Introduction

The well-logging inverse problem is an iterative method for predicting reservoir characteristics from recorded data, using local inversion technique. This linearized optimization approach minimizes overdetermination issues by limiting unknowns to recorded log data. (Dobróka and Szabó 2015). In addition. The layer borders are regarded unknown and cannot be calculated using local inversion. To address the constraints of local inversion, a series expansion-based inversion can be utilized to estimate reservoir properties across a predetermined interval. (Dobróka et al. 2016) refer to this approach as interval inversion. Interval inversion involves series expansion of polynomial function to describe petrophysical parameter changes over depth intervals. It ensures precision and reliability, allowing for parameter inclusion. The Damping Least Squares (DLSQ) approach solves ill-posed inversion issues by reducing a positive factor until it approaches zero.

This means that little changes in the input parameters (model attributes) might result in huge changes in the anticipated results. Essentially, the sensitivity matrix magnifies the influence of minor changes in the parameter space, making it difficult to correctly invert the issue (Van Rijn and Hutter 2018). When an inversion issue is ill-posed due to a large condition number, it might result in unstable solutions. This instability implies that tiny errors or uncertainties in observed data can lead to substantial inaccuracies in calculated model parameters. In practice, this might make it challenging to achieve consistent and relevant results from the inversion process. To overcome this issue, a variety of strategies are used, including regularization methods.

Theoretical background

The response function governs the recording of data and the estimation of parameters. These response functions are distinguished by their nonlinearity. The linearized inversion approach includes estimating the nonlinear connection between the observed data and the predicted parameters using a Taylor series truncated at first order. The relationship between the observed data and model parameters at z depth can be written as follow

$$\vec{d}^{(obs)}{}_k(z) = g_k(m_1(z), m_2(z), \dots, m_M(z)), \quad (1)$$

The discretization of the *i-th* spatial dependent model parameter can be written in the following form

$$m_i(z) = \sum_{q=1}^{Q} B_q^{(i)} \Psi_q(z), \quad (2)$$

On the contrary to the point-by-poitn inversion, the unknowns of the interval inversion are the discretization coefficients *B*. By using the DLSQ algorithm, the initial model can be iteratively refined, in case of interval inversion as follow

$$\delta\vec{B} = (G(B)^T G(B) + \lambda I)^{-1} G(B)^T \delta\vec{d}^B, \quad (3)$$

The ill-posed problem is based on the condition number which is function of the minimum and maximum eigenvalues of the sensitivity matrix. (Meju 1992) introduced an iterative algorithm based on factorizing the sensitivity matrix (Jacobian matrix) into three other matrices as follow

$$G(B^i) = USV^T, \quad (4)$$

where U is (NxQ_M) data eigenvector, V is (Q_MxQ_M) model parameters eigenvector, and S is (Q_MxQ_M) matrix with eigenvalues in its diagonal. The eigenvalues of $G(B^i)$ are positive numbers with i ≤ M (number of parameters). the ill-posed problem will arise in case of small eigenvalues of the term $G(B^i)^T G(B^i)$. Therefore, the SVD scheme recommends adding a positive bias to the eigenvalues of that term. By substituting, the final equation to update the model parameters provided by can be written as

$$B^{i+1} = B^{i} + V\, diag\left\{\frac{\eta_i}{\eta_i^2+\lambda_i^2}\right\}\, U^{T}\delta\vec{d}^{B}. \qquad (5)$$

The damping factor is determined through a process of comparing the misfit of the L-th test at any given iteration with that of the (L+1)-th test. If an improvement in misfit is observed, the new factor is accepted.

The standalone SVD-based interval inversion will test all the eigenvalues of the sensitivity matrix to use as a damping factor for the inversion process. However, the high overdetermination ratio of the interval inversion will lead to a time-consuming process. In this research we implement a hybrid optimization technique that integrates between the SVD and DLSQ optimization schemes. The algorithm begins using the SVD scheme which does the testing for all the eigenvalues to guarantee the stability of the convergence procedure while the initial model is so far from the solution. Then, after a data distance threshold value, Close to the optimum value, the algorithm is changed for a fast DLSQ scheme. This hybrid algorithm both increases the rate of convergence of the inversion procedure and provides a regularized scheme in the case of the far initial models cases.

Synthetic Data-driven hybrid SVD-based Interval Inversion

The synthetic model that is constructed based on the Heaviside basis function consists of 4 layers model varying in the volume of lithological constituents (shale-sand), porosity, and saturation content (hydrocarbon-water). Figure 1 shows the misfit between the actual and calculated data at iteration 60, respectively, while Figure (2a) shows the predicted petrophysical parameters. The SVD based inversion shows a smooth convergent pattern, but it shows a rapid convergent until data distance lower to 5% and too slow convergent until reach the zero data distance (Figure 2 (b and c)).

