New rock physical models describing the pressure dependence of seismic/acoustic dispersion characteristics

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Summary

In this paper, new rock physical models describing the pressure dependence of acoustic wave velocities and quality factors are presented. They are based on the consideration that the cause of changing velocities/quality factors under varying pressure is due to the changes in pore volume. After determining the model parameters by jointly inverting P and S wave velocity/quality factor data, they can be calculated for any pressure. Based on these results the pressure-dependent Lamé parameters and loss angles can be derived as well. To prove the applicability of the methods, literature data (velocities and quality factors) measured on coal samples were inverted. The results show that the misfits between measured and calculated data are small, and the model can be applied well in practice.

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Introduction

The knowledge of pressure dependence of acoustic wave velocities and elastic properties has great significance in seismic practice. It is observed that pressure has more influence on velocities in the beginning phase of loading, later it lessens and the velocities tend to have a limit value. After Birch's (1960) consideration the reason for the velocity increase is the decreasing pore volume with increasing pressure. A nonlinear relationship between velocity and pressure was proved by several empirical equations, however, they do not provide a physical explanation of the process.

Besides the P and S wave velocities, the pressure dependence of quality factors (Q_α, Q_β) (or absorption coefficients) are often investigated. There are several models in the international literature to explain the attenuation of elastic waves, among others the nonlinear friction model, the Biot model, the viscoelastic model, and the elastic dispersion model. The theories for the pressure dependence of velocities are suitable for the description of the relationship between quality factor and pressure. Previous studies revealed that the quality factors behave similarly to the velocities, a rapid nonlinear increase occurs at the beginning of loading. As the pore volume decreases with the increasing pressure, the contacts between the grains become better thus the measurable absorption coefficient decreases, and the quality factor increases.

The following rock physical models are formulated with the assumption of the constant (frequency-independent) Q model, where the velocities and the quality factors are rock stress-dependent. Using laboratory-measured velocity and absorption coefficient data the material parameters of the models are processed in a joint inversion procedure. After determining these values the phase velocities and the Q factors can be calculated at any pressure, so the pressure-dependent Lamé parameters and loss angles are deduced.

Modeling the pressure dependence of acoustic velocities

The new rock physical model explaining the physical relationship between the applied stress and the acoustic P and S wave velocities are based on the idea of Birch (1960). The model law of the velocity model can be formulated by Eq. (1)

$$dV = -\lambda_V V d\sigma \ , \tag{1}$$

where dV is the change of the closable partial pore volume, $d\sigma$ is the applied stress increase and λ_v is the proportionality factor, a new rock physical parameter. The negative sign indicates that the closable partial pore volume and stress are inversely proportional. We assume also a linear relationship between the infinitesimal change of the appropriate propagation wave velocity dv (substitutable with the longitudinal or shear wave velocities α or β, respectively) and dV

$$dv = -\kappa dV \ , \tag{2}$$

where the κ proportionality factor is a new material characteristic. The negative sign represents that the velocity and pore volume are inversely proportional. Combining Eqs. (1-2) and solving the differential equations as well as applying the notation $\Delta v_0=\kappa V_0$ one can obtain

$$v = v_0 + \Delta v_0 \left(1 - exp(-\lambda_V \sigma)\right) \ , \tag{3}$$

where v_0 is the propagation velocity at a stress-free state, while the quantity Δv_0 means the velocity drop caused by the presence of pores at a stress-free state (Ji et al. 2007) and can be considered as the difference between the velocities measured at maximum and zero stresses i.e. $\Delta v_0=v_{max}-v_0$.

