



# Interplay between bulk medium evolution and (D)GLV energy loss

Denes Molnar and Deke Sun

*Department of Physics and Astronomy, Purdue University, West Lafayette, IN 47907, U.S.A.*

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## Abstract

We study the consistency between high- $p_T$  nuclear suppression ( $R_{AA}$ ) and elliptic flow ( $v_2$ ) using Gyulassy-Levai-Vitev (GLV) energy loss or a simpler power-law  $dE/dL$  formula, for a variety of bulk evolution models. The results generally confirm our earlier work [1] that found suppressed elliptic flow for transversely expanding media. One exception is the set of hydrodynamic solutions used recently[2] by Betz and Gyulassy, which give significantly higher  $v_2$  but unfortunately assume unrealistic bag-model equation of state. On the other hand, we show that covariant treatment of energy loss introduces an interplay between jet direction and hydrodynamic flow of the medium, which largely counteracts elliptic flow suppression caused by transverse expansion.

*Keywords:* Heavy-ion collisions, elliptic flow, parton energy loss

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## 1. Introduction

An important crosscheck of parton energy loss calculations is the consistency between nuclear suppression ( $R_{AA}$ ) and differential elliptic flow  $v_2(p_T)$ . Recently we found[1] that in realistic applications of Gyulassy-Levai-Vitev (GLV) radiative parton energy loss[3] that include transverse expansion of the bulk medium, high- $p_T$  elliptic flow is reduced by nearly a half compared to transversely frozen evolution scenarios. This reinforced the conclusions[4] by the PHENIX Collaboration that perturbative QCD energy loss models generally fail to reproduce the azimuthal angle dependent neutral pion suppression. However, a recent work by Betz and Gyulassy claims[2] simultaneous reproduction of this set of observables with simple pQCD-motivated energy loss formulas. This apparent contradiction, on the other hand, may be due to important differences between the two calculations, especially in the energy loss model and bulk medium evolution assumed. Here we pinpoint the origin of the discrepancy, and show that the findings of Ref. [2] are largely due to the hydrodynamic solutions used in that calculation for bulk medium evolution. In addition, we show that covariant treatment of energy loss introduces an interplay between jet direction and hydrodynamic flow of the medium, which largely compensates the elliptic flow suppression we found earlier in [1].

## 2. Radiative energy loss and bulk medium evolution

### 2.1. Sensitivity to bulk medium model

Consider the parameterized energy loss model  $dE/dL = \kappa E^a L^b T^c$  by Betz and Gyulassy[2], with “pQCD-like” exponents  $a = 1/3$ ,  $b = 1$ , and  $c = 2 - a + b = 8/3$  ( $\kappa$  is then dimensionless). Here  $E$  is the jet energy,  $T$  is temperature of the medium, and  $L$  is the pathlength traveled by the jet. To study the sensitivity to the bulk medium evolution, we investigate five different dynamical models for Au+Au at  $\sqrt{s_{NN}} = 200$  GeV at RHIC, impact parameter  $b \approx 7.5$  fm. Four of these are solutions of boost-invariant 2+1D hydrodynamics using the VISH2+1 code[5], which are available in tabulated form from the TECHQM Collaboration website [6] in two data sets. Set 1 is for a “bag-model” like

