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Review Article

A comprehensive review of fiber-reinforced topology optimization for advanced polymer composites produced by automated manufacturing

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A B S T R A C T

This review paper focuses on Fiber-Reinforced Topology Optimization (FRTO) methods for automated manufacturing techniques, addressing topology and morphology optimization. Accordingly, the review introduces the main TO techniques and the common reinforcement path design strategies using concurrent and sequential optimization approaches. Furthermore, this paper examines the potential transformation of the conventional role of TO algorithms in structural optimization by integrating Artificial Intelligence (AI) into the optimization process [1]. We collected and categorized the most relevant papers from the past decade in the field of FRTO; comparisons were made based on appropriate metrics, including algorithm types, effectiveness, and validation environment. We emphasize practical considerations such as manufacturing constraints and algorithmic efficiency, addressing real-world usability aspects [2]. The analysis underscores the necessity for universally applicable benchmark methods and standardization to facilitate direct comparisons among various methodologies [3]. The main conclusions of the paper highlight the emerging trends in research, the potential of fiber-reinforced polymer composites designed by FRTO, the challenges facing the field, and the efficiency improvements and synergy with AI, indicating an evolving role for TO in structural optimization.

1. Introduction

In recent decades, minimizing material consumption and energy usage while at the same time meeting increasing volume demands has come to the forefront. Parts produced using traditional manufacturing methods and designed manually tend to be oversized, often due to the structural checks conducted after the design phase and the limitations of subtractive manufacturing processes. Since its invention in the mid-20th century, *additive manufacturing* (AM) has evolved a great deal, and now it is a widely used technology, which makes it possible to manufacture complex geometries. Simultaneously, *topology optimization* (TO) emerged as a distinct field, revolutionizing design methods and enhancing the utilization of structural materials. As industries strive to reduce material consumption and increase energy efficiency, the intersection of AM and TO offers a promising avenue for optimizing structural designs and refining manufacturing processes. Moreover, the exploration of automated manufacturing techniques reflects a broader trend toward integrating advanced technologies into design practices. Thus, the topics and methods chosen in this review study arise from their relevance in addressing contemporary challenges and advancing methodologies within composite manufacturing. Through our review, we focus on the evolving landscape of composite manufacturing, where innovation and efficiency converge to shape the future of structural design.

Automated manufacturing techniques in the composite industry involve processes where machinery and robotic systems autonomously perform manufacturing tasks or operate with minimal human intervention. These methods commonly include *automated fiber placement* (AFP), *automated tape placement* (ATP), and other automated processes for depositing composite materials in desired configurations. Hence, these methods fall in a subcategory of AM

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List of abbreviations

AFP	automated fiber placement
AI	artificial intelligence
AM	additive manufacturing
ATP	automated tape placement
BESO	bi-directional evolutionary structural optimization
CFAO	continuous fiber angle optimization
CFRP	continuous fiber-reinforced polymer
CM	conventional manufacturing
DCMO	discrete-continuous material orientation
DMO	discrete material orientation
EQS	equally-spaced
ESO	evolutionary structural optimization
FEA	finite element analysis
FMO	free material optimization
FRTO	fiber-reinforced topology optimization
FW	filament winding
GA	genetic algorithm
GPTO	geometry projection topology optimization
LSM	level-set method
MAT	medial axis extraction
MBB	Messerschmidt–Bölkow–Blohm
MEX	material extrusion
ML	machine learning
MMV	moving morphable void
NN	neural network
NURBS	non-uniform rational B-splines
PBF	powder bed fusion
SIMP	solid isotropic material with penalization

techniques. The complexity of automated manufacturing varies widely. Usually, 3D printers can only reinforce layers in-plane. In contrast, the more advanced AFP and ATP techniques employ multi-axis devices to lay fibers on complex shapes in curvilinear paths, extending beyond planar surfaces.

While AM methods have been present for years, they have only been widely adopted for structural components more recently, driven by advancements such as *powder bed fusion* (PBF) for metals and innovative techniques for polymers like *continuous fiber–reinforced 3D printing* (MEX CFRP) and other automated methods such as AFP and ATP. These advancements increased the importance of AM processes in producing structural elements and accelerated the development of technology, design methods, and materials [4].

However, transferring knowledge and design principles from *conventionally manufactured* (CM) composites to fiber-reinforced AM composites presents a significant challenge. The unique properties of composites, relying on strategically placed reinforcing fibers, require intricate structural design and analysis. The manufacturing of CM composites is based on ply-stacking methods, limiting fiber orientations within the lamina. In contrast, AM techniques allow continuously variable fiber laying paths, giving designers significant design freedom. To exploit the potential of this design freedom, algorithmic design methods are at the designers' disposal, allowing both optimal topology and reinforcement fiber placement.

Traditionally, part design involves separate iterative steps of strength calculations, usually involving *finite element analysis* (FEA). Algorithmic design methods such as TO integrate these steps. The technique introduced by Bendsøe and Kikuchi [5] revolutionized optimal topology formulation based on iterative structural analysis. This approach has evolved to encompass non-isotropic, multiphase optimization methods for various materials and lattice structures, marking a significant development in TO and overall structural design [6].

In the literature, TO methods and manufacturing technology are frequently treated as separate topics: Cheng et al. [7] presented the opportunities and main characteristics of continuous fiber–reinforced AM composites, but the creation of cellular microstructures marked the boundary of their work. Similarly, Rimkus et al. [8] focused on comparing the performance achievable with fiber-reinforced AM composites to that of CM composites, discussing the material and manufacturing aspects. Parmar et al. [9] concentrated on automated large-scale composite manufacturing methods and their limitations. However, they did not address the structural advancements introduced by AM that TO could exploit. Articles addressing multiscale optimization typically focus on isotropic materials and the related manufacturing technologies: Zhu et al. [10] summarized the state of the art isotropic TO, also addressing microstructure optimization and 3D printing related global orientation issues, albeit excluding fiber reinforcement as a local, microstructural orientation problem. Wu et al. [11] similarly focused on isotropic issues in their analysis, elaborating extensively on the methodology of microstructure optimization but neglecting the topic of material anisotropy.

The primary deficiency of publications addressing manufacturing technology and TO methods together typically lies in insufficiently comprehensive analysis. Wong. et al. [12], Tian et al. [13], Liu et al. [14], Hu [15], Schmidt et al. [16], and Yu et al. [4] all provided an excellent analysis of FRTO papers. However, these studies needed comprehensive comparisons of the algorithms behind the optimizations and other important metrics, such as efficiency or manufacturability considerations.

To address this issue, this article provides a comprehensive overview of optimization methods relevant to automated manufacturing techniques in the advanced composite industry. Various categories of TO methods are briefly introduced, with insights into primary methods such as density–based, homogenization–based, level-set, B-spline–based, free energy–based, and feature mapping–based TO. Additionally, reinforcement optimization methods focusing on morphology are discussed, such as oriented vector field–based reinforcement path design, curve fitting methods, and integrated reinforcement design methods. Concurrent and sequential TO and reinforcement path design are further explored, along with practical considerations, including manufacturing limitations, optimization algorithm efficiency, and objective comparison methodologies. Lastly, the future trend of integrating AI into TO is highlighted, and the paper concludes with a summary of key findings and future directions for research.

The primary value of this review article is an extensive and comprehensive summary of recent years' papers on non-isotropic TO and FRTO. The main objective of this review article is to compile a comprehensive collection of essential literature, assisting researchers in the field regarding the current state of the art in TO methods, potentially usable in the advanced composite industry, thus promote the shift of this novel field from academic to industrial use. However, despite the authors' best efforts, the limitation of the presented paper is the potential exclusion of recently published papers or emerging methodologies due to the review's timeframe and the rapid evolution of AI, leading to the possible introduction of new optimization techniques.

2. Research methodology

This comprehensive review systematically investigates FRTO methods tailored to advanced polymer composites manufactured through automated techniques. We conducted a thorough search of academic databases such as Scopus and Web of Science to identify relevant studies published mainly over the past decade (2014–2024). The keywords of the search included "topology optimization," "concurrent topology optimization," "reinforcement optimization," "automated manufacturing," "continuous fiber reinforcement," "path planning," "neural network," and "artificial intelligence." Papers were included based on their relevance to FRTO methods specifically applied to advanced polymer composites in the context of automated manufacturing techniques. We considered studies focusing on TO and reinforcement path design strategies, concurrent and sequential optimization approaches, and integration of AI. Relevant data from selected papers were systematically extracted and categorized. This includes details on TO techniques, reinforcement path design strategies, algorithm types, effectiveness metrics, validation environments, and practical considerations such as manufacturing constraints and algorithmic efficiency. The data were analyzed to identify trends, patterns, and emerging insights in FRTO methodologies for advanced polymer composites in automated manufacturing settings. Special attention was paid to emerging research directions, challenges facing the field, and potential efficiency improvements through AI integration. We critically evaluated papers to assess their methodological rigor, significance, and relevance to objectives of this review. Any discrepancies or limitations in the existing literature were identified and discussed. We then synthesized the findings to provide a comprehensive overview of FRTO methods, their applicability in the automated manufacturing of advanced polymer composites, and the evolving role of TO algorithms in structural optimization. This systematic approach ensures the review's credibility, rigor, and relevance, providing valuable insights for researchers, practitioners, and stakeholders in the field of advanced composite materials and automated manufacturing processes.

3. Topology optimization–compatible automated composite manufacturing technologies

CM composites are reaching their limit as the demand for volume and performance continues to grow. However, automated fiber laying methods began to develop at the beginning of the 21st century to address these issues. With the emergence and widespread use of universal robotic arms and AM methods, the possibility of full automation was introduced, making the Industry 4.0 framework feasible in composite manufacturing as well. The rise of advanced manufacturing technology has prompted the exploration of novel design methods to enhance its overall efficiency [17–19].Optimization methods and advanced strategies for designing reinforcement paths have always been a popular research area in the field of composites. However, the utilization of inherently designable anisotropy in composites has recently become significantly more effective with the above-mentioned advancements in AM technologies [20]. To illustrate the structural hierarchy in optimized parts, Fig. 1 shows how the strength-to-weight ratio of a simply loaded part's changes across various manufacturing technology and optimization levels. The red side of the figure shows the optimization of the reinforcement structure, while the blue side illustrates the influence of geometrical optimization. Fig. 1(a) shows CM technologies, whereas Fig. 1(b) and (c) present automated composite manufacturing methods. Fig. 1(e) shows a part manufactured from a conventional isotropic material produced by subtractive processes, and Fig. 1(d) displays a part that has undergone geometric optimization but lacks reinforcement.

