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P–type orbits in the Pluto–Charon system

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Abstract
The dynamical structure of the phase space of the Pluto–Charon system is studied in the model of the spatial circular restricted three-body problem by using numerical methods. With the newly discovered two small satellites S/2005 P1 and S/2005 P2, the Pluto–Charon system can be considered as the first known binary system in which celestial bodies move in P-type orbits. It is shown that the two satellites are in the stable region of the phase space and their origin by capture is unlikely.
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1 Introduction

In 1930 C. Tombaugh discovered Pluto, the ninth planet of the Solar system. Pluto's first moon, Charon was found by Christy & Harrington (1978). The Pluto–Charon system is remarkable, since in the Solar system Charon is the largest moon relative to its primary, with the highest mass-ratio 0.130137. Subsequent searches for other satellites around Pluto had been unsuccessful until mid May 2005, when two small satellites were discovered (Stern et al., 2005) provisionally designated as S/2005 P1 and S/2005 P2 (henceforth P1 and P2). With this observation Pluto became the first Kuiper Belt object known to have multiple satellites. These new satellites are much smaller than Charon, with diameters 61-167 km (P1) and 46-137 km (P2) depending on the albedo. Both satellites appear to be moving in nearly circular orbits in the same orbital plane as Charon, with orbital periods 38 days (P1) and 25 days (P2).

From a dynamical point of view the Pluto–Charon system corresponds to such a binary system whose mass parameter is approximately one tenth. The phase space of binaries and the Pluto–Charon system can be studied simultaneously. To survey the phase space of binaries is a fundamental task, since more and more exoplanetary systems are being discovered. The great majority of exoplanets have been observed around single stars, but more than 15 planets are already known to orbit one of the stellar components in binary systems (this type of motion is referred to as satellite or S-type motion).

There are some studies on planetary orbits in binaries. The so far discovered planets in binaries move in S-type orbits. Theoretically there is another possible type of motion, the so-called planetary or P-type, where a planet moves around both stars. There are several studies on S- and P-type motions using the model of the planar elliptic restricted three-body problem see e.g. (Dvorak, 1984, 1986; Holman & Wiegert, 1999; Pilat-Lohinger & Dvorak, 2002) and references therein. The three-dimensional case, that is the effect of the inclination was studied by Pilat-Lohinger, Funk & Dvorak (2003) for P-type orbits in equal-mass binary models.

The main goal of this paper is to investigate the dynamical structure of the phase space of the Pluto–Charon system which can be considered as the first known binary system in which celestial bodies, namely P1 and P2 move in P-type orbits. In Section 2 we describe the investigated model and give the initial conditions used in the integrations. The applied numerical methods are briefly explained in Section 3. The results are shown in Section 4. Section 5 is devoted to some conclusions.

2 Model and initial conditions

To study the structure of the phase space of the Pluto–Charon system we applied the model of the spatial circular restricted three-body problem. We integrated the equations of motion by using a Bulirsch–Stoer integrator with adaptive stepsize control. The orbits of the primaries were considered circular and their mutual distance A was taken as unit distance. The orbital plane of the primaries was used as reference plane, in which the line connecting the primaries at $t = 0$ defines a reference x-axis. We assume that the line of nodes of the orbital plane of the massless test particle (i.e. P1 or P2) coincides with the x-axis at

$t = 0$, thus the ascending node $\Omega = 0°$. The pericenter of the test particle's orbit is also assumed to be on the x-axis at $t = 0$, thus the argument of the pericenter $\omega = 0°$. Though P1 and P2 are in the orbital plane of Charon, still we study the problem more generally by considering the effect of non-zero inclinations on the orbital stability. Thus our results are applicable to a wider class of satellite or planetary systems around binaries for the mass parameter $\mu = m_2/(m_1+m_2) = 0.130137$, corresponding to the Pluto–Charon system (m_1 and m_2 being the mass of Pluto and Charon, respectively).

To examine the phase space and the stability properties of P-type orbits, we varied the initial orbital elements of the test particle in the following way (see Table 1):

- the semimajor axis a is measured from the barycentre of Pluto and Charon and it is varied from 1 to 5 A with stepsize $\Delta a = 0.005\ A$,
- the eccentricity e is varied from 0 to 0.3 with stepsize $\Delta e = 0.05$ ($\Delta e = 0.002$ in Fig. 2),
- the inclination i is varied from $0°$ to $180°$ with stepsize $\Delta i = 1°$,
- the mean anomaly M is given the values: $0°$, $45°$, $90°$, $135°$, and $180°$.