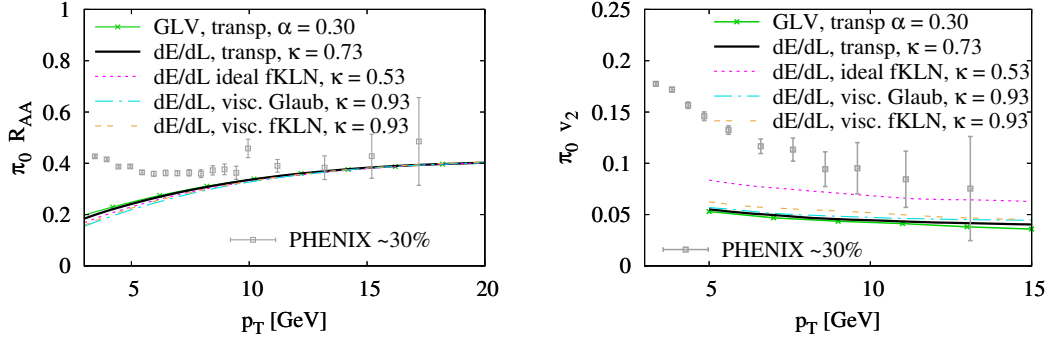


Figure 1. Neutral pion suppression factor  $R_{AA}$  (left) and differential elliptic flow  $v_2(p_T)$  (right) at midrapidity in mid-peripheral ( $b \approx 7.5 fm$ ) Au+Au at  $\sqrt{s_{NN}} = 200$  GeV, calculated using parton energy loss parameterization[2]  $dE/dL = \kappa E^{1/3} L^1 T^{8/3}$ . Results for four different bulk medium models are plotted (see text): i) ideal hydrodynamics with fKLN initial profile from “Set 1” (dotted); ii) viscous hydrodynamics with  $\eta/s = 0.08$  and fKLN initial profile (dashed-dotted); iii) viscous hydrodynamics with  $\eta/s = 0.08$  and Glauber initial profile from “Set 2” (double short dashes); and iv) covariant parton transport MPC as in Ref. [1] (solid lines). For comparison, results from Ref. [1] using MPC and GLV energy loss are also shown (solid lines with crosses). In all cases energy loss is scaled to set a fixed  $R_{AA} \sim 0.4$  at  $p_T \sim 15 - 20$  GeV. As in Ref. [1], data[8, 9] from PHENIX (boxes) are shown to guide the eye.

equation of state (EoS), “fKLN” initial profile motivated by the color glass condensate model, with zero viscosity or constant  $\eta/s = 0.08$ . The ideal and viscous versions of this set are practically identical for observables studied here, so we only show results for “ideal-fKLN”, which is the evolution used in Ref. [2]. Set 2 from TECHQM is a later calculation with more realistic lattice QCD EoS, constant  $\eta/s = 0.08$ , for fKLN or Glauber initial profile. The fifth model is the same covariant transport evolution as in Ref. [1], computed using Molnar’s Parton Cascade (MPC) [7].

Figure 1 shows the neutral pion  $R_{AA}$  (left plot) and  $v_2$  (right plot) in Au + Au at RHIC for these scenarios.  $R_{AA}$  is basically the same for all cases because  $\kappa$  is dialed to obtain the same suppression at high  $p_T$ . In all scenarios, elliptic flow is reduced to  $\sim 4 - 5\%$  at high  $p_T$ , much the same value as what we found earlier with GLV energy loss[1], *except* for the “ideal-fKLN” evolution studied by Betz and Gyulassy. Thus we confirm their result, but also demonstrate that transverse expansion does suppress  $v_2$  for a hydrodynamic medium as well if one includes a realistic equation of state.

## 2.2. Energy loss model

Next we test how well the power-law  $dE/dL \propto E^a L^b T^c$  formula captures perturbative QCD parton energy loss in the Gyulassy-Levai-Vitev (GLV) formulation[3]. The approach is identical to the one in Ref. [1], i.e., we use the average radiative energy loss along the path of a massless jet parton obtained via integrating the first-order (in opacity) GLV radiated gluon spectrum:

$$\langle \Delta E^{(1)} \rangle = \frac{C_R \alpha_s}{\pi^2} E \int_0^\infty d\tau \rho(\vec{x}_0 + \vec{v}\tau, \tau) \sigma_{gg}(\tau) \int dx d^2\mathbf{k} \int d^2\mathbf{q} \frac{\mu^2(\tau)}{\pi[\mathbf{q}^2 + \mu^2(\tau)]^2} \frac{2\mathbf{k}\mathbf{q}}{\mathbf{k}^2(\mathbf{k} - \mathbf{q})^2} \left( 1 - \cos \frac{(\mathbf{k} - \mathbf{q})^2 \tau}{2xE} \right), \quad (1)$$

where  $E$  is the jet parton energy,  $\mathbf{q}$  is the momentum transfer in scattering with the medium,  $\mu$  is the local Debye screening mass,  $\sigma_{gg} = 9\pi\alpha_s^2/2\mu^2$  is the scattering cross section in the medium for gluons, and the momentum integrals are performed observing finite energy and kinematic bounds ( $|k| \lesssim xE$ ,  $|q| \lesssim \sqrt{6ET}$ ,  $xE \gtrsim \mu$ ).

