PARAMETER SENSITIVITY ANALYSIS OF A SYNCHRONOUS GENERATOR

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A previously developed simple dynamic model of an industrial size synchronous generator is analyzed in this paper. The constructed state-space model consists of a nonlinear state equation and a bilinear output equation. It has been shown that the model is locally asymptotically stable with parameters obtained from the literature for a similar generator.

The effect of load disturbances on the partially controlled generator has been analyzed by simulation using a PI controller. It has been found that the controlled system is stable and can follow the set-point changes in the effective power well. The sensitivity of the model for its parameters has also been investigated and parameter groups have been identified according to the system's degree of sensitivity to them. This groups form the different candidates of parameters for subsequent parameter estimation.

The ways of applying the developed methods to other generators used in the automotive industry are also outlined.

Keywords: synchronous machine, dynamic state space model, parameter sensitivity

Introduction

Synchronous generators are popular and widely used electrical power sources in a wide range of applications including power plants and the automotive industry, too. Whatever size and application area, these generators share the most important dynamic properties, and their dynamic models have a similar structure.

In almost all power plants, both the effective and reactive components of the generated power depend on the need of the consumers and on their own operability criteria. This consumer generated time-varying load is the major disturbance that should be taken care of by the generator controller.

Therefore the final aim of our study is to design a controller that can control the reactive power such that its generation is minimized in such a way that the quality of the control of the effective power remains (nearly) unchanged.

Because of the specialties and great practical importance of the synchronous generators in power plants, their modelling for control purposes is well investigated in the literature. Besides of the basic textbooks (see e.g. [1] and [2]), there are papers that describe the modelling and use the developed models for the design of various controllers [3, 4]. These papers, however, do not take the special circumstances found in nuclear power plants into account that may result in special generator models.

The aim of this paper is to perform model verification and parameter sensitivity analysis of a simple dynamic model of a synchronous generator (SG) proposed in [5]

and [6]. The result of this analysis will be the basis of a subsequent parameter estimation step.

The model of the synchronous generator

In this section the bilinear state-space model for a synchronous generator is presented based largely on [5] that will be used for local stability and parameter sensitivity analysis in later sections.

Modelling assumptions

For constructing the synchronous generator model, let us make the following assumptions:
- a symmetrical tri-phase stator winding system is assumed,
- one field winding is considered to be in the machine,
- there are two amortisseur or damper windings in the machine,
- all of the windings are magnetically coupled,
- the flux linkage of the windings is a function of the rotor position,
- the copper losses and the slots in the machine are neglected,
- the spatial distribution of the stator fluxes and apertures wave are considered to be sinusoidal,
- the stator and rotor permeability are assumed to be infinite.

It is also assumed that all the losses due to wiring, saturation and slots can be neglected.

The six windings (three stators, one rotor and two dampers) are magnetically coupled. Since the magnetic coupling between the windings is a function of the rotor position, the flux linking of the windings is also a function of the rotor position. The actual terminal voltage v of the windings can be written in the form

$$v = \pm\sum_{j=1}^{J}(r_j \cdot i_j) \pm \sum_{j=1}^{J}(\dot{\lambda}_j),$$

where i_j are the currents, r_j are the winding resistances, and λ_j are the flux linkages. The positive directions of the stator currents point out of the synchronous generator terminals.

Thereafter, the two stator electromagnetic fields, both travelling at rotor speed, were identified by decomposing each stator phase current under steady state into two components, one in phase with the electromagnetic field and another phase shifted by 90°. With the above, one can construct an air-gap field with its maximal aligned to the rotor poles (d axis), while the other is aligned to the q axis (between poles). This method is called the Park's transformation.[4, 5]

