proper time derivatives:

$$\begin{aligned}
& \dot{s} + s\partial_a u^a + \partial_a J^a = \\
& \beta\dot{e} - \alpha\dot{n} + \beta w^a \dot{q}^a + \beta(hw^a - q^a)\dot{u}^a + s\partial_a u^a + \partial_a [\beta q^a + \beta w_b \Pi^{ab} - \alpha j^a] = \\
& (s - \beta h - \alpha n - \beta w_a q^a) \partial_b u^b + \beta w_a q^a \partial_b u^b + \Pi^{ab} (\partial_b (\beta u_a) + \partial_a (\beta w_b)) - \\
& j^a \partial_a \alpha + q^a \partial_a \beta + \beta w^a (h\dot{u}^a + \dot{q}_a + q^a \partial_b u^b + \partial_b \Pi_a^b) \\
& = \Pi^{ab} \partial_b \beta_a - j^a \partial_a \alpha + q^a \partial_a \beta + \beta w^a \left(\frac{\beta}{h} (h\dot{u}_a + \dot{q}_a + q^a \partial_b u^b + \partial_b \Pi_a^b) \right) \geq 0
\end{aligned} \tag{21}$$

A further transformation of the final expression with the help of the momentum balance and also by Gibbs-Duhem relation (15) results in a remarkable form:

$$\Sigma = (\Pi^{ab} - w^a q^b) \partial_b \beta_a + (q^a - hw^a) (\partial_a \beta - \beta w^b \partial_a u_b) + (nw^a - j^a) \partial_a \alpha \geq 0 \tag{22}$$

From this expression it is apparent that the entropy production is zero, whenever the fluid is ideal in the sense of (3), suggested by the kinetic theory. The other observation is that seen from the thermometer frame $w^a = 0$ and the entropy production reduces to

$$\Sigma = \Pi^{ab} \partial_b \beta_a + q^a \partial_a \beta - j^a \partial_a \alpha \geq 0. \tag{23}$$

This form of the entropy production does not contain the disturbing acceleration term in the thermal thermodynamic force. It was suggested to apply by a different argumentation in [32, 33]. However, the generic stability of this fluid is conditional, e.g. $\frac{\partial \beta}{\partial n}|_e > 0$ is a requirement.

The condition of kinetic equilibrium, $q^a = hw^a$, leads to another simplified expression:

$$\Sigma = \left(\Pi^{ab} - \frac{q^a q^b}{h} \right) \partial_b \beta_a + \left(\frac{n}{h} q^a - j^a \right) \partial_a \alpha \geq 0. \tag{24}$$

In this case only two additive terms occur in the entropy production formula above eq.(24), but for three undetermined constitutive functions j^a , q^a and Π^{ab} . Therefore a classical flow-frame fixing, like the Eckart or the Landau-Lifshitz one, closes the system of equations instead of restricting or contradicting to thermodynamical requirements. In the followings we prove that a rather general choice leads to a generic stable relativistic fluid theory.

4. CONSTITUTIVE THEORY AND GENERIC STABILITY IN A MIXED PARTICLE-ENERGY FLOW-FRAME

The fluid velocity in the Eckart flow-frame is given by the direction of the particle number density vector: $n^a = N^a/N$, in the Landau-Lifshitz flow-frame by the direction of the energy-momentum vector: $e^a = E^a/E$. Here $N = \sqrt{N^a N_a}$ and $E = \sqrt{E^a E_a}$ are the magnitudes of the particle number and the energy-momentum density vectors, respectively. Then we introduce a mixed flow, where the coefficients A and B characterize the weights of the Eckart and Landau-Lifshitz flows:

$$u^a = \frac{Ae^a + Bn^a}{O}, \quad \text{where} \quad O = \sqrt{A^2 + B^2 + 2AB\chi}. \tag{25}$$

Here $\chi = e^a n_a$. The energy and particle number densities in this flow are

$$e = u_a E^a = \frac{E}{O}(A + B\chi) \quad \text{and} \quad n = u_a N^a = \frac{N}{O}(B + A\chi). \tag{26}$$