Figure 2 shows neutral pion  $R_{AA}$  and  $v_2$  for the different bulk medium scenarios with GLV energy loss. Qualitatively the results are very similar to those in Fig. 1, confirming that the “pQCD-like” exponents in Ref. [2] are a reasonable approximation to GLV energy loss. After fixing  $R_{AA} \sim 0.4$  at high  $p_T$  (left plot), a residual sensitivity to the bulk evolution still remains in elliptic flow (right plot). The “ideal-fKLN” evolution used in Ref. [2] gives largest  $v_2$ , almost as large as the results with transversely frozen dynamics in Ref. [1] (solid line). Hydrodynamic solutions with lattice QCD EoS, on the other hand, give smaller  $v_2$ . There is a modest  $\sim 15\%$  difference between fKLN and Glauber profiles with viscous hydrodynamics (fKLN is higher), which may help constrain the initial geometry.

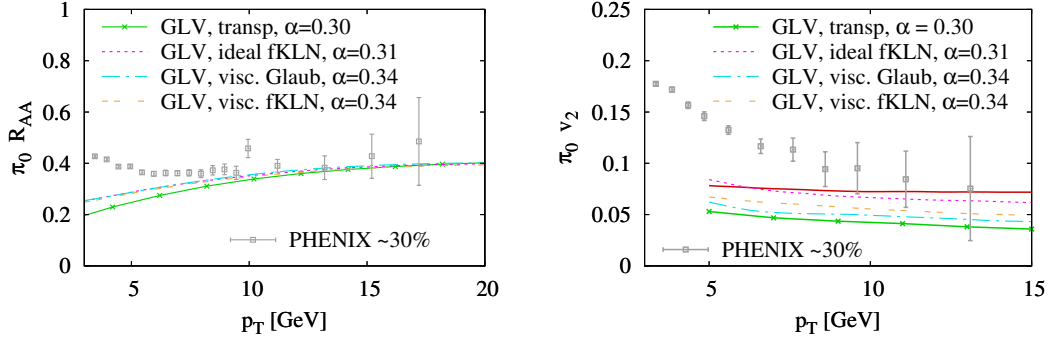


Figure 2. The same as Fig. 1, except calculated using GLV energy loss. The solid line (without crosses) in the right plot now shows  $v_2$  from Ref. [1] for transversely frozen, boost-invariant 0+1D medium evolution.

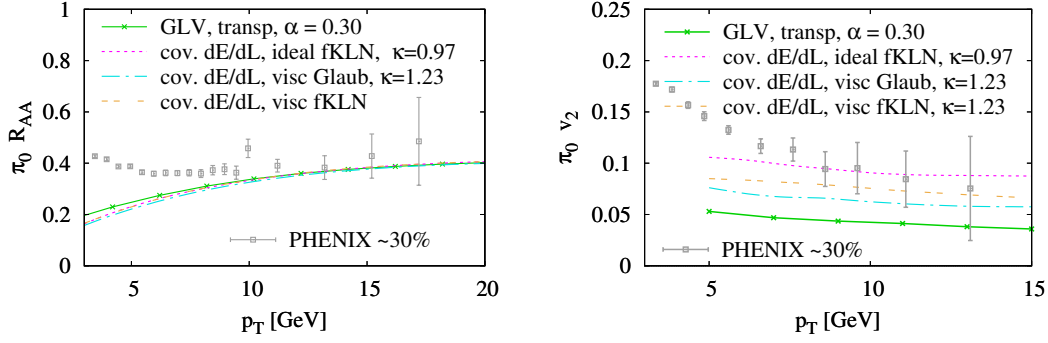


Figure 3. The same as Fig. 1, except for a covariant  $dE/dL$  calculation using Eq. (2).

### 2.3. Covariant energy loss

Neither of the above calculations observe proper Lorentz covariance, however, because both  $dE/dL \propto E^a L^b T^c$  and GLV energy loss Eq. (1) give frame dependent results. We can formulate a frame-independent prescription if we require energy loss contributions to be computed in the frame where the fluid is locally static along the path (LR frame). For massless partons produced at spacetime point  $(0, \vec{0})$ , scattering occurs at  $L(1, \vec{v})$ , which transforms the same way as the four-momentum  $E(1, \vec{v})$ . Therefore, in the massless case  $dE/dL$  is a Lorentz scalar, which means that for the  $dE/dL$  model we should have

$$\frac{dE}{dL} = \frac{dE_{LR}}{dL_{LR}} = \kappa E_{LR}^a L_{LR}^b T^c = \kappa [\gamma_F (1 - \vec{v} \cdot \vec{v}_F)]^{a+b} E^a L^b T^c, \quad (2)$$

while for GLV

$$dL_{LR} \rho_{LR} \sigma = dL \rho_{LR} \sigma \gamma_F (1 - \vec{v} \cdot \vec{v}_F) = dL \rho \sigma (1 - \vec{v} \cdot \vec{v}_F). \quad (3)$$

