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PARAMETER SENSITIVITY ANALYSIS OF AN INDUCTION MOTOR

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A simple dynamic model of an induction motor is presented in this paper based on engineering principles that describe the mechanical phenomena together with the electrical model. The investigated state space model consists of nonlinear state equations and linear output equations. The model has been verified under the usual controlled operating conditions when the speed is controlled. The effect of load on the controlled induction motor has been analyzed by simulation. The sensitivity analysis of the induction motor and the bridge of the inverter have been applied to determine the model parameters to be estimated.

Keywords: induction machine, dynamic state space model, parameter sensitivity analysis

Introduction

The induction motors are the most commonly used electrical rotating machines in several industrial applications including the automotive industry, too.

In the modern adjustable speed induction motor drives inverters are used to drive the three-phase motor as variable frequency voltage or current sources.

Whatever the size and the application area, these motors share the most important dynamic properties, and their dynamic models have a similar structure.

Therefore the final aim of our study is to design a controller that can control the speed and the torque of the induction motor.

Because of the specialties and great practical importance of the induction motor in industrial applications, their modelling for control purposes is well investigated in the literature. Besides of the basic textbooks (see e.g. [1-3]), there are several papers that describe the modelling and use the developed models for the design of different types of controllers: vector control [1] and [4], sensor less vector control [5] and direct torque control (DTC) [6].

The aim of this paper is to build a simple dynamical model of the induction motor together with the three-phase inverter and analyze the models sensitivity of its parameters. The result of this analysis will be the basis of a subsequent parameter estimation step. The state space model has been implemented in Matlab/Simulink environment which enables us to analyze the parametric sensitivity based on simulation experiments.

The model of the induction motor

In this section the state- space model for an induction motor is developed.

Modelling assumptions

For constructing the induction motor model the following assumptions are made:

- symmetrical three phase windings,
- the slotting effect and the copper losses are neglected,
- the permeability of the iron parts is assumed to be infinite with linear magnetic properties,
- flux density is radial in the air gap,
- the spatial distribution of fluxes and apertures wave are considered to be sinusoidal,
- the spatial distribution of the stator fluxes and apertures wave are considered to be sinusoidal.

According to the above modeling conditions the mathematical description of the induction motor is developed through the space vector theory. If the voltage of the stator is presumed to be the input excitation of the machine, then the spatial distribution along the stator of the *x* phase voltage can be described by the complex vector $U_{sx}(t)$. We can determine the orientation of the voltage vector U_s the direction of the respective phase axis and the voltage polarity.

$$i_s(t) = \frac{2}{3}\left(a^0 \cdot i_{sa}(t) + a \cdot i_{sb}(t) +\_a^2 i_{sc}(t)\right) = \\ = \sqrt{2} \cdot i_{eff}(t) \cdot e^{j\,\omega_0 t + \frac{\pi}{2} + \varphi_i} \quad (2.1)$$

$$i_{sa}(t) = Re\{i_s(t)\} \quad (2.2)$$

$$i_{sb}(t) = Re\{a^2 \cdot i_s(t)\} \quad (2.3)$$

$$i_{sc}(t) = Re\{a \cdot i_s(t)\}, \quad (2.4)$$

where $a = e^{j120°}$

In equation (2.1) *2/3* is a normalizing factor. The flux density distribution can be obtained by integrating the current density wave along the cylinder of the stator. The flux linkage wave as a system variable, because it contains detailed information about the winding geometry. The rotating flux density wave induces voltages in the individual stator windings. Thus stator voltage $U_s(t)$ can be represented in the overall distributed voltages in all phase windings:

$$u_s(t) = \frac{2}{3}\left(a^0 \cdot u_{sa}(t) + a \cdot u_{sb}(t) a^2 \cdot u_{sc}(t)\right) =$$

$$= \sqrt{2} \cdot u_{eff}(t) \cdot e^{j\omega_0 t + \frac{\pi}{2} + \varphi_u} \quad (2.5)$$

$$u_{sa}(t) = Re\{u_s(t)\} \quad (2.6)$$

$$u_{sb}(t) = Re\{a^2 \cdot u_s(t)\} \quad (2.7)$$

$$u_{sc}(t) = Re\{a \cdot u_s(t)\} \quad (2.8)$$

Considering the stator of the induction machine as the primer side of the transformer, then using the Kirchoff's voltage law the following equation can be written:

$$u_s(t) = i_s(t) \cdot R_s + \frac{d\Psi_s(t)}{dt} \quad (2.9)$$

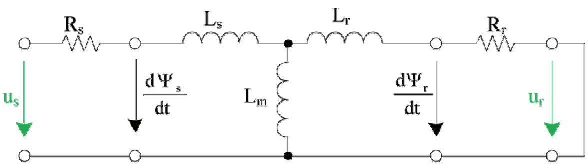

Figure 1: The equivalent circuit of the induction motor

As for the secondary side of the transformer, it can be deduced that the same relationship is true for the rotor side space vectors:

$$u_r(t) = i_r(t) \cdot R_r + \frac{d\Psi_r(t)}{dt} = 0 \quad (2.10)$$

Equations (2.9) and (2.10) describe the electromagnetic interaction as the connection of first order dynamical subsystems. Since four complex variables ($i_s(t)$, $i_r(t)$, $\Psi_s(t)$, $\Psi_r(t)$) are presented in these two equations, (2.1) and (2.5) flux equations are needed to complete the relationship between them.

$$\Psi_s(t) = i_s(t) \cdot L_s + i_r(t) \cdot L_m \cdot e^{j\rho(t)} \quad (2.11)$$

$$\Psi_r(t) = i_s(t) \cdot L_m \cdot e^{-j\rho(t)} + i_r(t) \cdot L_r \quad (2.12)$$

where angle $\rho(t)$ defines the position of the rotor compared to the axis of the stator, while

$$L_s = {}^3/_2\, l_s + {}^3/_2\, l_f \text{ and } L_r = {}^3/_2\, l_r + {}^3/_2\, l_f$$

are the three-phase inductances and l_s, l_r are the inductances of a stator and a rotor phase winding, $L_m = 3/2 * l_m$ is the mutual inductance between the stator and the rotor. By applying the following substitutions:

$$i_r(t) = i_r(t) \cdot e^{j\rho(t)} \quad (2.13)$$

$$\Psi_r(t) = \Psi_r(t) \cdot e^{j\rho(t)} \quad (2.14)$$

then the following equations are obtained with the flux connections in the model:

$$\Psi_s(t) = i_s(t) \cdot L_s + i_r(t) \cdot L_m \quad (2.15)$$

$$\Psi_r(t) = i_s(t) \cdot L_m + i_r(t) \cdot L_r \quad (2.16)$$

The mechanical energy $P_{mech}(t)$ of the system can be defined as:

$$P_{mech}(t) = \frac{dW_{mech}(t)}{dt} \quad (2.17)$$

where the mechanical energy $W_{mech}(t)$ in case of rotating motor can be given by:

$$\frac{dW_{mech}(t)}{dt} = T_e(t) \cdot \omega(t) \quad (2.18)$$

On the other hand, there is another expression for the mechanical energy:

$$W_e(t) = W_{mech}(t) + W_v(t) + W_{field}(t) \quad (2.19)$$

where

$$\frac{dW_e(t)}{dt} = \frac{3}{2} Re\{u_s \cdot i_s + u_r \cdot i_r\}$$

is the input electric power,

$$\frac{dW_v(t)}{dt} = \frac{3}{2} Re\{R_s \cdot |i_s|^2 + R_r \cdot |i_r|^2\}$$

is the resistive power loss, and

$$\frac{dW_{field}(t)}{dt} = \frac{3}{2} Re\left\{\frac{d\Psi_s}{dt} i_s + \frac{d\Psi_r}{dt} i_r\right\}$$

is the air gap power.

Using the above equations it can be concluded that:

$$P_{mech} = T_e \cdot \omega = \frac{3}{2}\frac{L_m}{L_r} \cdot \omega \Psi_r(t) \times i_s(t) \quad (2.20)$$

The transformer can be decomposed into d-axis and q-axis. Park's transformation converts the equations to a simplified and more tractable form.