Considering only the topology of the structure, isotropic optimization can be employed where metals in conjunction with PBF technologies are usually a popular choice [21,22]. In contrast, algorithms designed for non-isotropic optimization are paired with MEX on a desktop scale or AFP/ATP at an industrial level. *Filament Winding* (FW) is also a widely considered option. This is particularly relevant for highly anisotropic polymer composites, where the orientation of the reinforcing fibers must be considered during the design process. These techniques allow the precise positioning of fiber reinforcement, a prerequisite for implementing morphologically optimized reinforcement structures. In the aerospace industry, AFP, ATP, and FW are widely used, mainly to manufacture wings, fuselage elements, pressure vessels, and turbine covers that also serve structural functions [20,23,24].

Inspired by FW, AFP involves placing multiple pre-impregnated tows to form a band of material on a mold surface. On the other hand, ATP employs much wider unidirectional tapes for faster layup rates. While ATP is preferred for manufacturing large composite parts with relatively simple geometries, AFP is favored for producing complex aerostructures due to its ability to lay material on contoured surfaces. Collaborative robots have also emerged as a promising solution for reducing production time and costs by enabling rapid manufacturing of aircraft components through multiaxial AFP/ATP robots [9].

Manufacturing methods can be primarily ranked based on their productivity, complexity, and integrity, as well as the characteristic fiber content of the composite structure they can produce. In this classification, MEX systems occupy the lowest tier since the output is directly influenced by the diameter of the filament passing through the extruder. Additionally, MEX systems can only reinforce in-plane, significantly impacting design freedom (Fig. 2(c)). Fiber content is typically around 30%, mainly due to the two-step impregnation process (Fig. 2(a)), and the porosity of the components is relatively high compared to CM composites due to the lack of a consolidation step (typically ~10%). In contrast, robotic systems can follow curved, complex paths in all directions and, with ATP technologies, significantly enhance their output volume compared to systems working with prismatic filaments or tows. Another advantage of robotic methods is their ability to locally compact the deposited structure using a roller at the end of the nozzle (Fig. 2(b)). Moreover, their fiber content can compete with that of CM composites since the system directly lays the pre-impregnated tape/tow in a single technological step. FW technologies encompass all the advantages of robotic technologies, with the limitation that they can only produce axisymmetric components, making them irrelevant in terms of TO for this article.

4. Introduction of topology optimization

Topology optimization is a robust numerical procedure by which the distribution of material can be optimized in a predefined design space if the loads and the mechanical properties of the material are known [26]. This method makes it possible to design lighter and more efficient structures. The combination of TO and AM has opened new possibilities in designing and manufacturing complex structures. AM techniques, such as 3D printing, are well suited for creating structures with optimized geometry, which could not be produced economically with CM methods [27].

The origins of TO date back to the mid-20th century, when researchers first applied mathematical methods to the geometric optimization of bridges and other structures [6]. The practical applicability of the technique was made possible by the advent of

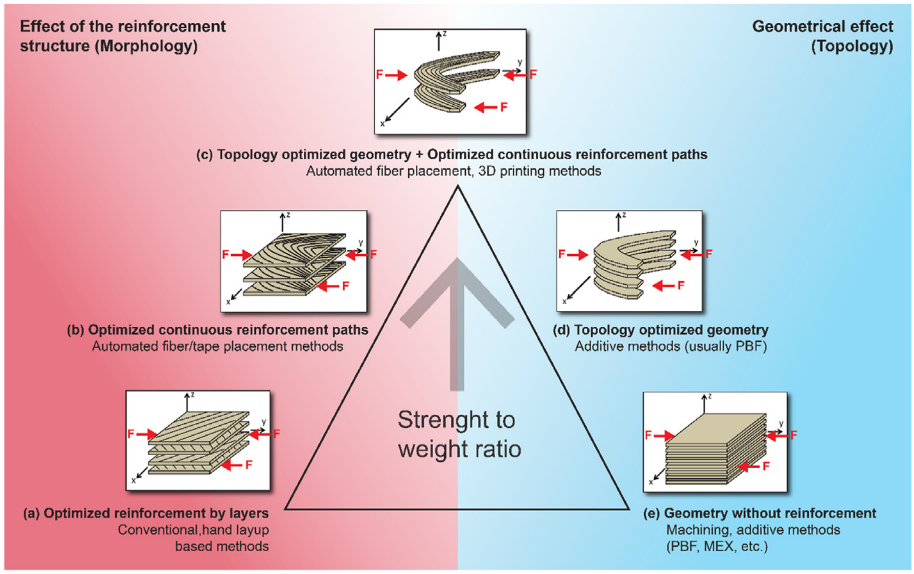


Fig. 1. Hierarchical graph of structural performance with the use of different design and manufacturing strategies showing changes in strength-to-weight ratio as a result of various manufacturing technologies and optimization levels. The left side (red) shows reinforcement structure optimization, while the right side (blue) shows geometrical optimization. (a)–(c) showcases different manufacturing methods, (e) displays a part made from conventional isotropic material via subtractive processes, and (d) shows a geometrically optimized part without reinforcement.

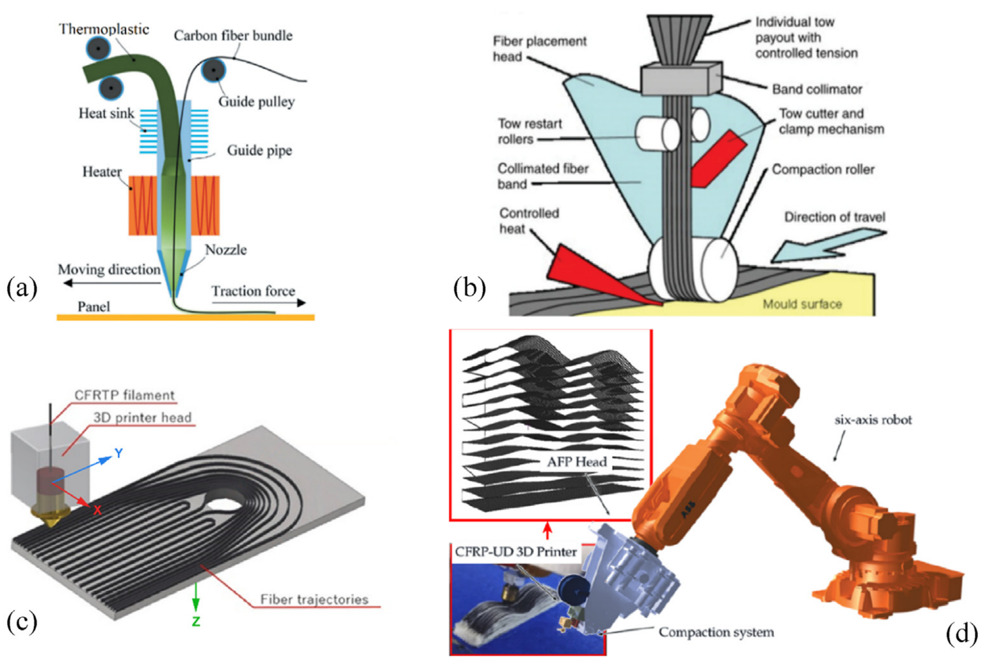


Fig. 2. Schematic representation of CFRP MEX (a, c) and AFP/ATP (b, d) systems. Due to the usually employed co-extrusion mechanism in the CFRP MEX system (a) [9], the achievable fiber content is lower compared to AFP/ATP systems working with prepreg tapes or tows (b) [9]. In the case of AFP/ATP systems, spatial freedom is higher thanks to the increased number of degrees of freedom (DOFs) of the robotic arms employed (d) [9,13], while MEX systems can only place reinforcement in a planar configuration (c) [25].

finite element analysis in the 1960s, and it gained new momentum in the past decade with the spread of AM processes.

Bendsøe and Kikuchi in 1988 developed the micromechanical homogenization method [5,6], which is followed by the density–based TO method called *Solid Isotropic Material with Penalization* (SIMP) by Bendsøe [28] in 1989. The SIMP method became the most popular and widely adapted algorithm for emerging TO algorithms. Parallel to the density and homogenization–based methods, Osher and Sethian developed the so-called *Level-set method* (LSM) [29] in 1988. Later, the first *Evolutionary Structural Optimization* (ESO) algorithms appeared in the 1990s, first published by Mattheck and Burkhardt in 1990 [26,30]. The first orthotropic optimization scheme was published by Bendsøe and Sigmund in 1999, called *Solid Orthotropic Method with Penalization* (SOMP) [31,32]. In the same year, *Bidirectional Evolutionary Structural Optimization* (BESO) was introduced by Xie and Yang [33], and the thermodynamic Free energy method was also introduced by Bendsøe and Sigmund in 1999 [31]. Multi-material and multiphase optimization methods appeared around the turn of the millennium [34,35], followed by algorithms using stochastic description [36] in the early 2010s, while *Neural Networks* (NN) and *Genetic Algorithms* (GA) prevailed in TO starting from the late 2010s [37,38].

The formulation of the TO problem can take many forms depending on the type of algorithm and the goal of the optimization, ranging from maximizing structural stiffness to optimizing specific eigenfrequencies [39], maximizing heat transfer capability [40], or maximizing the buckling load of a given structure [41]. According to Sigmund [26], the general TO problem is finding the spatial distribution of the material in a space where the objective function **F** has a minimum, and for which the following volumetric constraint is true: $G_0 \leq 0$, i = 1 … M. The material distribution is characterized by the density variable $\rho(x)$, which can take on values of either 0 (indicating void) or 1 (representing solid material) over the entire volume range Ω. The optimization problem can be mathematically described with equation (1):

$$\left.\begin{array}{l} \underset{\rho}{min} : F = F(\boldsymbol{u}(\rho), \rho) = \int_{\Omega} f(\boldsymbol{u}(\rho), \rho) dV \\ subject\ to : G_0(\rho) = \int_{\Omega} \rho(\boldsymbol{x}) dV - V_0 \leq 0 \\ : G_i(\boldsymbol{u}(\rho), \rho) \leq 0, j = 1, \ldots, M \\ : \rho(\boldsymbol{x}) = 0\ or\ 1, \forall \boldsymbol{x} \in \Omega \end{array}\right\} \quad (1)$$

where the state-field **u** complies with the linear or nonlinear equation of state and where one assumes that when the goal is maximizing the structural stiffness or minimizing compliance, the function can be calculated as the integral of the f (**u** (ρ), ρ) local function, which is also equivalent to the strain energy density. The extra parameter **M** indicated in the equation represents the limitations of the design criteria appearing in real-life applications, such as manufacturing constraints or load uncertainties [26].