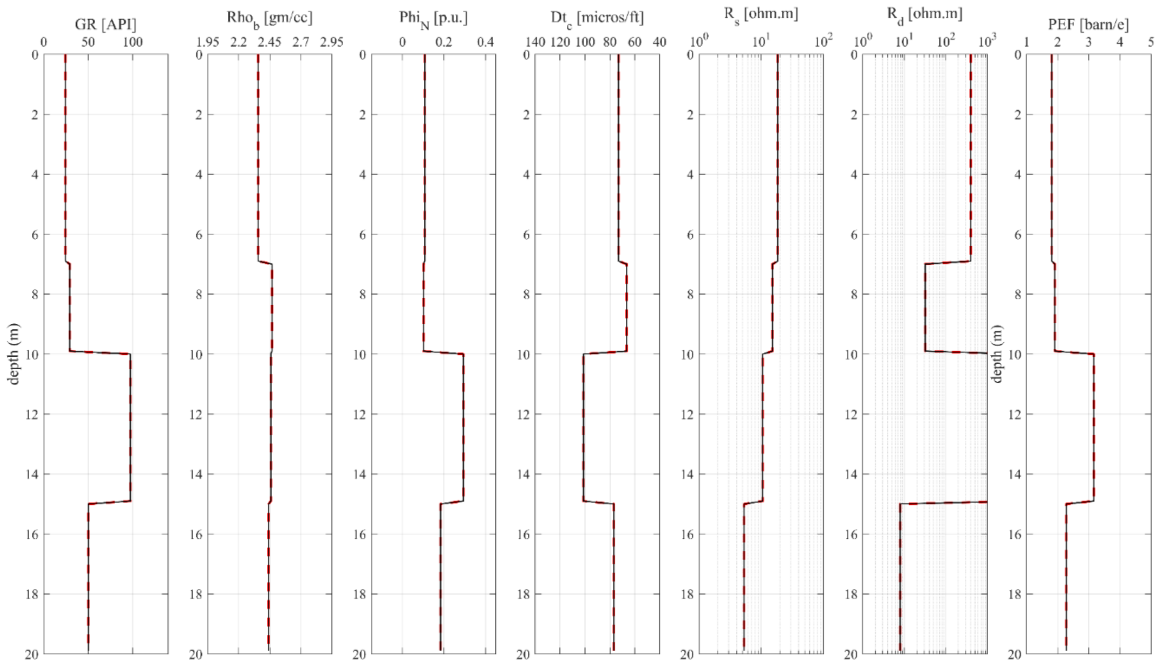

Figure 1. The fitting between the synthetic data and the calculated data (iteration 60); the red dashed lines represent the calculated data, while the solid black lines represent the synthetic data.

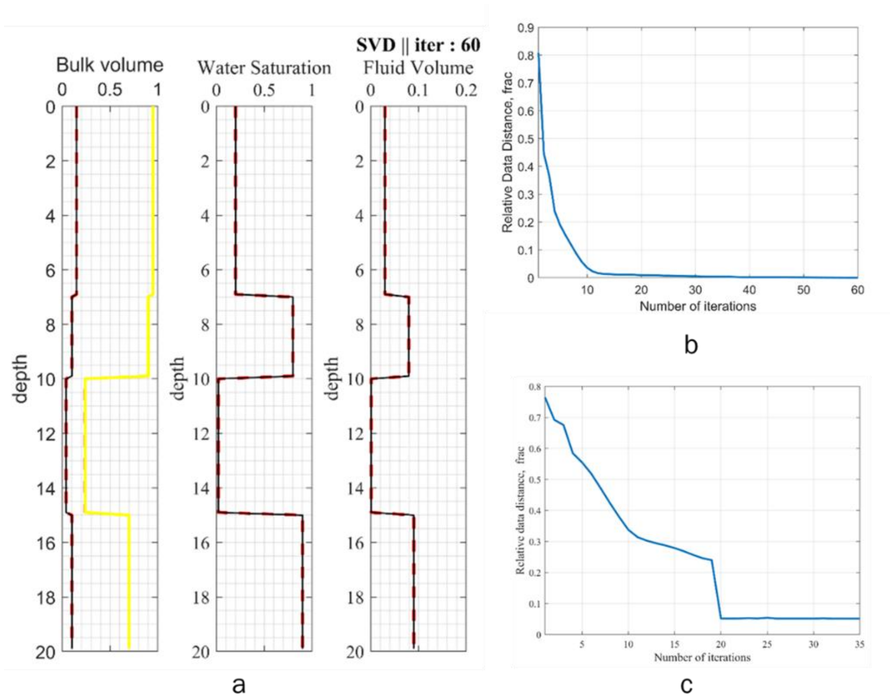

Figure 2. Shows the predicted petrophysical parameters (a), and the convergence of data distance. (b) the standalone SVD-based interval inversion, (c) Hybrid SVD-based interval.