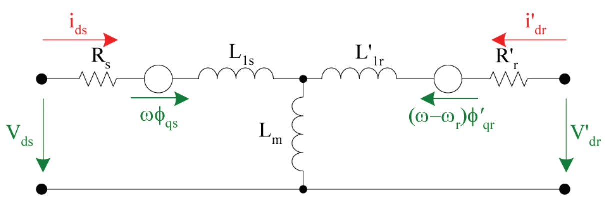

Figure 2: The equivalent circuit of the d axis of the induction motor

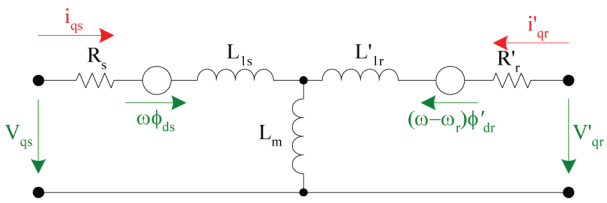

Figure 3: The equivalent circuit of the q axis of the induction motor

The actual terminal voltage *v* of the windings can be written in the following form

$$v = \pm\sum_{j=1}^{J}(R_j \cdot i_j) \pm \sum_{j=1}^{J}(\Psi_j) \quad (2.21)$$

where i_j are the currents, R_j are the winding resistances, and Ψ_j are the flux linkages. Assume, that the positive directions of the stator currents point out of the induction motor terminals.

By considering the d-axis and the q-axis of the induction motor, the following equations can be written:

$$v_{qs} = R_s i_{qs} + \frac{d\Psi_{qs}}{dt} + \omega\Psi_{ds} \quad (2.22)$$

$$v_{ds} = R_s i_{ds} + \frac{d\Psi_{ds}}{dt} - \omega\Psi_{qs} \quad (2.23)$$

$$v'_{qr} = R'_r i'_{qr} + \frac{d\Psi'_{dr}}{dt} + (\omega - \omega_r)\Psi'_{dr} \quad (2.24)$$

$$v'_{dr} = R'_r i'_{dr} + \frac{d\Psi'_{dr}}{dt} - (\omega - \omega_r)\Psi'_{qr} \quad (2.25)$$

$$T_e = 1.5p\left(\Psi_{ds} i_{qs} - d\Psi_{qs} i_{ds}\right), \quad (2.26)$$

where ω is the reference frame angular velocity and ω_r is the electrical angular velocity.

$$\Psi_{qs} = L_s i_{qs} + L_m i'_{qr} \quad (2.27)$$

$$\Psi_{ds} = L_s i_{ds} + L_m i'_{dr} \quad (2.28)$$

$$\Psi'_{qr} = L'_r i'_{qr} + L_m i_{qs} \quad (2.28)$$

$$\Psi'_{dr} = L'_r i'_{dr} + L_m i_{ds} \quad (2.29)$$

The above model can be summarized in a state-space model by expressing the fluxes from the voltage equations.

$$\frac{d}{dt}\begin{bmatrix}\Psi_{qs}\\ \Psi_{ds}\\ \Psi_{qr}'\\ \Psi_{dr}'\end{bmatrix} = \begin{bmatrix} \frac{-R_s}{L_s - \frac{L_m^2}{L_r'}} & -\omega & \frac{R_s \cdot L_m}{L_r'\left(L_s - \frac{L_m^2}{L_r'}\right)} & 0 \\ \omega & \frac{-R_s}{L_s - \frac{L_m^2}{L_r'}} & 0 & \frac{R_s \cdot L_m}{L_r'\left(L_s - \frac{L_m^2}{L_r'}\right)} \\ \frac{-R_r'}{L_m - \frac{L_r' \cdot L_s}{L_m}} & 0 & \frac{R_r' \cdot L_s}{L_m\left(L_m - \frac{L_r' \cdot L_s}{L_m}\right)} & -\omega - \omega r \\ 0 & \frac{-R_r'}{L_m - \frac{L_r' \cdot L_s}{L_m}} & \omega - \omega r & \frac{R_r' \cdot L_s}{L_m\left(L_m - \frac{L_r' \cdot L_s}{L_m}\right)} \end{bmatrix} \cdot \begin{bmatrix}\Psi_{qs}\\ \Psi_{ds}\\ \Psi_{qr}'\\ \Psi_{dr}'\end{bmatrix} + \begin{bmatrix}-v_{qs}\\ -v_{ds}\\ -v_{qr}'\\ -v_{dr}\end{bmatrix}$$

$$\frac{d\omega_m}{dt} = \frac{1}{2H} \cdot \frac{1.5 \cdot \sigma}{L_s - \frac{L_m^2}{L_r'}} \cdot \Psi_{qs} \cdot d\Psi_{qs} - \frac{1}{2H}\frac{1.5 \cdot \sigma}{L_s - \frac{L_m^2}{L_r'}} \cdot \Psi_{ds} \cdot d\Psi_{ds} + \frac{1}{2H}\frac{L_m}{L_r'}\frac{1.5 \cdot \sigma}{L_s - \frac{L_m^2}{L_r'}}\Psi_{ds} \cdot d\Psi_{dr}' - \frac{F}{2H} - \frac{T_m}{2H}$$