As a result of the derivation in [5] the vector voltage equation is as follows:

$$v_{dFDqQ} = -R_{RS\omega} \cdot i_{dFDqQ} - L\dot{i}_{dFDqQ} \tag{1}$$

with $i_{dFDqQ} = [i_d \; i_F \; i_D \; i_q \; i_Q]^T$ and
$v_{dFDqQ} = [v_d \; -v_F \; v_D = 0 \; v_q \; v_Q = 0]^T$, where v_d and v_q are the direct and the quadratic components of the stator voltage of the SG, v_D and v_Q are the direct and the quadratic components of the rotor voltage of the SG, i_d and i_q are the direct and the quadratic components of the stator current, i_D and i_Q are the direct and the quadratic components of the rotor current, while v_F and i_F are the exciter voltage and current. Furthermore, $R_{RS\omega}$ and L are the following matrices

$$R_{RS\omega} = \begin{bmatrix} r + R_e & 0 & 0 & \omega L_Q & \omega k M_Q \\ 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 \\ -\omega L_d & -\omega k M_F & -\omega k M_D & r + R_e & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{bmatrix}$$

$$L = \begin{bmatrix} L_d + L_e & k M_F & k M_D & 0 & 0 \\ k M_F & L_F & M_R & 0 & 0 \\ k M_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & L_q + L_e & k M_Q \\ 0 & 0 & 0 & k M_Q & L_Q \end{bmatrix}$$

where r is the stator resistance of the SG, r_F is the exciter resistance, r_D and r_Q are the direct and the quadratic part of the rotor resistance of the SG, L_d, L_q, L_D and L_Q are the direct and the quadratic part of the stator and rotor inductance, ω is the angular velocity, and M_F, M_D and M_R are linkage inductances (see later). The resistance R_e and inductance L_e represent the output transformer of the synchronous generator and the transmission-line.

The state-space model for the currents is obtained by expressing $\dot{i}_{dFDqQ}$ from (1), i.e.

$$\dot{i}_{dFDqQ} = -L^{-1} \cdot R_{RS\omega} \cdot i_{dFDqQ} - L^{-1} \cdot v_{dFDqQ} \tag{2}$$

The motion equation is the following

$$\dot{\omega} = \left[-\frac{L_d i_q}{3\tau_j} \quad -\frac{k M_F i_q}{3\tau_j} \quad -\frac{k M_D i_q}{3\tau_j} \quad \frac{L_q i_d}{3\tau_j} \quad \frac{k M_Q i_d}{3\tau_j} \quad -\frac{D}{\tau_j} \right] \bullet \tag{3}$$
$$\bullet [i_d \; i_F \; i_D \; i_q \; i_Q \; \omega]$$

Altogether, there are 6 state variables: i_d, i_F, i_D, i_q, i_Q and ω. The input variables (i.e. manipulatable inputs and disturbances) are: T_{mech}, v_F, v_d and v_q. Observe, that the state equations *are bilinear in the state variables* because matrix $R_{RS\omega}$ depends linearly on ω.

Note, that (3) can be used as an additional state equation for state space model (2).

The loading angle (δ) of the synchronous generator is

$$\delta = \delta_0 + \int_{t_0}^{t} (\omega - \omega_r) dt$$

that we can differentiate to obtain the time derivative of the δ in per unit notation

$$\dot{\delta} = \omega - 1 \tag{4}$$

The output active power equation can be written in the following form:

$$p_{out} = v_d i_d + v_q i_q, \tag{5}$$

and the reactive power is

$$q_{out} = v_d i_q - v_q i_d. \tag{6}$$

Note, that *output equations are bi-linear in the state and input variables*.

Model analysis

The above model has been verified by simulation against engineering intuition using parameter values of a similar generator taken from the literature [1]. After the basic dynamical analysis, the set of model parameters is partitioned based on the model's sensitivity on them.

Generator parameters

Because the above developed model uses pseudo-parameters that are composed from the original physical ones, mathematical expressions are needed to describe how these parameters depend on the physical ones.

The parameters are described only for a single phase "a" since the machine is assumed to have symmetrical tri-phase stator windings system. The stator mutual inductances for phase a are

$$L_{ab} = L_{ba} = -M_s - M_m \cos(2(\Theta - \frac{\pi}{6}))$$

where M_S is a given constant. The rotor mutual inductances are $L_{FD} = L_{DF} = M_R$, $L_{FQ} = L_{QF} = 0$ and $L_{DQ} = L_{QD} = 0$.

The phase a stator to rotor mutual inductances (from phase windings to the field windings) are given by:

$$L_{aF} = L_{Fa} = M_F \cos(\Theta)$$

where the parameter M_F is also a given constant.

The stator to rotor mutual inductance for phase a (from phase windings to the direct axis of the damper windings) is

$$L_{aD} = L_{Da} = M_D \cos(\Theta)$$

with a given parameter M_D.