The physical meaning of the λ_v parameter was derived by Dobróka and Somogyi Molnár (2012). It can be formulated as the logarithmic stress sensitivity of the velocity-drop

$$S(\sigma) = -\frac{1}{\Delta v}\frac{d\Delta v}{d\sigma} = -\frac{d\,ln(\Delta v)}{d\sigma} \ \rightarrow \ \lambda_V = -\frac{d\,ln(\Delta v)}{d\sigma} = S \ . \tag{4}$$

By substituting the appropriate velocities the model equations describing the pressure dependence of longitudinal (α) and shear (β) wave velocities can be obtained in the forms of Eqs. (5)

$$\alpha = \alpha_0 + \Delta\alpha_0\,(1 - exp(-\lambda_V \sigma)), \qquad \beta = \beta_0 + \Delta\beta_0\,(1 - exp(-\lambda_V \sigma)). \tag{5}$$

Note that λ_v is the same for both types of waves therefore if both P and S wave velocity data are available they can be processed in a joint inversion procedure.

Modeling the pressure dependence of quality factors

The base of the rock physical model describing the pressure dependence of quality factor is the same, the varying pore volume causes the changes in wave attenuation. As the effect of increasing stress the grain structure becomes more compact, e.g. the pore volume decreases, resulting increase in the measured quality factors. Assuming a linear relationship between the change of pore volume (dV) and the change of quality factors (ΔQ_α and ΔQ_β) we introduce Eq. (6) as model laws

$$dQ_\alpha = -\chi_\alpha dV\,, \qquad dQ_\beta = -\chi_\beta dV\,, \tag{6}$$

where the Q_α and Q_β represent the quality factors for P and S waves respectively, χ_α and χ_β are proportionality factors and the negative signs represent the inverse proportionality between pore volume and quality factor. Combining Eqs. (1) (with proportionality factor λ_Q) and (6) the following relations can be written

$$dQ_\alpha = \chi_\alpha \lambda_Q V_0 \exp(-\lambda_Q \sigma) d\sigma\,, \qquad dQ_\beta = \chi_\beta \lambda_Q V_0\, exp(-\lambda_Q \sigma) d\sigma\,. \tag{7}$$

The quality factors at a stress-free state ($Q_{\alpha 0}$ and $Q_{\beta 0}$) can be measured, thus the integration constants can be calculated (similarly to the velocity models). Introducing the notations $\Delta Q_{\alpha 0} = \chi_\alpha V_0$ and $\Delta Q_{\beta 0} = \chi_\beta V_0$, Eq. (7) take the forms

$$Q_\alpha = Q_{\alpha 0} + \Delta Q_{\alpha 0}(1 - exp(-\lambda_Q \sigma)), \qquad Q_\beta = Q_{\beta 0} + \Delta Q_{\beta 0}(1 - exp(-\lambda_Q \sigma)), \tag{8}$$

where λ_Q is a common material-dependent rock physical parameter. The exponential characteristic of quality factor change with increasing pressure can be seen from Eq. (8). $\Delta Q_{\alpha 0}$ and $\Delta Q_{\beta 0}$ can be considered as quality factor drop – similarly to the velocity – since they mean the differences between the quality factors at the stress-free state and maximal stress ($Q_{\alpha max}$, $Q_{\beta max}$).

The effect of pressure on elastic moduli and loss angles

In case of rapidly changing stresses, the rocks respond as perfectly elastic materials i.e. they suffer strain during loading but they perfectly recover their shapes after unloading. This is the assumption of the Hooke body, where the stresses are proportional to the deformations. The proportionality factors are called elastic moduli. One can distinguish static and dynamic moduli. Former ones are determined from stress-strain measurements letter ones can rather be derived from acoustic measurements. Here the dynamic elastic moduli and their pressure dependence are investigated. They can be deduced based on the previously introduced rock physical models.

In the general form of the Hooke body the two constants, the Lamé coefficients describe the stress-deformation relationship. They can be calculated from velocities as

$$\mu = \beta^2 \rho\,, \quad \lambda = \alpha^2 \rho - 2\mu\,, \tag{9}$$

where ρ is the density of the medium. The pressure dependence of density is assumed negligible in comparison to the pressure-dependent velocity. Therefore, one requires accurate velocity measurements to obtain pressure-dependent elastic moduli.