So far, TO methods have played a prominent role in the reinforcement design of aircraft wings, medical implants, suspension, and chassis parts. With the continued increase in available computing power and sustainability increasingly included in the design requirements, the popularity of TO is expected to continue growing in the coming years [42–44].

5. Non-isotropic topology optimization

The formulation of the TO problem depends on the elastic characteristics of the material or microstructure used, therefore it can be an isotropic or non-isotropic method in a single material case. Reviewing the history of TO methods, we can conclude that isotropic TO methods appeared in the 1980s [28]. In comparison, non-isotropic optimization became a popular research topic in the 1990s with the development of composite materials [45]. The fundamental contrast between isotropic (Fig. 3(a, b)) and non-isotropic TO methods (Fig. 3(c)) is that the latter can not only optimally distribute the material within the design domain but also adjust its orientation [6]. The choice of material for a design fundamentally dictates the category of algorithms capable of producing the optimal outcome. Metals typically exhibit isotropic properties, while polymers, undergoing orientation during manufacturing, tend to become anisotropic materials, resulting in anisotropic end geometries. This phenomenon is also observed in short fiber and continuous fiber–reinforced polymer composites, commonly utilized in additive technologies. These materials often have superior mechanical properties in specific directions, therefore orientation needs to be considered during optimization (Fig. 3(c)).

Non-isotropic topology optimization offers a distinct advantage over isotropic algorithms by taking into account material orientation, particularly emphasizing the superior direction. This characteristic makes non-isotropic optimization methods particularly well-suited for maximizing the performance of fiber-reinforced composites.

Optimizing orthotropic reinforcement structures involves thoroughly examining length scales and distinguishing between micro and macro levels. Methods like homogenization–based TO and specific cases of multi-material optimization operate on a microscale, refining material distribution locally (Fig. 4). These methods provide a natural approach to achieving continuous paths in the structure. Homogenization–based topology optimization methods yield structures resembling reinforcement (Fig. 4(c)). However, they are unsuitable for optimizing reinforcement structures due to their approach to interpreting density variables. Higher values in the density variable correspond to thicker trusses rather than denser reinforcing paths, posing challenges for manufacturing with fibrous reinforcements of constant diameter. Consequently, additional postprocessing steps are necessary to ensure manufacturability. The referenced literature contains further details about micromechanical reinforcement optimization [47,48].

6. Fiber-reinforced topology optimization

Fiber-reinforced Topology Optimization (FRTO) combines topology and morphology optimization within a single structure. Fig. 5 illustrates the initial cantilever problem (a) and a representative result of FRTO (b), where both the distributed material and the reinforcement contribute to maximizing structural stiffness.

Fig. 6 shows a flowchart summarizing important FRTO methods. The main steps are marked with numbers, and sub-processes are labeled with letters. The chart explains different ways to implement each step. The process involves two primary optimization phases; The first is topology optimization (1), and the second is morphology optimization (2). Each phase employs various algorithms and

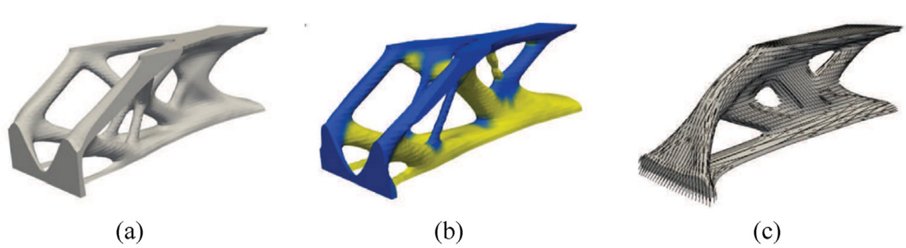


Fig. 3. Isotropic (a), isotropic multi-material (b), and anisotropic fiber-reinforced (c) structures generated with the same loading conditions. The resulting structures are slightly different in every case [46].

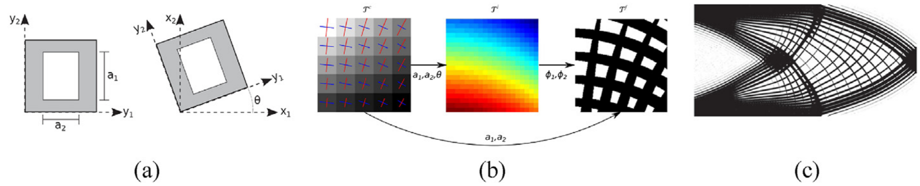


Fig. 4. Steps of Homogenization–based TO: Formation of the reinforcement structure involves varying the micro-scale unit cell edge size of a_1, a_2, and cell orientation θ (a), followed by the projection of density variables (b), where the lengths of red and blue lines locally represent a_1 and a_2. Finally, the macrostructure of reinforcement is formed (c) [47].

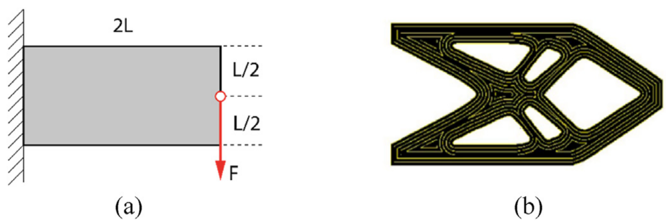


Fig. 5. Initial step of FRTO where loads and boundary conditions are defined (a) and the final optimized, fiber-reinforced structure (b). It is noticeable how the reinforcement goes alongside the trusses of the cantilever beam [49].

methods. For topology, options include density–based, homogenization–based, level-set function–based, B-spline–based, and Free energy–based methods. In the morphology phase, there are two options. The first is a two-step method, where an additional task involves curve fitting after determining discrete material orientation. The second is an integrated method forming the reinforcement paths based on either the geometry itself or on concentric level-set contours.

Determining reinforcement orientation can occur simultaneously (concurrent process) with optimizing topology (Fig. 6: Path of the red arrow). Alternatively, it can be a separate step, a sequential process (Fig. 6: Path of the black arrow). In the sequential approach, part morphology is determined separately, typically based on the inner stress states (see Section 7. for more details). Filters are often applied between these main phases to ensure smooth transitions and filter out numerical errors (see Section 6.2.1 for more information). These methods will be presented in more detail in the upcoming sections.

6.1. Introduction of the primary topology optimization methods

TO algorithms are crucial in shaping resultant geometry and other non-structural properties, including convergence and efficiency. This section categorizes and briefly introduces TO algorithms, offering insight into the opportunities and core concepts behind methods developed in the (FR)TO literature.

6.1.1. Density–based topology optimization

The first important method under consideration is a density–based TO method called the *Solid Isotropic Material with Penalization* (SIMP) method. The principle of the SIMP method is that during optimization, the original material properties are replaced by a density variable, which can take a value from 0 to 1, where 0 denotes a void, and 1 signifies solid material. During optimization, the algorithm minimizes the amount of material depending on performance criteria. For further information, see Refs. [5,6].

Introducing orthotropic material properties to broaden the applicability of the SIMP method gives rise to the *Solid Orthotropic Material with Penalization* (SOMP). This technique incorporates the optimization of local material orientation by rotating its elastic tensor, rendering it especially valuable in optimizing fiber-

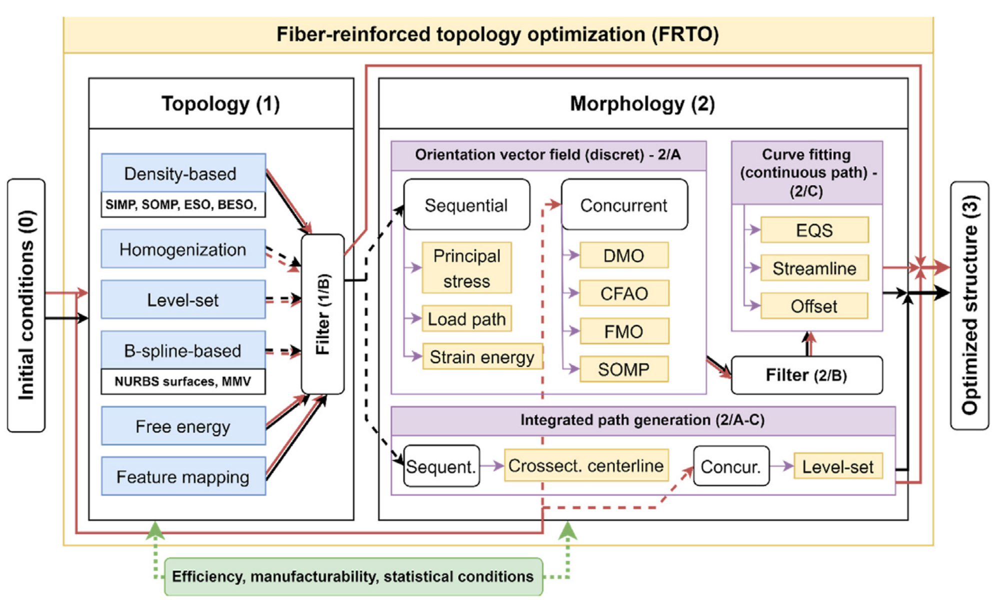


Fig. 6. Topology Optimization of Fiber-Reinforced Composites Flowchart. Solid lines denote essential steps, while dashed lines signify optional steps. Black arrows indicate sequential optimization steps, while red arrows show concurrent optimization steps.

reinforced composites. Further details on this method can be found in the referenced literature [31,32,50].

The *Evolutionary Structural Optimization* (ESO) method is noteworthy among density–based algorithms. It begins with a randomly generated geometry, systematically eliminates the least utilized elements via fitness functions, and iteratively enhances the design until optimal outcomes are attained. Its counterpart, the *Bidirectional Evolutionary Structural Optimization* (BESO) method, differs in its ability to remove material from areas with low stress and add material to those subjected to high stress. For further information, see Refs. [26,30,33].