Moreover, the momentum density and the diffusion current are expressed as

$$q^a = \frac{EB}{O^2}((B + A\chi)e^a - (A + B\chi)n^a), \quad (27)$$

$$j^a = -\frac{NA}{O^2}((B + A\chi)e^a - (A + B\chi)n^a). \quad (28)$$

By this, these current densities are parallel with a proportionality factor z :

$$j^a = -\frac{NA}{EB}q^a = -zq^a. \quad (29)$$

The entropy production (24) in fact relies on only two constitutive quantities, Π^{ab} and q^a :

$$\Sigma = \left(\Pi^{ab} - \frac{q^a q^b}{h} \right) \partial_b \beta_a + \left(\frac{n}{h} + z \right) q^a \partial_a \alpha \geq 0 \quad (30)$$

We can identify the corresponding thermodynamic forces and currents as follows:

	Diffusive-Thermal	Viscous
Fluxes	$\left(\frac{n}{h} + z \right) q^a$	$\Pi^{ab} - \frac{q^a q^b}{h}$
Forces	$\nabla^a \alpha$	$\nabla^{(b} \beta^{a)}$

Table 1. Thermodynamic fluxes and forces in the particle-energy flow-frame.

Here $\nabla^a = \Delta^{ab} \partial_b$ is the u-space derivative. The corresponding linear response relations for isotropic continua are:

$$\left(\frac{n}{h} + z \right) q^a = \lambda \nabla^a \alpha, \quad (31)$$

$$\Pi^{ab} - \frac{q^a q^b}{h} = \zeta \Delta^{ab} \partial^c \beta_c + 2\eta \Delta^{(bc} \nabla^{a)} \beta_c. \quad (32)$$

Here $\langle \dots \rangle$ denotes the symmetric traceless part in the bracketed indices, λ is the heat conduction coefficient, ζ is the bulk viscosity and η is the shear viscosity. Because of the nonnegative entropy production $\lambda \geq 0$, $\zeta \geq 0$, $\eta \geq 0$, reflecting very natural conditions.

Finally the linear stability of the classical homogeneous equilibrium of equations (16), (18), (19) with the constitutive equations (31)-(32) has to be discussed. The entropy production formula (30) indicates that the equilibrium is characterized by zero viscous stress and heat current (momentum density). The linearization of the above equations results in:

$$0 = \dot{\delta n} + n \partial_a \delta u^a - z \partial_a \delta q^a, \quad (33)$$

$$0 = \dot{\delta e} + h \partial_a \delta u^a + \partial_a \delta q^a, \quad (34)$$

$$0 = h \delta \dot{u}^a + \Delta^{ab} \partial_b \delta p + \delta \dot{q}^a + \Delta_c^a \partial_b \delta \Pi^{cb}, \quad (35)$$

$$0 = \left(\frac{n}{h} + z \right) \delta q^a - \lambda \Delta^{ab} \partial_b \delta \alpha, \quad (36)$$

$$0 = \delta \Pi^{ab} - \zeta \partial_c \delta \beta^c \Delta^{ab} - \eta \Delta^{ac} \Delta^{bd} (\partial_c \delta \beta_d + \partial_d \delta \beta_c - \frac{2}{3} \partial_f \delta \beta^f \Delta_{cd}). \quad (37)$$

One inspects solutions in the form $\delta Q = Q_0 e^{\Gamma t + i k x}$ with a general complex Γ . Then the condition of asymptotic stability of the above equations is that the solutions of the characteristic equation $\det \mathbf{M} = (\det \mathbf{N})(\det \mathbf{R})^2 = 0$ give $\text{Re } \Gamma < 0$.

Here

$$\mathbf{R} = \begin{pmatrix} h\Gamma & \Gamma & ik & 0 \\ 0 & 1 & 0 & 0 \\ ik\tilde{\eta} & ik\frac{\tilde{\eta}}{h} & 1 & 0 \\ ik\tilde{\zeta} & ik\frac{\tilde{\zeta}}{h} & 0 & 1 \end{pmatrix}, \quad (38)$$

couples the perturbation fields $\delta u^a, \delta q^a, \delta \Pi^{xa}, \delta \Pi^{aa}$, $a = y, z$ and