If measured data of quality factors besides velocities are also available, the pressure-dependent dissipative parameters (loss angles – ε and ε') can be also deduced. With the assumption of the constant Q model the Lamé coefficients are complexes

$$\mu = \mu^*(1 + i\varepsilon), \qquad \lambda = \lambda^*(1 + i\varepsilon'), \tag{10}$$

where μ^*, λ^* are the real part of the Lamé coefficients, ε, ε' are dissipative parameters, the so-called loss angles for which

$$tg\delta = \frac{Im\{\ \mu\ \}}{Re\{\ \mu\ \}} = \varepsilon\ , \qquad tg\delta' = \frac{Im\{\ \lambda\ \}}{Re\{\ \lambda\ \}}) = \varepsilon'\ . \tag{11}$$

For small dissipations $tg\delta \approx \delta$. Solving the wave equations for body waves and rearranging the equations, the pressure-dependent loss angles can be expressed as

$$\varepsilon = \frac{1}{Q_\beta}\ , \qquad \varepsilon' = \frac{\lambda + 2\mu}{\lambda Q_\alpha} - \frac{2\mu}{\lambda Q_\beta}\ . \tag{12}$$

To prove the applicability of the presented models, laboratory-measured data were processed in a joint inversion procedure, then the pressure-dependent Lamé coefficients and loss angles were calculated.

Application of the models on coal samples

Longitudinal and transverse velocity and quality factor data measured on coal samples were published by Yu et al. (1993). Sample Nr. 16 was selected to present the applicability of the rock physical models. The Upper Permian black coal sample was homogeneous and micro-banded in the central locality. The pulse transmission and spectral ratio techniques were used to measure P and S wave velocities and quality factors.

Data were inverted utilizing joint inversion processing. The calculated model parameters with their estimation errors can be seen in Table 1. With the estimated parameters the velocities and quality factors can be determined at any pressure using the developed model equations Eq. (5) and Eq. (8). Fig. 1 represents the results (calculated Lamé coefficients and loss angles are produced by Eq. (9) and (12)). The calculated curves are in good accordance with the measured data, which is strengthened by the calculated low RMS values (Table 1). RMS and mean spread were calculated after Menke (1984). In the case of quality factors RMS values are higher than those at the velocities which can be explained by the difficulty of quality factor measurements. Even so, the noise in data space is small-scale, which confirms the accuracy of the inversion results and the feasibility of the suggested rock physical models for the explanation of the exponential relationship between the P and S wave velocities/quality factors and rock pressure. The moderate (S=0.48) mean spread value (mean correlation between the estimated model parameters) confirms also that the inversion results are reliable.

Table 1 *Model parameters (and their errors) estimated by joint inversion method. RMS values.*

Velocities				**Common parameter**	**Quality factors**				**Common parameter**
P wave		***S wave***			***P wave***		***S wave***		
α_0 ***(km/s)***	$\Delta\alpha_0$ ***(km/s)***	β_0 ***(km/s)***	$\Delta\beta_0$ ***(km/s)***	λ_V ***(1/MPa)***	$Q_{\alpha 0}$ ***(-)***	$\Delta Q_{\alpha 0}$ ***(-)***	$Q_{\beta 0}$ ***(-)***	$\Delta Q_{\beta 0}$ ***(-)***	λ_Q ***(1/MPa)***
2.23 (±0.02)	0.35 (±0.02)	1.02 (±0.01)	0.17 (±0.01)	0.1494 (±0.0123)	10.92 (±1.12)	53.66 (±4.31)	14.09 (±1.02)	66.58 (±4.50)	0.0293 (±0.0043)
RMS = 0.54 %					RMS = 7.18 %				

Conclusions

This research aimed to develop rock physical models which provide a theoretical connection between acoustic wave velocities/quality factors and pressure. The models are based on the idea that the pore volume of the rock is decreasing with increasing pressure. After determining the model parameters by joint inversion these quantities can be calculated for any pressure. Furthermore, the pressure dependence of other parameters such as elastic moduli (Lamé coefficients) and dissipative parameters (loss angles) can be derived too. To prove the applicability of our models they were tested on a data set measured on a coal sample. The accuracy of the inversion estimates and the reliability of the suggested petrophysical model was proved.