6.1.2. Homogenization–based topology optimization

Homogenization–based methods break down the macrostructure into a fine, uniformly sized cellular structure. Local density control is attained by adjusting the geometric scaling of the microstructure, independently modifying the void size parameters of the cell in Cartesian directions. This process generates orthotropic material characteristics at the microstructural level. The relative density and orientation of the unit cell dictate the material's local behavior, which can be tailored at each spatial point based on the objective function, ultimately resulting in the optimal macrostructure. Homogenization–based methods were the first to address orientation problems beyond the optimal distribution of material in a spatial domain, laying the foundation for the emergence of non-isotropic optimization methods. For further information, see Refs. [47,51,52].

6.1.3. Level-set topology optimization

One of the most significant TO algorithms is the *Level-set method* (LSM), widely used in structural optimization problems, fluid mechanics, image processing, and materials science. This method employs a level-set function to describe the optimized domain topologically, where positive values of the level-sets correspond to material regions and negative values to voids. At the same time, zero levels represent the boundaries of the structure. The level-set function represents the three-dimensional coordinates with scalar functions, and the zero level-set evolves in accordance with the Hamilton-Jacobi equation. The imaginary velocity-field depends on the structure's shape and/or phase derivatives. The surface of the geometry is captured implicitly; therefore, the local manipulation of level-sets is one of the biggest challenges in the optimization process. For further information, see Refs. [29,39].

6.1.4. B-spline–based topology optimization

Introducing the primary B-spline–based methods, we begin with *Moving Morphable Voids* (MMV) optimization. This TO method effectively utilizes B-spline curves to represent and manipulate the geometry in the presence of intersecting voids, offering a practical approach for TO, particularly in 2D cases. The method addresses challenges associated with void intersections and provides a means to describe the resulting geometry explicitly. During the process, these morphable voids can unite but not nucleate. This poses some limitations to the MMV method. Another important method to mention is the NURBS–based algorithm, which uses *Non-Uniform Rational B-Splines* (NURBS) to represent and manipulate geometric shapes. NURBS provide a flexible and smooth parametric representation, enabling precise control over the spatial distribution of material within a design domain. This approach combines the benefits of NURBS–based geometric modeling with the efficiency of TO, offering a powerful tool for advanced engineering design. Both B-spline–based TO methods use parametric design variables such as control points and local weights to manipulate the geometry explicitly. For further information, see Refs. [53,54].

6.1.5. Free energy–based topology optimization

Among the most essential TO methods is the thermodynamic approach, employing the free energy method, which is worth mentioning. This technique involves minimizing the potential energy of the structure throughout optimization, guided by chosen objective functions. The system's free energy is a combination of volumetric and surface energy. Volumetric energy represents the potential energy of the bulk material, while surface energy represents the energy required to form new surfaces between different material phases. The algorithm balances these two terms to satisfy the objective function. For further information, see Refs. [31,46].

6.1.6. Feature mapping–based topology optimization

Feature mapping–based topology optimization involves using mappings to transform design spaces into density fields, enabling the projection of geometric primitive designs onto fixed analysis grids. Unlike traditional density–based TO methods, this approach offers flexibility in mapping techniques. Feature mapping methods are known for their adaptability and robustness, making them valuable in optimizing structures. The *Geometry Projection Topology Optimization* (GPTO) method stands out within this framework. GPTO uses feature mapping techniques to construct optimized structures using discrete elements of various geometric primitives. By leveraging a differentiable mapping, GPTO transforms design spaces into density fields, allowing for the projection of diverse geometric primitive designs onto fixed analysis grids. GPTO's versatility extends to accommodating the removal of elements, handling overlapping cases, and supporting multiple length scales. This makes it valuable for achieving manufacturability while avoiding complexities associated with free-form shapes. Additionally, GPTO enables modifications, penalization, and inclusion of predefined attributes tailored to different shapes, enhancing its applicability across various design scenarios. For further information, see Refs. [129,130].

6.2. Reinforcement optimization methods (morphology)

In reinforcement optimization, there are two principal methodologies to follow. One is the class of integrated methods, while the second is the class of (discrete) vector field–based methods. Integrated methods naturally result in continuous reinforcement paths, while orientation field–based methods necessitate additional refining steps. These involve converting vectors into continuous paths using curve-fitting methods or filters to ensure continuity across micro and macro levels. The significance of continuous paths cannot be overstated, especially for manufacturability considerations. Structures optimized by discrete orientation methods, lacking continuous paths, may be challenging or impossible to construct, reducing them to a theoretical realm rather than practical useability.

6.2.1. Oriented vector field–based reinforcement path design

In non-isotropic optimization, the usual concern is the local orientation of the elastic tensor of the homogenized material at each point in space (Fig. 7(a)), determining the path for fiber laying (Fig. 7(c)). Before curve fitting, the discrete vector field is typically "cleaned" with the use of a suitable filter (Fig. 7(b)). This process removes numerical errors and abrupt changes in direction from the vector field, usually resulting in numerical singularities, preventing the creation of problematic or unmanufacturable structures. The filters essentially interpolate the selected element based on vectors beyond the immediate vicinity of the given element, ensuring a more continuous integration into the system [49].

In fiber orientation optimization, the primary input metrics include the strain energy field, principal stress field, and load path

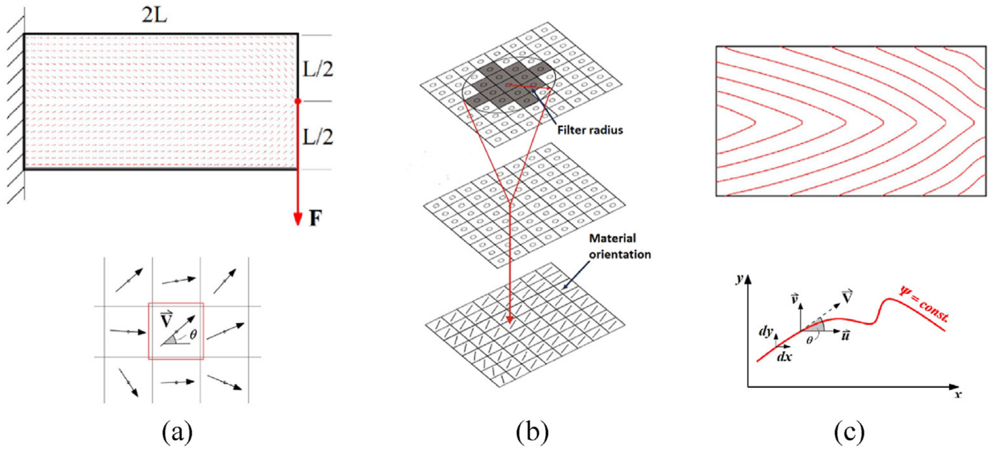


Fig. 7. Illustration depicting the sequential steps in determining the path of optimized reinforcement. This includes the initial problem setup and the formation of the orientation vector field in response to specified conditions (a), the application of a filter to control the vector field (b), and the subsequent continuous curve fitting process (c) [49,55].

directions. These metrics guide the determination of direction vectors across the design domain through various methods and considerations [56]. Important methods in this regard include *Continuous Fiber Angle Orientation* (CFAO), *Free Material Optimization* (FMO) and *Discrete Material Optimization* (DMO). CFAO employs techniques like the *Method of Moving Asymptotes* (MMA) to optimize material orientation in each element considered a continuous design variable [59]. However, the direct use of fiber angles as design variables in CFAO often led to convergence issues and sensitivity to initial guesses, resulting in multiple local minima [57]. To mitigate the problem of numerous local minima, the FMO method introduced design variables as elements of the material stiffness tensor [58]. While theoretically optimal, FMO often produced physically unfeasible structures. Subsequently, the DMO method was proposed to address local minima and angle identification challenges in CFAO. The Discrete DMO method selects discrete candidate angles from prescribed sets and calculates the effective elastic tensor via a weighted summation, utilizing these weights as design variables. This approach offers a more robust optimization strategy, particularly advantageous for non-convex objective functions related to density and orientation variables. However, it is essential to note that DMO may not be optimal for scenarios requiring a continuous fiber distribution. While DMO methods generally exhibit improved convergence and straightforwardness, a potential drawback exists as they might lead to discontinuous fibers [59].

Liu et al. [60] introduced a SOMP–based TO method, where the optimal reinforcement path is determined by projecting the spatial wave function created from the stress field. Their approach involves separating the compression and tension components of the principal stresses, resulting in two reinforcement patterns placed alternately in the layers of the final structure (Fig. 8). Yang et al. [61] adopted a similar approach, further developing the SOMP method and adjusting it to distinguish printable layers by the principal stresses.

Li et al. [62] devised a BESO–based TO method suitable for more than two materials. Their fundamental concept involves segregating materials that are more resistant to tension and compression and categorizing them into distinct product parts. Using their approach, they achieved a remarkable 76.3% increase in stiffness compared to single-material optimization. Applying the method to general bridge geometry design, they demonstrated its effectiveness in utilizing materials with distinct tension and compression behaviors, particularly advantageous in suspension bridges compared to traditional arched systems.

In addition to methods based on principal stresses, algorithms that consider load paths and strain energy distributions are also popular in structural analysis and design. These approaches capitalize on the characteristics of fibers used in composites, which are typically shear-weak materials. Such materials exhibit high stiffness and strength parallel to their fibers and crystal orientation, making them particularly well-suited for efficient load transfer along the same direction as the principal stress vectors [63,64]. Brown et al. [32] investigated the impact of shear modulus on the stiffness of structures generated in short fiber–reinforced TO with materials exhibiting different orthotropy (such as carbon fiber and silica-reinforced epoxy). They observed that varying modulus values over a wide range did not significantly alter the resulting geometry's stiffness, highlighting the structure's independence from shear properties. A similar conclusion was reached by Fedulov et al. [65] in their research, where they noted that the generated shape often aligns with the longitudinal axis of the optimal reinforcement path. This suggests that geometry and fiber reinforcement can be aligned independently during optimization.

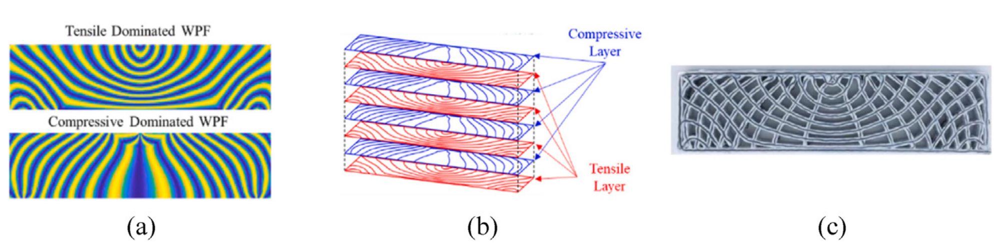


Fig. 8. Separating the principal stresses into compressive and tensile components. Compressive and tensile wave functions shaped according to the principal stresses (a), the corresponding alternating reinforcement paths generated (b), and the resulting final reinforced structure [60].