$$\mathbf{N} = \begin{pmatrix} \Gamma & 0 & ikn & -ikz & 0 \\ 0 & \Gamma & ikh & ik & 0 \\ ik\partial_n p & ik\partial_e p & \Gamma h & \Gamma & ik \\ -ik\tilde{\lambda}\partial_n \alpha & -ik\tilde{\lambda}\partial_e \alpha & 0 & 1 & 0 \\ 0 & 0 & ik\tilde{\eta} & ik\tilde{\eta}/h & 1 \end{pmatrix}. \quad (39)$$

couples $\delta n, \delta e, \delta u^x, \delta q^x, \delta \Pi^{xx}$. Here $h = e + p$, $\tilde{\eta} = \beta(\zeta + \frac{4}{3}\eta)$, $\tilde{\zeta} = \beta(\zeta - \frac{2}{3}\eta)$ and $\tilde{\lambda} = \lambda(z + n/h)^{-1}$. $\det \mathbf{R} = 0$ gives the condition

$$h\Gamma + \tilde{\eta}k^2 = 0, \quad (40)$$

which results in negative real Γ . $\det \mathbf{N} = 0$, on the other hand, results in a more complicated polynomial

$$\Gamma^3 h + \Gamma^2 k^2 (\tilde{\eta} + \lambda h \partial_n \alpha) + \Gamma k^2 (h \partial_e p + n \partial_n p + k^2 \tilde{\eta} \lambda \partial_n \alpha) + k^4 \lambda h (\partial_e p \partial_n \alpha - \partial_n p \partial_e \alpha) = 0.$$

According to Routh-Hurwitz criteria the real parts of the roots of the polynomial equation $a_0 \Gamma^3 + a_1 \Gamma^2 + a_2 \Gamma + a_3 = 0$ are always negative if the coefficients are all non-negative and $a_1 a_2 - a_0 a_3 > 0$ [34]. It is straightforward to check that these inequalities are fulfilled, if the transport coefficients $\lambda > 0, \eta > 0, \zeta > 0$ and the inequalities of the thermodynamic stability are satisfied,

$$\partial_e \beta < 0, \quad \partial_n \alpha > 0, \quad \partial_e \beta \partial_n \alpha + (\partial_n \beta)^2 \leq 0. \quad (41)$$

or equivalently $\partial_e^2 s < 0$, $\partial_n^2 s < 0$ and $(\partial_e \partial_n s)^2 \leq \partial_e^2 s \cdot \partial_n^2 s$.

5. DISCUSSION

Flow-frames are related to thermodynamics and thermodynamics is related to flow-frames. This is inevitable in a relativistic spacetime, where motion separates momentum and energy. Here we have analysed the relation of flow-frames and thermodynamics in case of relativistic fluids, researching the consequences of a motion related thermodynamic framework, where the velocity of the thermometer is not parallel to the flow. In this case also the momentum becomes a thermodynamic variable and w^a appears as a conjugated intensive variable. It constitutes the flow-orthogonal part of the temperature vector: $w^a = T \Delta^{ab} \beta_b$.

In this case

- the conventional thermodynamic forces and currents are interdependent, viscosity becomes entangled with heat conduction and diffusion.
- Kinetic equilibrium is compatible with a preferred frame, where $w^a = q^a/h$.
- A flow-frame fixing selects the proper thermodynamic forces and currents; a dissipative hydrodynamics can be fully determined.
- This dissipative hydrodynamics is generic stable in a mixed particle-energy flow-frame, therefore also in the particular cases of Eckart ($z = 0$) and Landau-Lifshitz ($z = \infty$).

All previous investigations introduced a flow that was fixed to the thermometer, or, equivalently, a thermodynamics, where the energy density was an extensive variable, but the momentum density was not. It is no wonder, that the Landau-Lifshitz flow-frame appeared to be distinguished for a deeper phenomenological analysis [19, 35] and also for the kinetic theory.

Stability calculations indicate that w^a cannot be arbitrary. In particular the usual thermometer related choice, $w^a = 0$, leads to violently unstable theories [5], when the additionally introduced flow-frame is incompatible, say the Eckart one.

The general form of the entropy production (22) shows that the choice of a flow-frame is not merely a possibility, but it is a necessity for fixing the constitutive relations with all its implications to the described physical system.

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