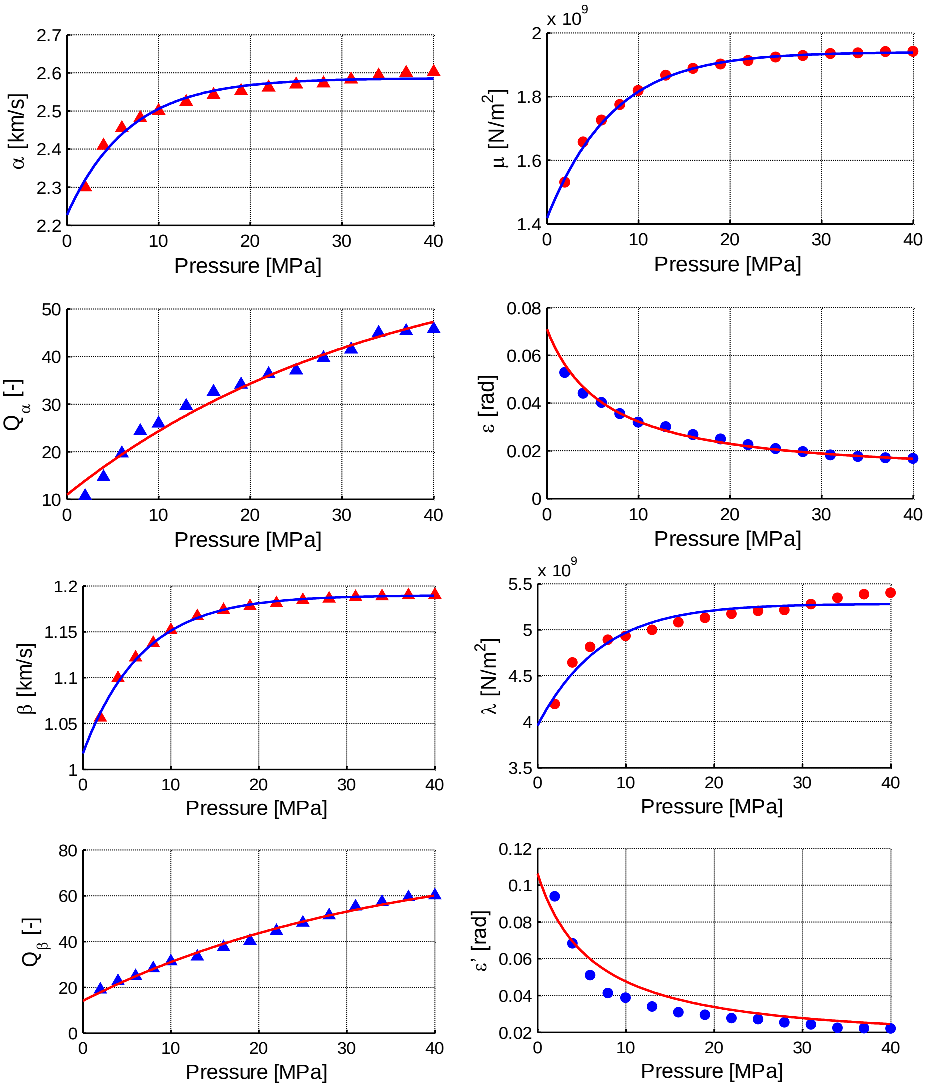

Figure 1 Left: Velocities and quality factors of P and S waves vs. uniaxial pressure (solid line – calculated data produced by the developed models, symbols – measured data). Right: Lamé coefficients (μ, λ) and loss angles (ε, ε') vs. uniaxial pressure. Measured data obtained by Yu et al. (1993).

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