6.2.2. Curve fitting methods

Three main techniques, *Equally Spaced* (EQS), *Offset*, and *Streamline*, are frequently employed to fit a continuous curve to an orientation vector field. Each method offers distinct advantages and drawbacks in optimizing fiber paths within the solid domain. The Offset method creates fiber paths parallel to the boundary of the solid domain, allowing for efficient and robust optimization, albeit with the drawback of potential acute corners. Conversely, the EQS method generates continuous fiber paths at equally spaced distances along the boundary, providing smoother trajectories and load distribution but with limitations on complex geometries. Finally, the Streamline method produces smooth paths based on the optimized fiber orientation vector field, although it is computationally more expensive and may require post-processing to address discontinuities caused by intersecting voids [49]. Fig. 9 illustrates these three methods. The Offset method is commonly employed when it is challenging to define reinforcement paths explicitly. This method can determine the reinforcement structure on geometric bases, layer by layer, using the contour of the subsequent cross-section. It follows the theory that the most efficient reinforcement paths should align with the principal stress vectors by definition. Commercially available slicing software like Eiger and Aura often incorporates this reinforcing strategy. For further information, see Ref. [66].

Fernandes et al. [72] used SOMP and Level-set methods to compare these methods, assessing the performance of the leading fiber laying strategies (EQS, Offset method). They focused on exploring the differences between the theoretical and practical optimum, considering technological constraints during optimization. The results indicated that the EQS method proved to be a more efficient fiber-laying method without the constraints of the manufacturing process. However, when manufacturing technology limitations were considered, the Offset method resulted in a better-performing structure.

6.2.3. Integrated reinforcement design methods

Integrated methods such as the *Medial Axis Extraction* (MAT) reinforcement path design method [67] are built on recognizing that fiber reinforcement aligned with the principal stress directions provides the most efficient reinforcement structure. Therefore, there is no need to establish a local orientation vector field. The reinforcement structure is created based solely on the final geometry, following the local centerlines of the structure. Fig. 10 shows a schematic diagram of MAT.

Another geometry–based method introduced by Catapano et al. [68] takes advantage of the fact that particular slicers can lay fibers based on the geometrical contours of the subsequent layers. Therefore, there is no need to define reinforcement a priori at all; the structure's shape can control the reinforcement path implemented in this step in the manufacturing process.

Vector-field–based and integrated methods mainly differ in how they calculate the reinforcement paths in the structure. In vector-field–based methods, the discrete orientation vector field is created concurrently with the topology of the part. It needs to be connected by filters to finally transform into continuous lines representing the reinforcement paths. On the other hand, geometry–based methods form the reinforcement structure in the post-processing stage after the topology is settled. Therefore, geometric–based curve fitting is more straightforward and less computationally demanding than vector-field–based methods as they handle orientation as an optimization variable. However, vector-field–based methods offer more accurate results.

7. Concurrent and sequential topology optimization and reinforcement path design

TO methods can be categorized based on their complexity, determined by their ability to handle multiple variables simultaneously and whether these variables are optimized interdependently or independently. These variables in FRTO are the spatial position and orientation of the material or reinforcement. The orientation of the reinforcement can be defined in a process together with TO, in which case, it is called a concurrent method, or in a separate step when it is called a sequential method. In the sequential case, the two variables are optimized independently, while concurrent algorithms simultaneously consider the interaction of the local orientation and optimal material distribution (Fig. 11) [32].

Remacle et al. [69] developed a TO algorithm that can optimize fiber direction and topology simultaneously and efficiently perform fiber direction filtering required for manufacturing. They compared the algorithm's simultaneous concurrent and sequential versions in different typical optimization cases. They concluded that the sequential algorithm is better for simple loads, while the concurrent method gives better results in more complex, multi-load environments. In a study, Brown et al. [32] conducted a comparison between the SIMP (sequential) and SOMP (concurrent) methods regarding computational demand and mechanical performance. The concurrent method generated a structure with higher stiffness, albeit with a marginal 2% difference. In comparison, the sequential method exhibited a 19% reduction in computational cost during the iteration process, further enhanced with a more efficient SIMP code.

Tables 1 and 2 display the list of publications in which the structure's topology and morphology (reinforcement) were considered in the optimization process. The tables indicate both optimization levels, and a separate column specifies the curve fitting and reinforcement path planning strategy. The publications have been divided into two tables based on whether they included the final curve-fitting step. The most representative figure in each case's has been included from the analyzed publication to aid visual interpretation.

In the initial section of the table (highlighted in yellow), we present the critical parameters of the algorithm. In the “TO algorithm” column, we specify the type of topology optimization algorithm (Section 6.1). The column “Fiber orientation algorithm” indicates the method for determining local orientation vectors, and the “Continuous path generation” column indicates the curve-

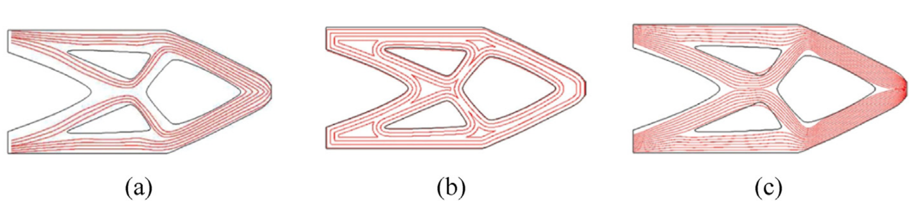


Fig. 9. Fiber reinforcement strategies optimized separately from the geometry in a cantilever TO problem. The Streamline method (a) uses streamlines tangent to the vector field, the Offset method (b) employs lines parallel to the boundary, and EQS (c) creates continuous paths parallel to the structure's boundaries [66].

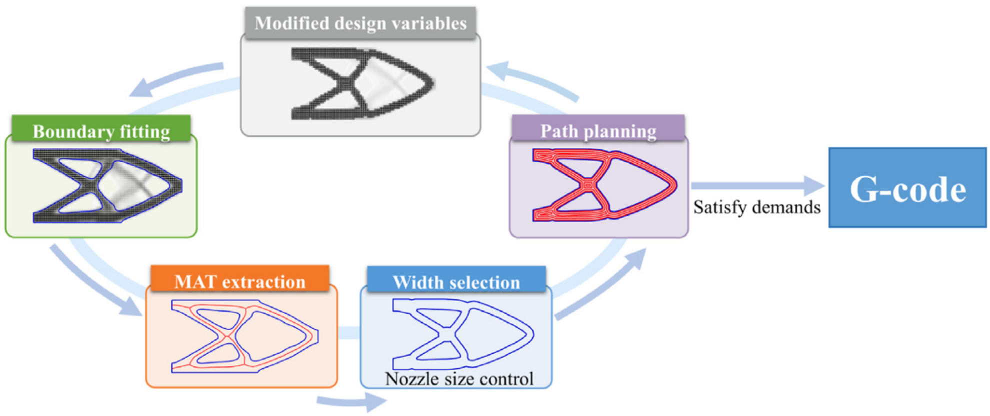


Fig. 10. The flowchart of the Medial Axis Extraction (MAT) reinforcement path design method. The medial axis is determined with the use of only the geometric boundaries of the structure. The process starts with boundary fitting and medial axis extraction. Then, adaptive width selection for each segmented central axis is performed, followed by offset path planning and the modification of design variables. These steps integrate the nozzle size control into the optimization process, leading to final optimization results after iterative adjustments [67].

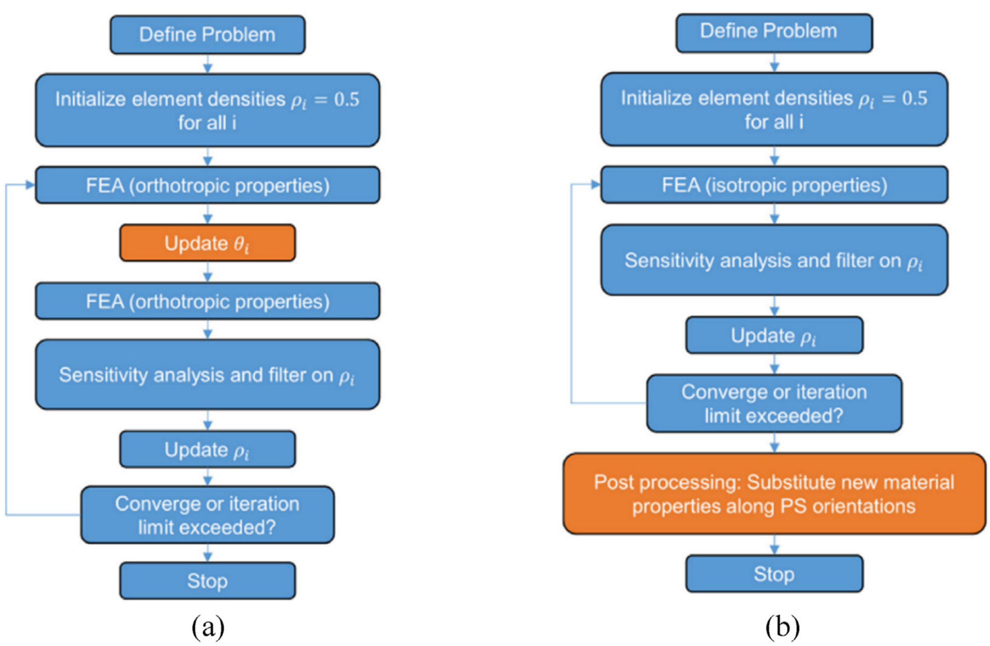


Fig. 11. Flowcharts of concurrent (a) and sequential (b) optimization methods. ρ_i represents local material density, and θ_i is local material orientation. Those two variables are optimized throughout the process [32].

fitting methodology (Section 6.2). In the "Algorithm architecture" column, we specify whether it is a concurrent or sequential method (Section 7). In the columns marked with blue, we highlight the characteristics of the generated geometry shown. The "Geometry dimensions" column clarifies whether the algorithm in the given publication was tested on planar geometry (2D), extruded profile (2.5D), or spatial (3D) geometry. In the "Benchmark environment" column, we discuss the typical optimization problems on which the algorithm was tested (1 = MBB; 2 = Cantilever beam; 3 = L-shear; 4 = Michell; 5 = Kirsch). Further details about these benchmark geometries are provided in Section 9.