The phase a stator to rotor mutual inductances (from phase windings to the damper quadratic direct axis) are given by:

$$L_{aQ} = L_{Qa} = M_Q \cos(\Theta)$$

The parameters L_d, L_q, M_F, M_D and M_R used by the state space model (2 3, 5, 6) and by the above inductance equations are defined as

$$L_d = L_S + M_S + \frac{3}{2}L_m \quad L_d = L_S + M_S + \frac{3}{2}L_m$$

$$M_D = \sqrt{\frac{3}{2}}L_{AD} \qquad M_F = \sqrt{\frac{3}{2}}L_{AD}$$

$$M_R = L_{AD} \qquad M_Q = \sqrt{\frac{3}{2}}L_{AQ}$$

Using the initial assumption of symmetrical tri-phase stator windings we get the resistance of stator windings of the generator, where r_F denotes the resistance of the rotor windings, and r_D and r_Q represent the resistance of the d and q axis circuit.

In order to avoid working with numerical values that are in order of magnitude different, the equations have been normalized to a base value. We choose the base for rotor quantities and normalize the voltage and the torque accordingly. The variables in the normalized equations are then in „per units".

The parameters of the synchronous generator were obtained from the literature [1]. The stator base quantities, the rated power, output voltage, output current and the angular frequency base values are:

$$
\begin{aligned}
S_B &= 160\,\text{MVA}/3 = 53.333\,\text{MVA} \\
V_B &= 15\,\text{kV}/\sqrt{3} = 8.66\,\text{kV} \\
I_B &= 6158\,\text{A} \\
\omega_e &= 2\pi f \text{ rad/s}
\end{aligned}
$$

The physical parameters of the synchronous machine and the external network in dimensionless values are:

$L_d = 1.700$	$l_d = 0.150$	$L_{MD} = 0.02838$
$L_q = 1.640$	$l_q = 0.150$	$L_{MQ} = 0.2836$
$L_D = 1.605$	$l_F = 0.101$	$r = 0.001096$
$L_Q = 1.526$	$l_D = 0.055$	$r_F = 0.00074$
$L_{AD} = 1.550$	$l_Q = 0.036$	$r_D = 0.0131$
$L_{AQ} = 1.490$	$r_Q = 0.054$	$R_e = 0.2$
$V_\infty = 0.828$	$L_e = 1.640$	$D = 2.004$

Local stability analysis

The steady-state values of the state variables can be obtained from the steady-state version of state equations (2, 3) using the above parameters. Equation (1) implies that the expected value to i_D and i_Q are 0, that coincide with the engineering intuition. The equilibrium point of the system is:

$$
\begin{aligned}
\omega &= 0.9990691 \\
i_d &= -1.9132609 \\
i_q &= 0.66750001 \\
i_F &= 2.97899982 \\
i_D &= -8.6242856 \cdot 10^{-9} \\
i_Q &= -5.3334899 \cdot 10^{-10}
\end{aligned}
$$

The state matrix A of the locally linearized state-space model $x = Ax + Bu$ has the following numerical value in this equilibrium:

$$
\begin{bmatrix}
-0.0361 & 0.0004 & 0.0142 & -3.4851 & -2.5455 & -2.3285 \\
0.0124 & -0.0049 & 0.0772 & 1.2011 & 0.8773 & 0.8025 \\
0.0228 & 0.0044 & -0.0964 & 2.2057 & 1.6110 & 1.4737 \\
3.5855 & 2.6464 & 2.6464 & -0.0361 & 0.0901 & 1.0247 \\
-3.5009 & -2.5839 & -2.5839 & 0.0352 & -0.1234 & -1.0005 \\
-8 \cdot 10^{-6} & -0.0002 & -0.0002 & -0.0008 & -0.0005 & -0.0011
\end{bmatrix}
$$

The eigenvalues of the state matrix are:

$$
\begin{aligned}
\lambda_{1,2} &= -3.619088 \cdot 10^{-2} \pm j0.997704 \\
\lambda_3 &= -0.100024 \qquad \lambda_4 = -1.67235 \cdot 10^{-3} \\
\lambda_5 &= -4.724291e \cdot 10^{-4} \qquad \lambda_6 = -0.123426
\end{aligned}
$$

It is apparent that the real parts of the eigenvalues are negative but their magnitudes are small, thus the system is on the boundary of the stability domain.

