Computing Balanced Solutions for Large International Kidney Exchange Schemes When Cycle Length Is Unbounded

Extended Abstract

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ABSTRACT

In kidney exchange programmes (KEP) patients may swap their incompatible donors leading to cycles of kidney transplants. Countries try to merge their national patient-donor pools leading to international KEPs (IKEPs). Long-term stability of an IKEP can be achieved through a credit-based system. The goal is to find, in each round, an optimal solution that closely approximates this target allocation. We provide both theoretical and experimental results for the case where the cycle length is unbounded.

KEYWORDS

cooperative game; partitioned permutation game; kidney exchange

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1 BALANCING KIDNEY EXCHANGE

The most effective treatment for kidney failure is transplant from a deceased or living donor, with better outcomes in the latter case. A kidney from a family member or friend might be medically incompatible with the patient. Therefore, many countries run Kidney Exchange Programmes (KEPs) [6], where patient-donor pairs are placed together in one pool. If for patient-donor pairs (p, d) and (p', d'), d, p are incompatible, as well as d', p', but d and d' could be donor for p' and p, respectively, then we obtain a 2-*way exchange*.

We now generalize. We model a pool of patient-donor pairs as a directed graph $G = (V, A)$ (the *compatibility graph*) in which V consists of the patient-donor pairs, and A consists of every arc (u, v) such that the donor of u is compatible with the patient of v. In a directed cycle $C = u_1u_2 \dots u_ku_1$, for some $k \geq 2$, the kidney of the donor of u_i could be given to the patient of u_{i+1} (with $u_{k+1} := u_1$). This is a k-*way exchange* using the *exchange cycle* C. To prevent exchange cycles from breaking, (and a patient from losing their willing donor), hospitals perform the k transplants in a k-way exchange simultaneously. Hence, KEPs impose a bound ℓ (the *exchange bound*) on the maximum length of an exchange cycle, typically $2 \leq \ell \leq 5$. An ℓ-*cycle packing* of G is a set C of directed cycles, each of length at most ℓ, that are pairwise vertex-disjoint. The *size* of C is the number of arcs that belong to a cycle in C.

KEPs operate in rounds. A solution for round r is an ℓ-cycle packing in the associated compatibility graph G^r. To help as many patients as possible in each round r, we seek an *optimal* solution, i.e., a *maximum (size)* ℓ-cycle packing of G^r. After a round, some patients have received a kidney or died, and other patient-donor pairs may have arrived, resulting in a compatibility graph G^{r+1}. The main computational issue is to find an optimal solution.

Theorem 1.1 ([1]). *If $\ell = 2$ or $\ell = \infty$, we can find an optimal solution for a KEP round in polynomial time; else this is* NP-*hard.*

As merging pools of national KEPs leads to better outcomes, *international* KEPs (IKEPs) are formed [8, 17]. *How can we ensure long-term stability of an IKEP to avoid countries from leaving?*

A *(cooperative) game* is a pair (N, v), where N is a set of n *players* and $v : 2^N \to \mathbb{R}$ is a *value function* with $v(\emptyset) = 0$. A subset $S \subseteq N$ is a *coalition.* If for every possible partition $(S_1, \dots, S_r)$ of N it holds that $v(N) \geq v(S_1) + \cdots + v(S_r)$, then players will benefit most by forming the *grand coalition* N. The problem is then how to fairly distribute $v(N)$ amongst the players of N. An *allocation* is a vector $x \in \mathbb{R}^N$ with $x(N) = v(N)$ (we write $x(S) = \sum_{p \in S} x_p$ for $S \subseteq N$). A *solution concept* prescribes a set of fair allocations for a game (N, v). In particular, the *core* consists of all allocations $x \in \mathbb{R}^N$ with $x(S) \geq v(S)$ for all $S \subseteq N$. Core allocations ensure N is stable, as no subset S will benefit from leaving. However, a core may be empty.

For a directed graph $G = (V, A)$ and subset $S \subseteq V$, we let $G[S] = (S, \{(u, v) \in A \mid u, v \in S\}$. An ℓ-*permutation game* on $G = (V, E)$ is the game (N, v), where $N = V$ and for $S \subseteq N$, the value $v(S)$ is the maximum size of an ℓ-cycle packing of $G[S]$. We have a *matching game* if $\ell = 2$, whose core may be empty, and a *permutation game* if $\ell = \infty$, whose core is always nonempty [16]. A *partitioned* ℓ-*permutation game* on a directed graph $G = (V, A)$ with a partition $(V_1, \dots, V_n)$ of V is the game (N, v), where $N = \{1, \dots, n\}$, and for

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$S \subseteq N$, the value $v(S)$ is the maximum size of an ℓ-cycle packing of $G[\bigcup_{i \in S} V_i]$. We obtain a *partitioned matching game* [2, 7] if $\ell = 2$, and a *partitioned permutation game* if $\ell = \infty$. The *width* of (N, v) is $c = \max\{|V_i| \mid 1 \leq i \leq n\}$.

For a round of an IKEP with exchange bound ℓ, let (N, v) be the partitioned ℓ-permutation game on the compatibility graph $G = (V, A)$, where $N = \{1, \ldots, n\}$ is the set of countries in the IKEP, and V is partitioned into sets $V_1, \ldots, V_n$ such that every V_p consists of the patient-donor pairs of country p. The game (N, v) is the *associated* game for G. We use a solution concept $\mathcal{S}$ for (N, v) to obtain an *initial allocation* y, where y_p prescribes the initial number of kidney transplants country p should receive in this round.