In columns marked with green, we explore applicability parameters that were investigated, such as resource requirements ("Optimized for efficiency" column) and manufacturing limitation considerations ("Manufacturing constraints considered" column). In addition, validation of the reported results through tests is indicated in the "Validated with tests" column.

Tables 1 and 2 reveal insightful trends regarding various TO methods and fiber orientation algorithms. The data suggests that approximately 80% of the TO algorithms employed include variants of the density–based method, such as SIMP and SOMP. These methods are favored for their effectiveness in determining discrete direction vectors of reinforcement, mainly through principal stress-field–based algorithms like SOMP. Additionally, the analysis highlights a balanced use of fiber orientation algorithms, with DMO, CFAO, Strain energy, principal stress, SIMP, and SOMP being commonly employed. Researchers often opt for commercially available software for curve fitting, leading to a prevalence of more straightforward methods (Offset) compared to Streamline and EQS. Most notably, around 80% of the algorithms are concurrently

Table 1
Reviewed FRTO papers with continuous, manufacturable reinforcement paths [32,48,49,60,61,66,69–83].

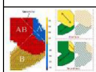
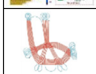
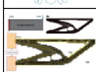
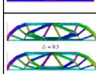
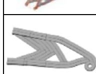
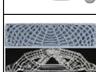
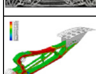
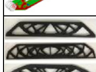
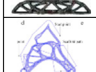
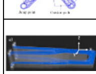
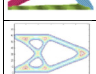
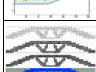
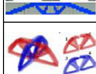
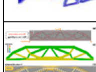
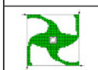
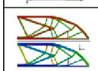
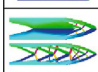
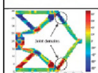
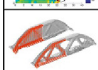
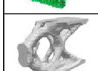
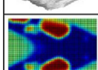
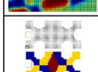
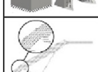
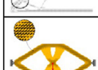
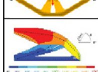
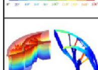
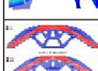
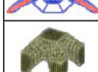
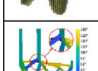
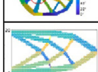
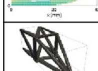
Thumbnail of the publication	TO algorithm	Fiber orientation algorithm	Continuous path generation	Algorithm architecture	Geometry dimensions	Benchmark environment	Efficiency considered	Manuf. constraints considered	Validated with tests	Reference
	SOMP	Orthogonal SOMP	Streamline + Offset	Concurrent	2,5D	-	✗	✓	✓	[61]
	SOMP	Load path vector field	Load depended path planning	Sequential	2D	1,2,3	✗	✓	✗	[66]
	SOMP	CFAO	Streamline + Hamilton path	Concurrent	2D	1.2	✗	✗	✓	[70]
	SIMP	Load path vector field	Streamline + Offset	Sequential	2,5D	1,2,4	✗	✗	✓	[71]
	SOMP	SOMP	EQS; Offset	Sequential	2,5D	1,2,3	✗	✓	✓	[72]
	SOMP; Level-set	SOMP; Level-set; Energy-method	Offset; EQS; Streamline	Concurrent & Sequential	2D	1,2,3	✗	✓	✗	[49]
	SIMP (bimat.)	SIMP (bimat.)	SIMP (bimat.)	Concurrent	2D	1	✗	✓	✗	[73]
	polar-SOMP	SOMP	Cartesian comp. of orient. vector	Concurrent & Sequential	2D	1.2	✗	✗	✗	[69]
	SIMP (bimat.); SOMP	SIMP (bimat.); SOMP	De-homogenization mapping	Concurrent	3D	1.2	✗	✗	✗	[48]
	(B)ESO	Principal stress (segmentat.)	Streamline	Sequential	2,5D	2	✗	✓	✗	[74]
	SOMP	Pricipal stress	Wavefunction projection	Sequential	2,5D	1	✓	✗	✓	[60]
	SOMP	Principal stress	Offset (Anisoprint Aura)	Concurrent	3D	2	✗	✓	✗	[65]
	SIMP	CFAO	Offset (MakerBot Desktop)	Concurrent	3D	1	✗	✓	✗	[75]
	SOMP; SIMP	SOMP	Offset (Ultimaker Cura)	Concurrent	2,5D	1	✓	✗	✓	[32]
	SIMP	Principal stress	Commercial (PD-3D printing slicer)	Sequential	2,5D	5	✗	✗	✓	[76]
	SIMP	none	Offset (Markforged Eiger)	Sequential	3D	-	✗	✓	✓	[77]
	SIMP	Hipercube to simplex (segmentat.)	EQS	Concurrent	2D	2	✗	✗	✗	[78]
	Level-set	Level-set	Offset	Concurrent	2D	1.2	✗	✗	✗	[79]
	SIMP	CFAO	Streamline	Concurrent	2D	1	✗	✗	✗	[80]
	SIMP	DMO	Streamline	Concurrent	2,5D	1.2	✗	✗	✗	[81]
	SOMP	CFAO	Streamline	Concurrent	2,5D	1	✗	✓	✓	[82]
	NN	NN-CFAO	NN-Streamline	Concurrent	2D	1.4	✓	✓	✗	[83]

Table 2
Reviewed FRTO papers that are lacking the continuous reinforcement path conversion [2,55,65,84–102].

Thumbnail of the publication	TO algorithm	Fiber orientation algorithm	Algorithm architecture	Geometry dimensions	Benchmark environment	Efficiency considered	Manuf. constraints considered	Validated with tests	Reference
	(B)ESO	Principal stress	Concurrent	2D	2	✗	✓	✗	[87]
	modified SIMP	Principal stress (ESO)	Concurrent	3D	1,2,3	✗	✗	✗	[84]
	SIMP	Strain energy (Cellular automata)	Concurrent	2D	2	✓	✗	✗	[85]
	modified SIMP	Strain energy	Concurrent	2D	-	✗	✗	✗	[86]
	polar-SOMP	SOMP	Concurrent & Sequential	3D	1	✗	✗	✗	[88]
	SIMP	CFAO	Concurrent	3D	1,2	✓	✓	✗	[2]
	SOMP	SOMP	Concurrent	2D	1.3	✗	✗	✗	[89]
	SIMP	DCMO	Concurrent	2D	1,2,3,4	✗	✗	✗	[55]
	(B)ESO	Hybrid stress-strain method	Concurrent	3D	1,2,4	✗	✗	✗	[65]
	Free energy	CFAO	Concurrent	3D	2	✗	✗	✗	[90]
	SIMP	Raster	Sequential	2,5D	2,5	✗	✓	✗	[91]
	SIMP	DMO + Heaviside	Concurrent	2D	-	✗	✗	✗	[92]
	SIMP	DMO	Concurrent	3D	2	✗	✓	✗	[93]
	SOMP	SOMP	Concurrent	2,5D	1	✗	✗	✓	[94]
	SIMP	DMO	Concurrent	2D	1,2	✗	✗	✗	[95]
	SOMP	Segmentation	Concurrent & Sequential	3D	1,2	✗	✗	✗	[96]
	NURBS	NURBS	Concurrent	2D	1,2,3	✓	✗	✗	[97]
	SIMP	DMO	Concurrent	2D	1,3	✗	✗	✗	[98]
	SIMP	CFAO	Concurrent	3D	1,2,3,4	✗	✗	✗	[99]
	SIMP	DMO	Concurrent	2D	1,2,3	✗	✗	✗	[100]
	Polar-SIMP	DMO	Concurrent	2D	2	✓	✗	✗	[101]
	MMC	Geometric centerline/segment.	Concurrent	3D	1,2	✗	✗	✗	[102]

implemented, showcasing the preference for comprehensive optimization approaches.

Furthermore, the investigation reveals interesting insights into the geometric complexity of the specimens considered. While approximately half of the investigated cases focus on 2D planar problems, there is a notable presence of extruded profiles (2.5D) and actual 3D geometries, each accounting for around 25% of the cases. Moreover, the most prevalent TO environments include the MBB, Cantilever, and L-shear configurations, indicating widespread adoption in the literature.

However, only a minority of the papers, approximately 20%, consider efficiency concerns, suggesting a potential gap in addressing optimization process efficiency. Similarly, around 40% of the papers highlight manufacturing concerns, underscoring the need for closer integration of manufacturing constraints into optimization processes. Additionally, approximately 25% of the papers are validated with tests, indicating that a significant portion of the literature focuses on theoretical or simulation–based analyses rather than experimental validation.

8. Practicality in topology optimization

TO methods can be classified according to their practical applicability. Thus, it becomes imperative to consider real-world constraints when formulating the TO problem. These constraints usually restrict design freedom, but neglecting them can result in unfeasible and unstable constructions, as seen in Refs. [103–107]. These considerations encompass manufacturing-related limitations, load uncertainties [105], and algorithm efficiency [108]. The computational demand of TO algorithms is also a significant issue, as these methods involve complex iterative processes that require substantial computing resources, especially with larger structures. Optimizing for multiple loads simultaneously for enhanced robustness introduces added complexity, reflecting real-world challenges where uncertainties and diverse loading conditions are expected.

8.1. Manufacturing limitations in reinforcement path design

Manufacturing limitations can manifest in various ways, particularly in fiber-laying processes. Typical limitations include the closest proximity at which the reinforcement structure can be placed, the minimum bending radius of the fiber bundle, the need for support material, the minimum cutting length of fibers, and the total number of reinforcement cuts required during manufacturing (Fig. 12). Wang et al. [66] devised a method for generating uninterrupted paths characteristic of automated fiber placing processes. The focus was on laying continuous fibers aligned with load lines, compared with results from commercially available slicer software using MBB and cantilever-beam reference geometries. They also compared their method with other fiber-laying strategies from the literature. Their method facilitated the creation of intersecting paths, enhancing optimization efficiency through layer interaction, and they assessed accuracy in matching the resulting reinforcement structure path with principal stress vector field directions. Results indicated that their method demonstrated the best match across all examined problems and loading environments.

Jantos et al. [2] combined the SIMP method with the Hamilton method, using principles of thermodynamics to determine material orientation. They introduced an algorithm that maximizes local angular change to assist in fiber laying while optimizing manufacturing-related constraints like the minimum bending radius of reinforcing fibers. Comparing the performance with unreinforced (SIMP) structures using an isotropic model, they observed that their method did not significantly change computational costs. However, it produced mechanically weaker results due to limited reinforcement path radii. The study incorporated real-life relevant parameters, such as manufacturability and resource requirements, into the optimization.