PI controller

The control scheme of the synchronous machine is a classical PI controller that ensures stability of the equilibrium point under small perturbations [4]. The controlled output is the speed (ω), the manipulated input is the mechanical torque T_{mech}. The proportional parameter of the PI controller of the speed is 0.05 and the integrator time is 0.1 in per units.

Model validation

The dynamic properties of the generator have been investigated in such a way that a single synchronous machine was connected to an infinite bus that models the electrical network. The response of the speed controlled generator has been tested under step-like changes of the exciter voltage. The simulation results are shown in *Fig. 1*, where the exciter voltage v_F and the torque angle δ are shown.

When the exciter voltage is increased the loading angle must be decreased as it can be seen in the Fig. 1.
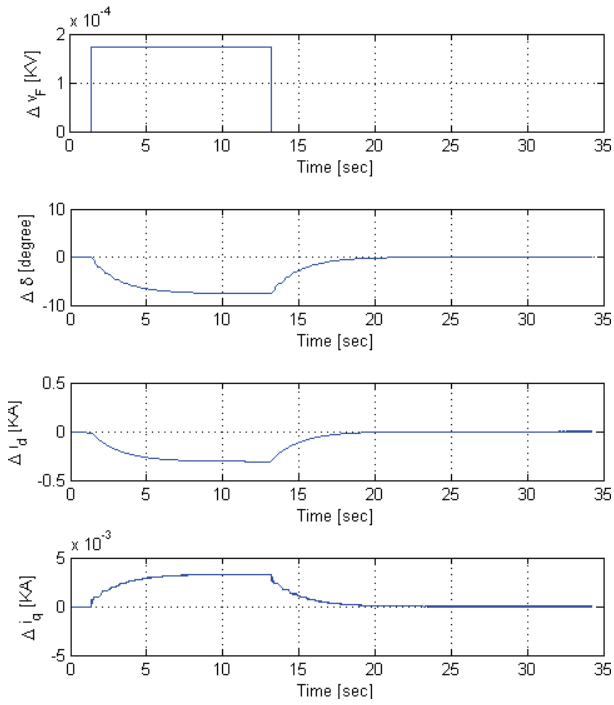

Figure 1: Response to the exciter voltage step change of the controlled generator
(Δ means the deviation form the steady-state value)

Sensitivity analysis

The aim of this sub-section is to define parameter groups according to the system's sensitivity on them.

Linkage inductances l_d, l_q, l_D, l_Q, L_{MD} and L_{MQ} are not used by the current model, only by the flux model [5]. It is not expected that the output and the state variables of system change when these parameters are perturbed, see *Fig. 2*. As it was expected, the model is not sensitive to these parameters. Note, that the linkage inductance parameters are only used for determining the fluxes of the generator.

Sensitivity of the model to the controller parameters P and I and the dumping constant D has also been investigated. Since the PI controller controls ω by modifying the value of T_{mech}, the controller keeps ω at synchronous speed. This is why the output and the steady state value of the system variables do not change (as it is apparent in *Fig. 3*) even for a considerably large change of D.

Sensitivity analysis of the resistance of the stator and the resistance of the transmission line led to the same result. A ±20% perturbation in them resulted in a small change in currents i_d, i_q and i_F. This causes the change of the effective and the reactive power of the generator, as shown in *Fig. 4*.