Parameter sensitivity analysis

Thirteen parameters of the state space model of the induction motor and the bridge have been selected for sensitivity analysis (collected in *Table 1*), and the sensitivity of the state variables: voltage, phase a current, speed, electric torque, and outputs has been investigated for all of them by means of Matlab/Simulink dynamical simulation.

Some simulation results are shown in *Figs 4-7*. The blue signal represents the simulation result with the nominal parameter values and the red signal represents the simulation result with the modified parameter values.

Fig. 4 shows the model responses for changing a critically sensitive parameter (stator self inductance L_S). It is apparent, that the speed diverges even for a 10% change of the parameter value. It can be seen that the speed of the motor becomes minus infinity and the electronic torque is zero. The case of a sensitive parameter (stator resistance R_S) can be seen on Fig. 6.

Table 1: The parameters of the induction motor and the bridge

Parameter	Initial value	Dimension	Name of the parameter
R_s	0.435	Ohm	Stator resistance
L_{ls}	0.002	H	Stator leakage inductance
R_r	0.816	Ohm	Rotor resistance
L_{lr}	0.002	H	Rotor leakage inductance
M	0.0693	H	Mutual inductance
P	2	-	Number of pole pairs
In	0.089	$kg \cdot m^2$	Inertia of the motor
E_d	1000	V	Voltage of the inverter
R_{sn}	10^5	Ohm	Resistance of the snubber circuit
C_{sn}	10^{10}	F	Capacitor of the snubber circuit
R_{br}	10^{-3}	Ohm	Resistance of the bridge
L_s	$M+L_{ls}$	H	Stator self inductance
L_r	$M+L_{lr}$	H	Rotor self inductance