Wang et al. [106] developed a SIMP–based optimization process capable of generating a structure without support material, considering the critical printing radius. The novelty lies in the algorithm's concurrent nature, allowing for optimal structure design considering manufacturing conditions. Numerical tests on various 2D and 3D reference geometries revealed a 10–20% reduction in stiffness compared to structures not optimized for manufacturing.

In summary, as the focus shifts from theoretical to feasible, real-world optimization, the role of production criteria becomes more significant.

8.2. Efficiency of the optimization algorithms

The computational efficiency of TO methods, particularly their resource requirements, is a crucial non-mechanical aspect. The effectiveness of a TO method is significantly hindered if it lacks computational efficiency. Since these methods involve a complete FEA in each optimization iteration, efficiency becomes a crucial parameter in algorithm development. Ferrari et al. [108] addressed this issue by devising an eigenfrequency optimization method capable of replacing computationally intensive spectral analysis. Their process involves conducting frequency response–based calculations at lower resolution levels, which are then projected back to higher resolution levels for structure optimization. 2D and 3D layout testing demonstrated an impressive 90% and 95% reduction in resource requirements, respectively.

Xia et al. [109] enhanced the slicing algorithm for MEX technology, specifically focusing on the layer creation path and aligning it with the principal stress field in the component. Their approach aimed to capitalize on orientation during printing and the resultant anisotropy, opting for intentionally directed space filling instead of a raster–based strategy. They partitioned each layer into regular hexagons and devised a path that efficiently filled the space using the Dijkstra algorithm. Comparative analysis of hole-weakened tensile specimens demonstrated that their method achieved a substantial strength increase ranging between 37.2% and 98.1%, with minimal impact on printing times.

Jantos et al. [46] refined the thermodynamic model–based TO method, enhancing its computational efficiency. The researchers introduced an adaptive mesh algorithm into the model, accelerating geometry formulation and yielding mesh-independent

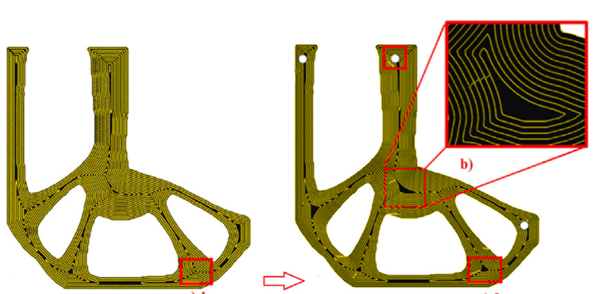


Fig. 12. Integrating limitations of the printing process into optimization involves actions such as eliminating fibers shorter than the minimum cutting length (a) and transforming closed loops into continuous paths that meet manufacturing requirements (b) (based on Ref. [72]).

results. The algorithm operates without the need for post-processing and allows for flexible adjustment of optimization complexity. Comparative analysis with methods utilizing the convolutional operator and Helmholz partial differential equation involved varying mesh resolutions, filter radii, and minimum element sizes. Results indicated that the resource requirements of the proposed method are proportional to the number of elements, the square of element size, and the square of filter radius. Notably, while competing methods show a faster increase in resource requirements based on these variables, the presented method demonstrates a significantly slower growth rate. This positions it as a more suitable approach for addressing complex problems.

Yang et al. [110] expedited the multiphase TO process using an adaptive finite element mesh and the Isogeometric Analysis approach. Their method incorporates three essential procedures. Firstly, they utilize quadratic elements in optimization, claiming improved handling of the continuous shape function of geometry with significantly reduced numerical errors. Secondly, they implement an adaptive resolution finite element mesh, strategically allocating higher resolution near phase boundaries and intensively loaded areas. Despite fewer elements, their method produces comparable results to high-resolution homogeneous processes. The third approach involves reducing multi-phase optimization to a two-phase problem, systematically comparing only two materials at each optimization level. Testing on various classical TO benchmarks (MBB, L-shear, and cantilever beam) demonstrated resource savings ranging from 38 to 54%, compared to traditional *Multiresolution Topology Optimization*, where mesh refinement occurs at phase boundaries and within the phases.

Despite the listed advancements, balancing thorough design exploration with computational efficiency remains an ongoing challenge. The algorithm must navigate the extensive design space effectively while ensuring convergence to an optimal solution. Dealing with sensitivity to initial conditions and avoiding local minima entrapment further underscores the need for robust and reliable TO algorithms.

8.3. Errors in topology optimization

In topology optimization, various errors can arise, including discretization, modeling, and convergence errors. Discretization errors occur due to the finite element mesh used to represent the design domain, leading to inaccuracies in the optimization results. Modeling errors stem from simplifications or assumptions made in the mathematical models used, which may not fully capture the real-world behavior of the structures. Convergence errors occur when the optimization algorithm fails to converge to an optimal solution within a reasonable number of iterations.

To mitigate these errors, several strategies can be employed. Increasing mesh density can reduce discretization errors, although it comes with increased computational cost [111]. Employing adaptive mesh refinement techniques can help focus computational resources where they are most needed [53,110], reducing errors while maintaining efficiency. Addressing modeling errors involves validating and refining the mathematical models to ensure they accurately represent the physical behavior of optimized structures. This may involve experimental validation [61,70] or the use of more advanced material models [112].

Convergence errors can be mitigated by selecting appropriate optimization algorithms and parameters and performing sensitivity analyses to ensure the stability and robustness of the optimization process [113,114].

To ensure accurate and reliable optimization results, mitigating topology optimization errors requires careful model development, numerical techniques, and algorithmic strategies.

9. Objective comparison of optimization results

For a proper analysis papers in TO research and a comparison of the provided results, it is necessary first to discuss the fundamental benchmark TO environments. While there is no standardized approach for comparing algorithms, there is a tendency in the literature to associate the evaluation of these methods with a few characteristic mechanical problems. These environments with prescribed boundary conditions make these algorithms comparable, assuming identical manufacturing parameters. However, a significant challenge is the need for manufacturing standards, making comparing results from different research groups difficult, even when tested on similar mechanical problems. Table 3 organizes these benchmark problems based on typical loads, mechanical boundary conditions, and representative result geometry.

In TO research, while numerical results are widely used for comparison, real-world application discrepancies often emerge due to model inaccuracies and overlooked manufacturing considerations. We emphasize that a series of steps is essential to compare TO algorithms effectively. These include establishing a coherent mathematical model, ensuring consistent utilization of hyperparameters, conducting stability analysis, defining convergence criteria, and more. The numerical results alone cannot adequately represent physical tests, yet they are insufficient for guiding optimization algorithm comparisons. We suggest that once a TO algorithm is validated through numerical parameters, its utility can be further demonstrated by incorporating manufacturing constraints. This step assesses the realizability of the optimized structure in practical applications, representing a distinct aspect of the validation process. Incorporating physical tests in TO research is crucial for enhancing validation and ensuring alignment with real-world conditions. Hence, future research efforts should prioritize increased validation through physical experimentation to bolster the reliability and applicability of TO methodologies.

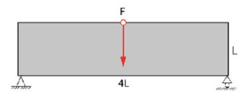
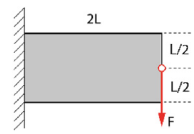
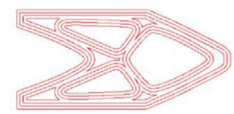
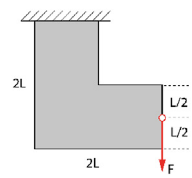
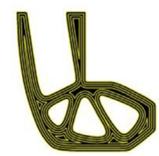
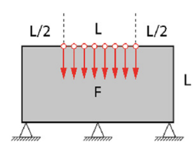
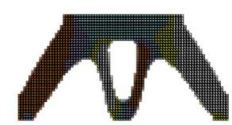
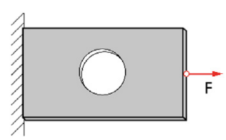
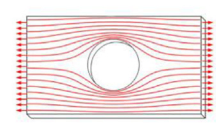
Despite the absence of a universal standard in the structures made by TO, Ole Sigmund's guidelines [3] provide a valuable framework for conducting TO research. His insights can assist researchers in navigating the complexities of algorithm comparison and the interpretation of results, contributing to the advancement of the field.

10. Artificial intelligence in topology optimization

In computer science, AI development has emerged as a prominent field, aiming to achieve intelligent behavior through methods that recognize and synthesize samples from extensive databases. While the concept of AI has been present for decades, a surge in related research projects occurred mainly in the 2010s.

Among AI–based processes in TO, NNs, GAs, and *Generative Design* (GD) methods have gained prominence as they can offer multiple enhancements across various stages of the optimization pipeline. At present, AI-driven pre-processing techniques facilitate data-driven exploration of design spaces, enabling engineers to navigate vast parameter landscapes efficiently [116,117]. Subsequently, during optimization iterations, NNs serve as surrogate models, accelerating convergence by predicting optimal solutions and guiding search strategies. NNs can also learn intricate relationships between inputs and outputs, creating Pareto-optimal solutions and enabling optimizations with multiple goals [118]. Concurrently, GANs facilitate the generation of diverse design candidates, fostering creativity and exploration within specified constraints. Similarly, GD methods can drastically reduce computational costs by generating thousands of designs in seconds, facilitating the exploration of a more extensive design space [1,119,120]. Moreover, post-processing stages benefit from AI-

Table 3
Validation environments for TO methods. The Messerschmidt Bölkow–Blohm (MMB) structure (a) extends the concept of 3-point bending, where the increased distance between the lower and upper sides generates internal loads that excellently reveal the characteristics of the optimization algorithm. This makes it one of the most popular reference geometries among TO researchers [115,132]. The cantilever beam (b) can be described as a spatially extended hanger with a shear-dominant load case. It is the second most popular reference geometry in the processed literature [49]. The L-shear structure (c) is an L-shaped geometry with its top clamped and vertically loaded at the end of its horizontal section. This geometry is also a prevalent structure for comparison, as it can be used to examine shear, tensile, and bending stresses simultaneously [49]. The Michell-truss is a compression-dominated structure (d), which less frequently appears in the literature than the first three types, but it is also a regularly recurring geometry [55]. The Kirsch geometry (e) is usually a hole-weakened geometry in tension [76]. This type of geometry can be used to test the stress-homogenizing performance of fiber reinforcement. Other types of TO validating environments, such as two-position, bi-stable compliant structures, are used for testing deformation or work caused by a given force [89]. Since this is a dynamic problem not covered in this article, it was not included in the table.