The analysis of the effect of the rotor resistance r_F showed, that the ±20% perturbation of r_F kept the quadratic component of the stator current (i_q) constant, but currents i_d and i_F were changed. The output of the generator also changed, as it is shown in *Fig. 5*.
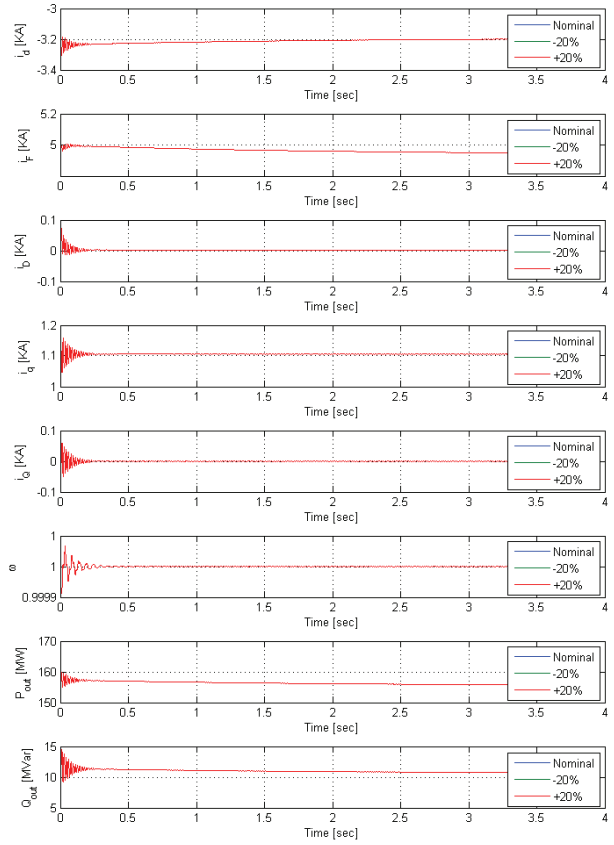

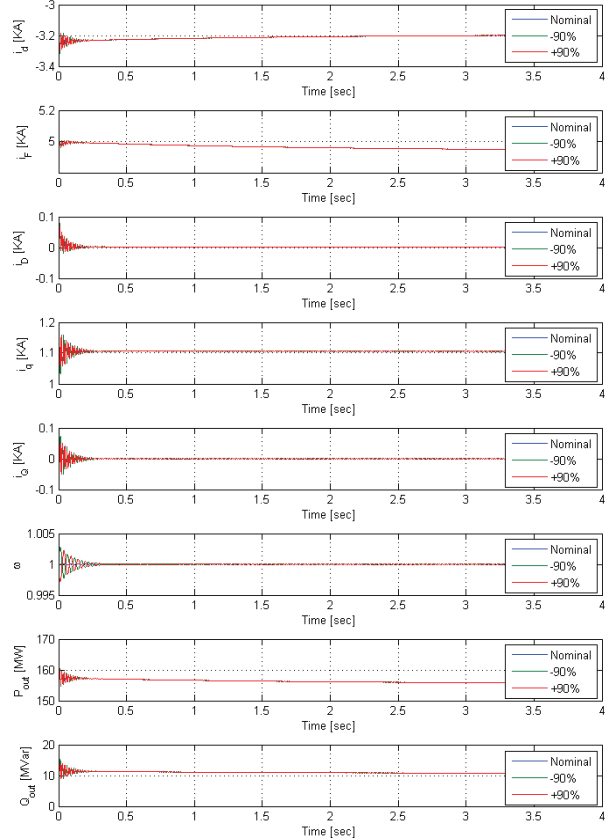*Figure 2:* The model states and outputs for a ±20% change of l_d

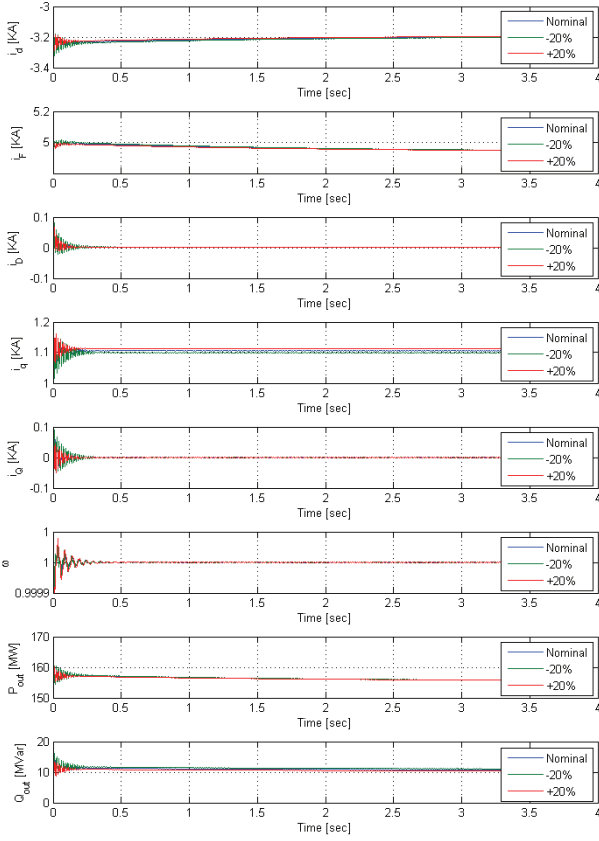*Figure 3:* The model states and outputs for a ±90% change of D