Here,  $\vec{v}_F$  is the local three-velocity of fluid flow, while  $\gamma_F \equiv (1 - v_F^2)^{-1/2}$ . In both cases, a new factor appears that couples the motion of the jet to that of the fluid. For GLV this is very similar to the term introduced in Ref. [10], however, in contrast to the results there we find that jet-medium flow coupling has significant effect on observables.

Figure 3 shows neutral pion  $R_{AA}$  and  $v_2$  in Au+Au at RHIC with  $b \approx 7.5$  fm, calculated using covariant  $dE/dL$  energy loss. Two features are noticeable immediately. First, with covariant energy loss one needs higher scaling

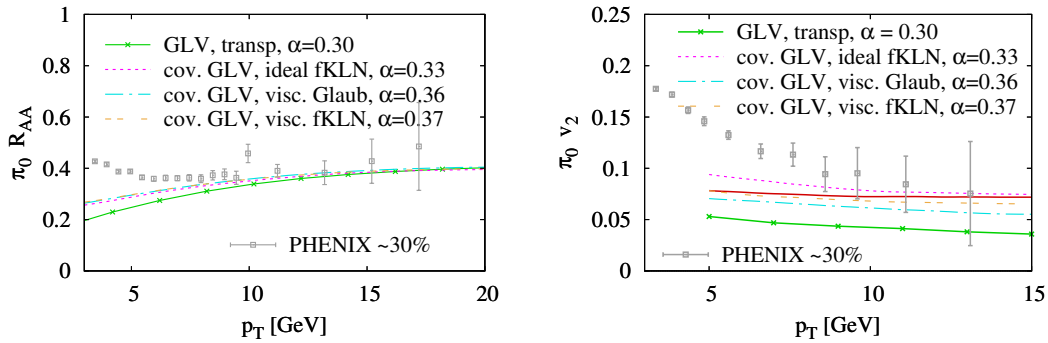


Figure 4. The same as Fig. 2, except with the covariant opacity factor Eq. (2) in the GLV energy loss formula Eq. (1).

factors  $\kappa$  to obtain the same  $R_{AA}$ . Second, even after setting  $\kappa$  to  $R_{AA}$  at high  $p_T$ ,  $v_2$  is larger with covariant energy loss and shows strong dependence on bulk dynamics. Qualitatively the reason is that jet-medium flow coupling reduces energy loss for jets moving in the same direction as the medium flow, the more the larger the flow velocity. For jets that move in-plane (short direction), flow tends to be larger, so the reduction is stronger. The resulting  $v_2$  enhancement largely cancels out the flow suppression due to transverse expansion found in Ref. [1]. We find the largest  $v_2$  for the “ideal-fKLN” profile used in Ref. [2].

Very similar results follow with covariant GLV energy loss, as shown in Figure 4. Elliptic flow is a little bit smaller than for the covariant  $dE/dL$  model but otherwise it shows the same ordering between the various scenarios.

At the conference we also showed preliminary results for charm and bottom quarks with Djordjevic-Gyulassy-Levai-Vitev (DGLV) energy loss[11]. Due to space constraints these results will be presented elsewhere.

### 3. Conclusions

We study the consistency between high- $p_T$  nuclear suppression ( $R_{AA}$ ) and elliptic flow ( $v_2$ ) using Gyulassy-Levai-Vitev energy loss or a simpler power-law  $dE/dL$  formula, for a variety of bulk evolution models. The results generally confirm our earlier work [1] that found suppressed elliptic flow for transversely expanding media. However, one exception is the set of hydrodynamic solutions used recently[2] by Betz and Gyulassy, which give significantly higher  $v_2$  but unfortunately assume unrealistic bag-model equation of state. On the other hand, we also find that covariant treatment of energy loss introduces an interplay between jet direction and hydrodynamic flow of the medium, which largely compensates for the elliptic flow suppression we found earlier in [1].

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