To ensure IKEP stability, we use the model of Klimentova et al. [12], which is a *credit-based* system. For round $r \geq 1$, let G^r be the computability graph with associated game (N^r, v^r); let y^r be the initial allocation (as prescribed by some solution concept $\mathcal{S}$); and let $c^r : N^r \rightarrow \mathbb{R}$ be a *credit function*, which satisfies $\sum_{p \in N^r} c_p^r = 0$; if $r = 1$, we set $c^r \equiv 0$. For $p \in N$, we set $x_p^r := y_p^r + c_p^r$ to obtain the *target allocation* x^r for round r (which is indeed an allocation, as y^r is an allocation and $\sum_{p \in N} c_p^r = 0$). We choose some maximum ℓ-cycle packing C of G^r as optimal solution for round r (out of possibly exponentially many optimal solutions). Let $s_p(C)$ be the number of kidney transplants for patients in country p (with donors both from p and other countries). For $p \in N$, we set $c_p^{r+1} := x_p^r - s_p(C)$ to get the credit function c^{r+1} for round $r+1$ (note that $\sum_{p \in N} c_p^{r+1} = 0$). For round $r+1$, a new initial allocation y^{r+1} is prescribed by $\mathcal{S}$ for the associated game (N^{r+1}, v^{r+1}). For every $p \in N$, we set $x_p^{r+1} := y_p^{r+1} + c_p^{r+1}$, and we repeat the process.

We must also determine how to choose in each round r a maximum ℓ-cycle packing C (optimal solution) of the corresponding compatibility graph G. We will choose C, such that the vector $s(C)$, with entries $s_p(C)$, is *closest* to the target allocation x for round r. Let $|x_p - s_p(C)|$ be the *deviation* of country p from its targe x_p if C is chosen. We order the deviations $|x_p - s_p(C)|$ non-increasingly as a vector $d(C) = (|x_{p_1} - s_{p_1}(C)|, \ldots, |x_{p_n} - s_{p_n}(C)|)$. We say C is *strongly close* to x if $d(C)$ is lexicographically minimal over all optimal solutions. If we only minimize $d_1(C) = \max_{p \in N}\{|x_p - s_p(C)|\}$ over all optimal solutions, we obtain a *weakly close* optimal solution.

Related Work. Benedek et al. [3] proved that, for partitioned matching games (the "matching" case), an optimal solution that is strongly close to a given target allocation x can be found in polynomial time; in contrast, Biró et al. [7] showed that it is NP-hard to find a weakly close maximum matching even for $|N| = 2$ once the games are defined on edge-weighted graphs. Benedek et al. [3] performed simulations for up to fifteen countries for $\ell = 2$. Using the Shapley value for the initial allocation yielded best results (slightly overtaken by the Banzhaf value in the full version [4] of [3]). This is in line with the results of Klimentova et al. [12] and Biró et al. [5] for $\ell = 3$. Due to Theorem 1.1, the simulations in [5, 12] are for up to four countries and use weakly close optimal solutions.

For $\ell = 2$, we refer to [15] for an alternative model based on so-called selection ratios using lower and upper target numbers. IKEPs have also been modelled as non-cooperative games in the *consecutive matching* setting, which has 2-phase rounds: national pools in phase 1 and a merged pool for unmatched patient-donor pairs in phase 2; see [9, 10, 14] for some results in this setting.

Our Results. Permutation games, i.e. partitioned permutation games of width 1, have a nonempty core [16], and a core allocation can be found in polynomial time [11]. We generalize these two results, and also show a dichotomy for testing core membership, which is in contrast with the dichotomy for partitioned matching games, where the complexity jump is at $c = 3$ [7].

Theorem 1.2. *The core of every partitioned permutation game is non-empty, and it is possible to find a core allocation in polynomial time. Moreover, for partitioned permutation games of fixed width* c*, the problem of deciding if an allocation is in the core is polynomial-time solvable if* $c = 1$ *and* coNP*-complete if* $c \geq 2$*.*

Due to Theorem 1.1, we cannot hope to generalize Theorem 1.2 to hold for any constant $\ell \geq 3$. Nevertheless, Theorem 1.1 leaves open the question if Theorem 1.2 is true for $\ell = \infty$ instead of only for $\ell = 2$. We show that the answer is no (assuming P $\neq$ NP).

Theorem 1.3. *For partitioned permutation games even of width* 2*, the problem of finding an optimal solution that is weakly or strongly close to a given target allocation* x *is* NP*-hard.*

Our last theoretical result is a randomized XP algorithm with parameter n; derandomizing it requires solving the notorious Exact Perfect Matching problem [13] in polynomial time.

Theorem 1.4. *For a partitioned permutation game* (N, v) *on a directed graph* $G = (V, A)$*, the problem of finding an optimal solution that is weakly or strongly close to a given target allocation* x *can be solved by a randomized algorithm in* $O(|A|^{O(n)})$ *time.*

We now turn to our simulations. Due to Theorem 1.3 and because the algorithm in Theorem 1.4 is not practical, we formulate the problems of computing a weakly or strongly close optimal solution as integer linear programs, and use an ILP solver. We still exploit the fact that for $\ell = \infty$ (Theorem 1.1) we can find optimal solutions and values $v(S)$ in polynomial time. In this way we can perform simulations for IKEPs up to *ten* countries, so more than the four countries in the simulations for $\ell = 3$ [5, 12], but less than the fifteen countries in the simulations for $\ell = 2$ [3].

For the initial allocations we use benefit value, contribution value, Banzhaf value, Shapley value, and nucleolus. Our simulations show, just like those for [3], that a credit system using strongly close optimal solutions makes an IKEP the most balanced without decreasing the overall number of transplants. Our simulations indicate that the Banzhaf value yields the best results: on average, a deviation of up to 0.90% from the target allocation. Moreover, moving from $\ell = 2$ to $\ell = \infty$ yields on average 46% more kidney transplants, but cycles may be very large, in particular in the starting round.

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