Figure 4: The model states and outputs
for a ±20% change of r_{resist}

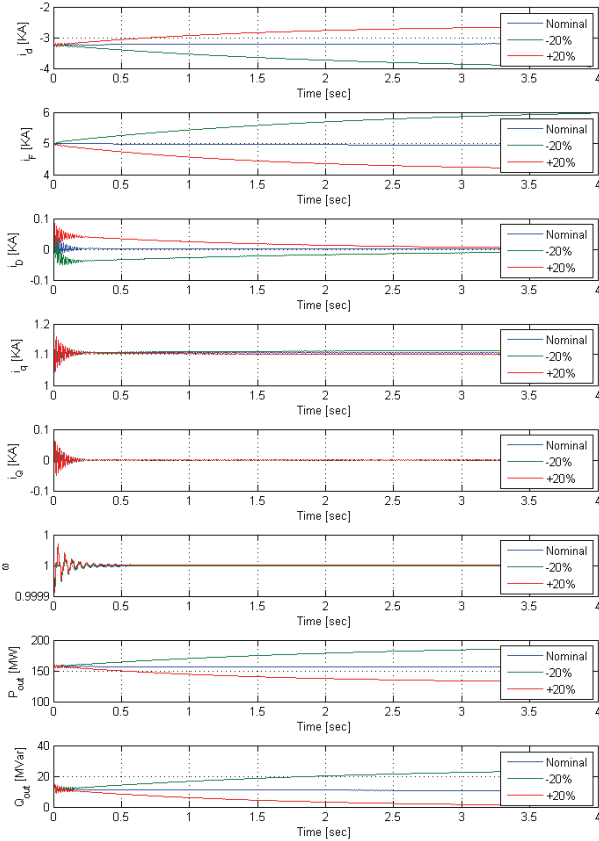*Figure 5:* The model states and outputs
for a ±20% change of r_F

The sensitivity of the model states and outputs to the inductance of the rotor (L_F) and the inductance of the direct axis (L_D) has also been analyzed. The results show only a moderate reaction in i_d and i_F to the parameter perturbations, and the equilibrium state of the system kept unchanged. However, decreasing the value of the parameters to the 90 percent of their nominal value destabilized the system. The results of a ±9% perturbation in L_F are shown in *Fig. 6*. A small perturbation of the outputs is noticeable.
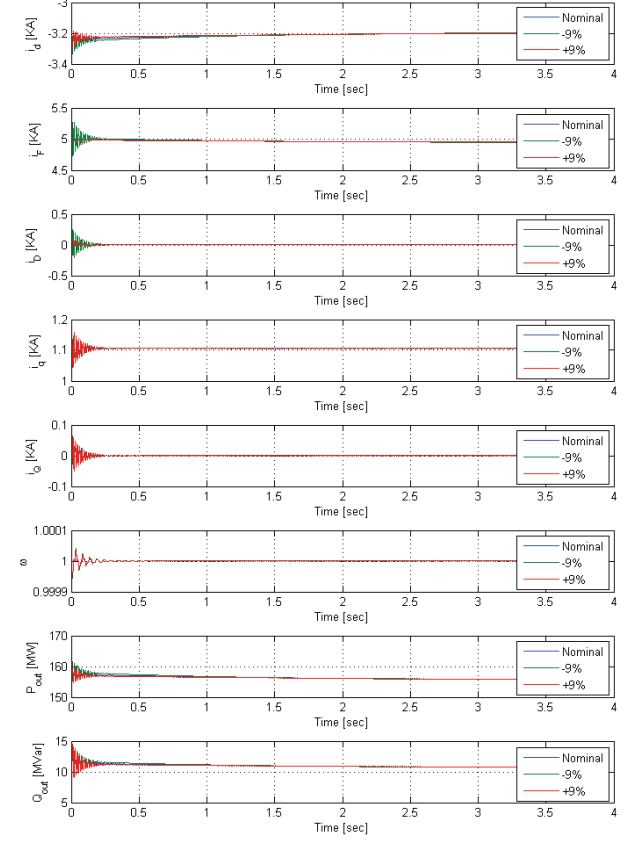

Figure 6: The model states and outputs
for a ±9% change of L_F

Finally, the sensitivity of the model (2, 3, 5, 6) to the linkage inductance L_{AD} has been examined. When the parameter has been changed ±5%, currents i_D and i_F changed only a little. On the other hand, the steady-state of the system has shifted as it can be seen in *Fig. 7*. A parameter variation of more than 5% destabilized the system.