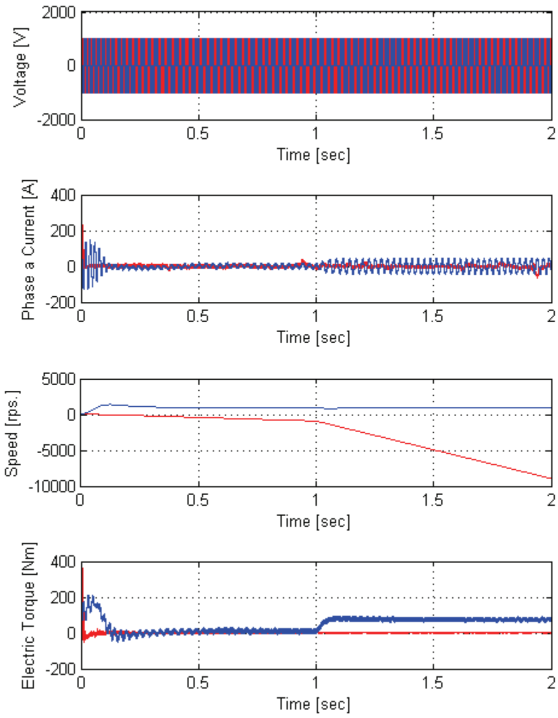

Figure 4: The -10% changing of parameter L_s

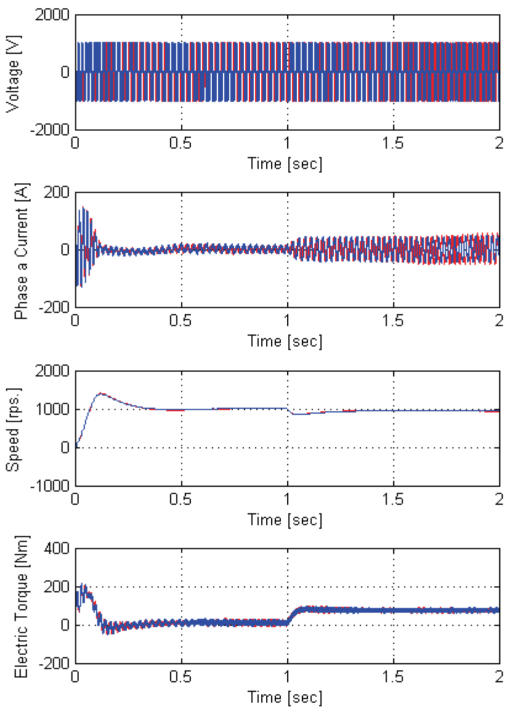

Figure 5: The +50% changing of parameter *M*

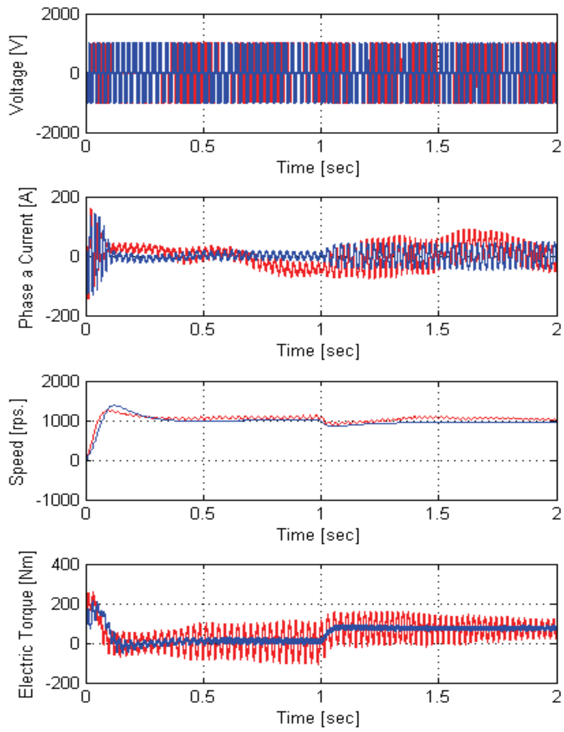

Figure 6: The -90% changing of parameter R_s

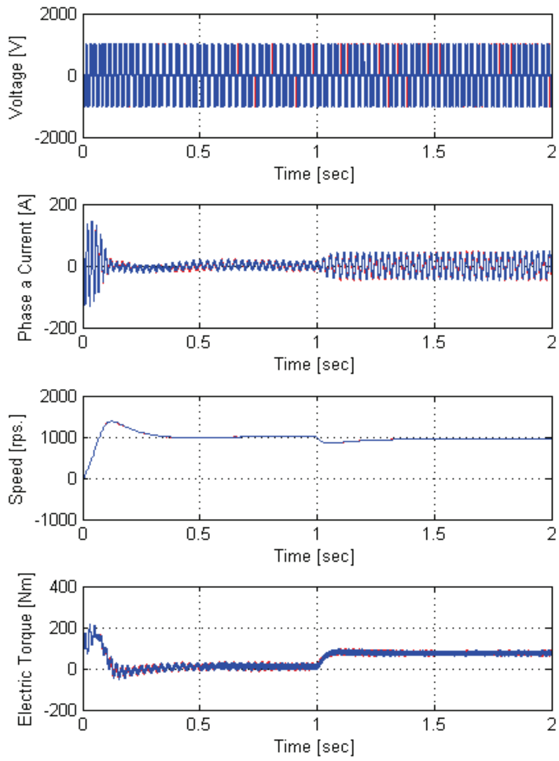

Figure 7: The -90% changing of the resistance of the snubber circuit

As a result, the model parameters have been partitioned to four groups:

- **Critically sensitive:**
  The self inductance of the rotor widings (L_r) and the self inductance of the stator widings (L_s)
- **Sensitive:**
  The inertia (In) and the resistance of the stator (R_s)
- **Less sensitive:**
  The stator leakage inductance (L_{ls}) and the resistance of the rotor (R_r), the rotor leakage inductance (L_{lr}) and the Mutual inductance (M)
- **Not sensitive:**
  The Resistance of the bridge (R_{Br}), the capacitor of the snubber circuit (C_{Sn}) and the resistance of the snubber circuit (R_{sn}).

Conclusions and future works

Based on the results presented here, it is possible to select the candidate parameters for model parameter estimation based on real data that is a further aim of the authors, the four parameters are rotor self inductance (L_r), stator self inductance (L_s), Inertia and rotor resistance (R_s). The final aim of is to develop a simple yet detailed state space model of the induction motor for control purposes which gives us the possibility to develop and analyze different control strategies for the induction motor.

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