Designation		Dominant load-case	Boundary condition arrangement	Representative optimization result
(a)	Messerschmidt Bölkow–Blohm (MBB)	Bending		
(b)	Cantilever-beam	Shear + bending		
(c)	L-shear	Shear + bending		
(d)	Compression-Michell	Compression		
(e)	Kirsch	Tension		

enabled analysis tools, which extract actionable insights and iteratively refine designs based on performance metrics [37,38].

A significant practical distinction between AI–based and deterministic algorithms lies in their continuously improving capability, therefore TO methods employing various forms of AI could offer advantages over traditional optimization techniques. However, it is essential to acknowledge the drawbacks associated with integrating AI into optimization processes. High resource requirements and extended training times pose significant challenges, demanding substantial computational power and datasets of considerable size and quality, acquiring notable expenses. Despite these challenges, once a model is trained, the cost of generating new designs is drastically reduced, allowing for efficient and rapid exploration of design possibilities. These characteristics underscore the potential of AI to revolutionize the optimization landscape, making it more accessible and streamlined for designers and researchers alike.

10.1. Neural network–based reinforcement path design methods

Based on the literature, NNs play a role in TO across two primary categories: accelerating TO through NN and directly employing NN for topology computations. In the former category, researchers have explored methods to train NNs for the real-time prediction of gradients, accelerating TO processes [121]. Convolutional NNs, acquainted with extensive databases of optimized topologies and associated loads, have enhanced the efficiency of predicting topologies for new loading scenarios [122]. Similar strategies have been presented in various studies [37,123]. Another approach involves using generative adversarial networks to train generators to predict topologies under diverse loading scenarios [38,124]. NNs have also been employed to recognize and replace evolving features, streamlining TO iterations [125]. Autoencoders trained with data relevant to homogenized microstructures have found application in TO frameworks, removing the necessity for homogenization during the optimization process [126,127]. Alternatively, NNs can be directly employed to represent topology as an implicit function of spatial coordinates, taking advantage of their capability to handle intricate signal representation. The NN–based TO framework introduced by Chandrasekhar et al. [83] decouples design representation from FEA through NNs, allowing detailed geometric queries while maintaining computational efficiency. NN activation functions and weights manage design variables independently, including matrix density and fiber morphology, offering flexibility. At the same time, the algorithm concurrently optimizes matrix topology and fiber morphology, adhering to volume constraints (Fig. 13). This innovative approach enhances computational efficiency and ensures functionally graded structure optimization accuracy.

Patel et al. [1] developed a multiscale TO procedure utilizing AI based on NNs to design microstructures in geometry. The method involves breaking down the initial macro-level geometry created

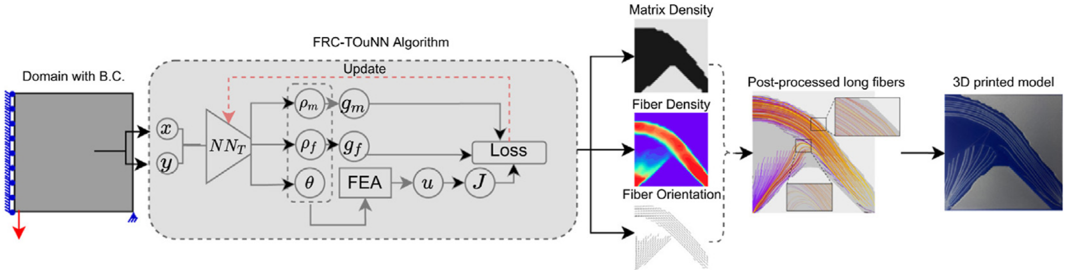


Fig. 13. Flowchart of Neural Network–based topology and reinforcement path optimization, using FEA only for training the model. The variables are represented by NN. Utilizing a neural network (NN) representation for variables in the optimization process offers several advantages. Firstly, it allows for simultaneous optimization of matrix topology, fiber density, and fiber orientation, independent of mesh structure. This NN–based approach facilitates end-to-end automatic differentiation, eliminating the need to derive sensitivity expressions manually. Additionally, the implicit nature of this representation enables the realization of continuous fibers at high resolution during post-processing, exploiting the trained NN. Consequently, the optimized fiber-reinforced composite (FRC) part can be readily fabricated with automated fiber laying techniques [83].

with the SIMP method into smaller units, determining the mechanical load system for each elementary unit from nodal displacements within the structure. The optimal macrostructure can then be selected from a pre-generated database with the desired volume ratio for the given location, enabling microstructure generation faster by an order of magnitude.

11. Future trends

Based on the literature, we can conclude that FRTO methods can be categorized into three main groups. The first group is the most computationally demanding, where non-isotropic TO methods determine topology and fiber orientation. These concurrent methods will likely find the global optimum, achieving the highest structural performance. Most of the processed literature consisted of these complex methods, showing a trend toward such comprehensive approaches where there is an observable tendency that with the maturation of TO methods, researchers are increasingly focusing on complex, mathematically challenging problems. These include non-linear methods, optimization for dynamic loads, and multiscale or simultaneous multi-material and microstructure processes. However, as issues become more complex, computational costs rise significantly, posing a severe challenge to the widespread adoption of TO design methods. High resource requirements and extended running times have increased interest in more efficient optimization approaches.

Efficiency improvement is pursued in the other two main directions in the literature, defining the remaining two classes of FRTO. The first involves integrated fiber placement methods, which avoid the resource-intensive local orientation arrangement during optimization. These sequential methods follow the shape of the geometry pre-determined by isotropic TO methods, arranging reinforcing fibers longitudinally to the emerging trusses in the structure or designing fiber reinforcement based on similar considerations. While these methods undoubtedly result in some performance loss in terms of the global optimum, the balanced consideration of their structural performance and computational efficiency could often be a suitable compromise in industrial situations.

The second, technologically more advanced groups of optimization involve the integration of AI into TO algorithms, where the role of TO is poised to shift. This is because an AI–based system pretrained on sufficient TO samples is anticipated to streamline the design process by providing iteration-free results.

It is important to note that while AI brings benefits, TO methods remain essential for achieving precise solutions and detailed tunability. Unlike TO methods, AI models operate as 'grey boxes,' making the cause-and-effect relationship less transparent. Since TO procedures are intricate and well-defined, they are valuable tools, especially when precision and detailed control over the design process is crucial. As the industry navigates this evolution, the synergy between TO algorithms and AI–based systems holds promise for a more efficient and adaptable approach to composite structure design.

In conclusion, addressing prevailing challenges to boost FRTO utilization is crucial as the field progresses. This involves reshaping designers' mindsets and deepening their understanding of the introduced processes to seamlessly integrate AM technologies with the potential of TO. Thus, future research should enhance the robustness, user-friendliness, and accessibility of the methodology to facilitate the widespread FRTO adoption. AI can automate this intricate process, simplifying its complexity. Exploring innovative approaches and techniques that enhance the adaptability of FRTO methodologies to various manufacturing constraints offers a promising avenue for future exploration, ensuring alignment with evolving real-world manufacturing scenarios.

12. Conclusion

In this paper reviewing the state of the art, we analyzed and summarized various *Fiber-reinforced Topology Optimization* (FRTO) methods in the literature supplemented by related manufacturing technologies, where automated robotic arms, fiber and tape placement, and high-performance polymer composites play a crucial role. We discussed the possibilities of topology and morphology optimization, examining their sequence and highlighting the advantages and disadvantages of each methodology. We addressed the integration of manufacturing constraints into optimization and explored filters and curve-fitting algorithms that link different optimization steps, resulting in manufacturable parts. Furthermore, we delved into the significance of *Artificial Intelligence* (AI) within the domain of *topology optimization* (TO). Subsequently, we consolidated our findings into a list of conclusions:

1. Trends in research: The prevalence of density–based methods like SIMP and SOMP highlights their effectiveness in determining reinforcement direction vectors. At the same time, the balanced use of fiber orientation algorithms indicates a versatile approach to optimizing the morphology of composite structures. Moreover, the widespread implementation of these algorithms concurrently emphasizes the importance of comprehensive optimization strategies. A noticeable trend is

emerging towards comprehensive approaches, reflecting the maturation of TO methods. There is a growing focus on addressing complex, mathematically challenging problems such as nonlinear optimization and multiscale processes, along with considering manufacturing constraints and robustness.
2. Fiber-reinforced polymer composites designed by FRTO: Non-isotropic TO methods offer the potential for achieving the global optimum and highest structural performance when combined with polymer composite materials. Optimization in these methods occurs not only at the macro level but also aligns with the morphology of the part, further enhancing its efficiency.
3. Research opportunities in FRTO: The analysis of TO methods and fiber orientation algorithms underscores the need for continued research and development in optimizing geometric complexity and addressing real-world constraints. Understanding the geometric complexity of specimens and prevalent TO environments provides valuable insights for future research directions, encouraging further exploration into optimizing efficiency, manufacturing constraints, and experimental validation methods.
4. Challenges in the field of TO: Increasing computational costs are challenging the widespread adoption of TO and FRTO design methods, driving interest in more efficient optimization approaches. Furthermore, the absence of international standards poses a significant challenge in objectively comparing results and methodologies within the field.
5. Efficiency Improvement, synergy with AI, and the evolving role of TO: Efforts to enhance efficiency include integrated fiber placement methods and the incorporation of AI into FRTO algorithms, both offering promising paths for streamlining the design process. The synergy between TO algorithms and AI—based systems shows potential for a more efficient and adaptable approach to composite structure design, indicating a potential shift in the role of TO in structural optimization. Despite the benefits of AI, TO methods remain essential for achieving precise solutions and detailed tunability, especially when transparency and control are essential.

Data availability

No data was used for the research described in the article.

CRediT authorship contribution statement

Bence Szederkenyi: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Norbert Krisztian Kovacs:** Writing – review & editing, Writing – original draft, Supervision, Software, Methodology, Data curation, Conceptualization. **Tibor Czigany:** Writing – review & editing, Writing – original draft, Supervision, Project administration, Funding acquisition, Conceptualization.

Declaration of competing interest

Tibor Czigany is an editorial board member for Advanced Industrial and Engineering Polymer Research and was not involved in the editorial review or the decision to publish this article. The authors declare that they have no known competing financial interests or personal relationships that could influence the work reported in this paper.

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