As a result of the sensitivity analysis, it is possible to define the following groups of parameters:

● Not sensitive inductances l_d, l_q, l_D, l_Q, L_{MD}, L_{MQ}, L_{AQ}, L_Q damping constant D and the controller parameters P and I. Since the state space model of interest is insensitive for them, the values of these parameters cannot be determined from measurement data using any parameter estimation method.

● Less sensitive: resistances of the stator r and the transmission-line R_e.

● More sensitive: resistance r_F of the rotor and the inductance of transmission-line L_e. These parameters are candidates for parameter estimation.

- Critically sensitive: linkage inductance L_{AD}, inductances L_D and L_F. These parameters can be estimated very well.
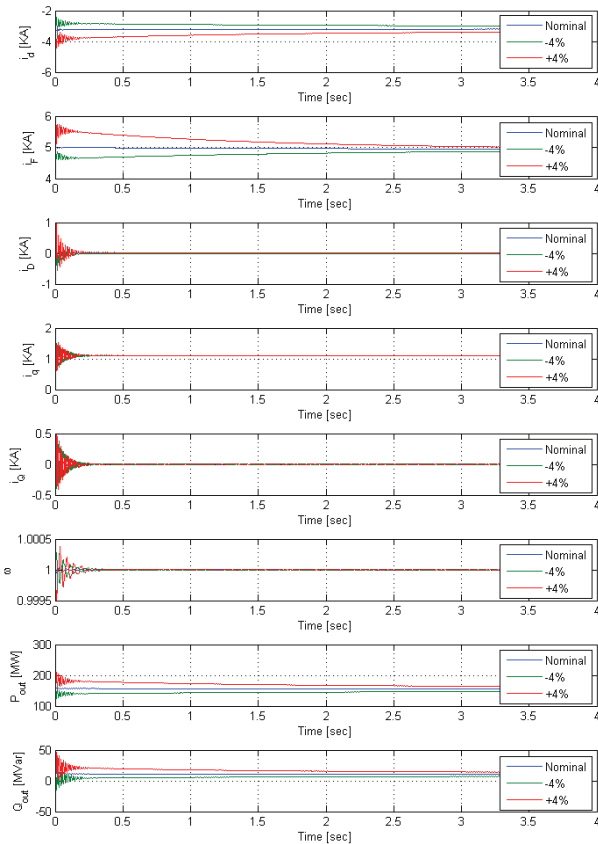

Figure 7: The model states and outputs for a ±4% change of L_{AD}

Conclusion and further work

The simple bilinear dynamic model of an industrial size synchronous generator described in [5] and [6] has been investigated in this paper.

It has been shown that the model is locally asymptotically stable around a physically meaningful equilibrium state with parameters obtained from the literature for a similar generator. The effect of load disturbances on the partially controlled generator has been analyzed by simulation by using a traditional PI controller. It has been found that the controlled system is stable and can follow the setpoint changes in the effective power well.

Eighteen parameters of the system have been selected for sensitivity analysis, and the sensitivity of the state variables and outputs has been investigated for all of them. As a result, the parameters have been partitioned to four groups.

Based on the results presented here, the further aim of the authors is to estimate the parameters of the model for a real system from measurements. The sensitivity analysis enables us to select the candidates for estimation that are r_F, L_e, L_{AD}, L_D and L_F.

It is important to emphasize that this model can be and will be used as a starting point for the model development of a permanent magnet synchronous motor (PMSM) which board spectrum, that is widely used in the automotive industry. This becomes possible by changing the exciter coil of the classical synchronous machine to a permanent magnet: this way the model of the PMSM is obtained, which is one variant of the brushless direct current motors (BLDC motor).

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