Field application of the hybrid SVD-based interval inversion

The viability of both algorithms was tested using a well-logging dataset from a gas-bearing reservoir in an Egyptian field in the northwestern region of Egypt's Western Desert. The reservoir is part of a highly heterogeneous Jurassic sandstone deposit.

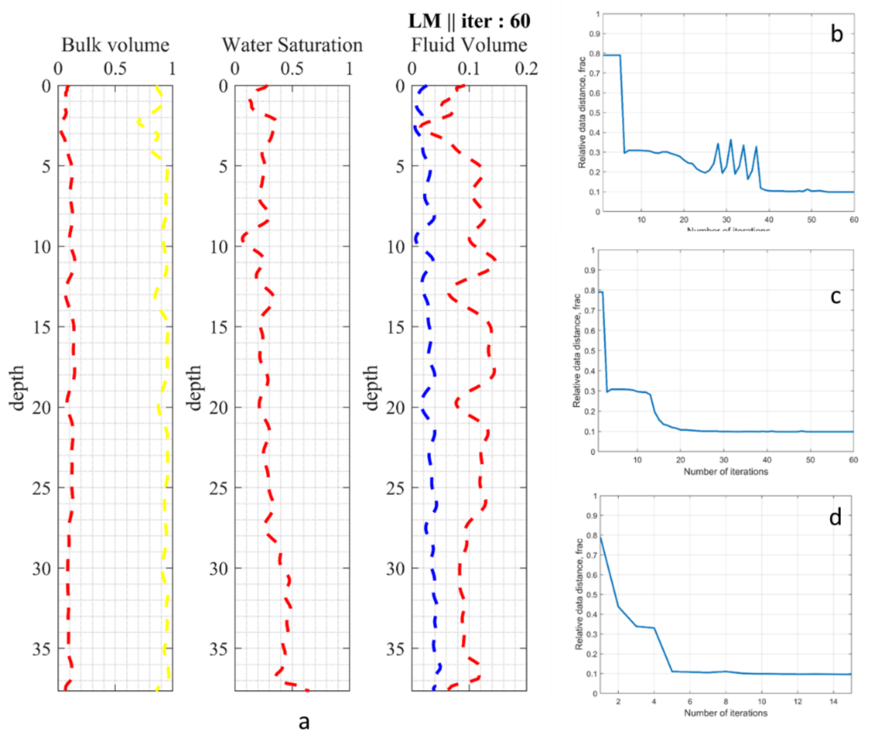

Figure 3. Shows the predicted petrophysical parameters (a), and the convergence of data distance. (b) interval inversion, (c) the standalone SVD-based interval inversion, (d) Hybrid SVD-based interval.

Figure (3a) demonstrates that the reservoir is mostly composed of sandstone layers with minor shale laminations that impact both storage and flow capacity. The study's findings highlight an important observation: reservoir parameters vary depending on reservoir quality. This means that these critical characteristics are not uniformly distributed, but rather adapt and modify in response to varied levels of reservoir quality. Figures (3b, 3c, and cd) show the data distance convergence pathway of traditional interval inversion, SVD-based interval inversion, and the hybrid SVD-based interval, respectively.

Conclusions

The hybrid Singular Value Decomposition (SVD)-based interval inversion algorithm is a promising solution for ill-posed inversion problems and computational efficiency. It integrates the SVD scheme for initial iterations, ensuring stability and convergence, and then transitions to the faster Damped Least Squares (DLSQ) method near the optimum solution. The algorithm accurately predicts petrophysical parameters across multiple layers with varying reservoir characteristics, and its performance was validated in a field application on a gas-bearing reservoir in Egypt's Western Desert. The hybrid approach ensures stable and relevant results even with uncertainties or errors in input data, and its ability to capture variations in reservoir quality and adapt to underlying petrophysical changes is significant for improved characterization and decision-making in reservoir development. The field well logging dataset shows that the reservoir is not homogeneous and consists mainly of sandstone layers separated by small laminations from shale. The data distance convergence is smooth in the case of the hybrid and normal SVD based schemes but it's fast only in the case of the hybrid scheme. The data distance converge to 9.5 %.

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