The above orbital elements refer to a barycentric reference frame, where the mass of the barycentre is $m_1 + m_2$. By the usual procedure we calculated the barycentric coordinates and velocities of the test particle and then transformed them to a reference frame with Pluto in the origin. In the numerical integrations we used the latter coordinates and velocities.

In total almost six million orbits were integrated for 10^3 Charon's period (hereafter T_C) and approximately 500 thousand for 10^5 T_C.

3 Methods

To determine the dynamical character of orbits we used three methods. The method of the relative Lyapunov indicator (RLI) was introduced by Sándor, Érdi & Efthymiopoulos (2000) for a particular problem, and its efficiency was demonstrated in a later paper (Sándor et al., 2004) for 2D and 4D symplectic mappings and for Hamiltonian systems. This method is extremely fast to determine the ordered or chaotic nature of orbits.

For an indication of stability a straightforward check based on the eccentricity was used. This action-like variable shows the probability of orbital crossing

Table 1: *In the first three rows the orbital elements from unrestricted fits (epoch = 2452600.5) are listed (Buie et al., 2005): a, e, i, ω, Ω and M denote the semimajor axis, eccentricity, inclination, argument of the pericenter, longitude of the ascending node, and mean anomaly. In the last column the orbital periods are given in days. The orbital elements for P-type orbits are given with the corresponding stepsizes.*

Object	a [A]	e	$i[°]$	$\omega[°]$	$\Omega[°]$	$M[°]$	T [day]
Charon	1.0	0	96.145	–	223.046	257.946	6.387
P2	2.487	0.0023	96.18	352.86	223.14	267.14	25
P1	3.31	0.0052	96.36	336.827	223.173	122.71	38
P-type	0.55–5	0–0.3	0–180	0	0	0–180	
Δ	0.005	0.05	1	–	–	45	

and close encounter of two planets, and therefore its value provides information on the stability of orbits. We examined the behaviour of the eccentricity of the orbit of the test particle along the integration, and used its largest value ME as a stability indicator; in the following we call it the maximum eccentricity method (hereafter MEM). This simple check was already used in several stability investigations, and was found to be a powerful indicator of the stability character of orbits (Dvorak et al., 2003; Süli, Dvorak & Freistetter, 2005).

The maximum difference of the eccentricities method (MDEM). We developed this new method and applied it for the first time in this investigation. Two initially nearby trajectories emanating from a chaotic domain of the phase space will diverge according to the strength of chaos. The divergence manifests itself in the differences between the eccentricities of the orbits and in the angle variables. The more chaotic the system is, the faster the difference in the eccentricities grows. This difference is sensible to the variations around the running average of the eccentricity and depends also on the position along the orbit. Thus if the positions along the two orbits change chaotically, the eccentricities of the two orbits also behave differently and their momentary differences can be large even if the average value of the eccentricity of each orbit remains small. This method characterises the stability in the phase space, whereas the MEM does it in the space of orbital elements. We define the stability indicator MDE as:

$$\mathtt{MDE}(t) = \mathtt{max}|e(t, x_0) - e(t, x_0 + \Delta x)|, \qquad (1)$$

where x_0 is the initial condition of the orbit and Δx is the distance of the nearby orbit in the phase space. The method of the MDE has the advantage with respect to the MEM that in the case of chaotic orbits the MDE grows more rapidly than the ME, and while the difference between the ME for regular and chaotic motions is only 1-2 orders of magnitude, this can be 4-7 orders of magnitude for the MDE and therefore can be detected more easily.

4 Results

We show the results of our investigations in Figs. 1 – 2. These were obtained as follows. By varying the initial orbital elements as described in Section 2, we performed the integration of each orbit for five different initial values of the mean anomaly: $M = 0°, 45°, 90°, 135°$, and $180°$. For each M the indicators $I^{(M)}(a,e,i)$ were determined, where $I^{(M)}$ stands for RLI, ME, and MDE, respectively. Any value, plotted in the figures, is an average over M:

$$\bar{I}(a,e,i) = \frac{1}{5}\sum_{M} I^{(M)}(a,e,i). \tag{2}$$

We note, that this averaging in the case of the RLI and the MDE emphasises the chaotic behaviour of an orbit, while in the case of the ME is not so drastic.

The three methods are not equivalent, however they complete each other. The ME detects macroscopic instability (which may even result in an escape from the system), whereas the RLI and the MDE are capable to indicate microscopic instability.

In most of the simulations the $I^{(M)}$ values were calculated for 10^3 T_C. To decide whether this time interval is enough to map the real structure of the phase space, several test runs were done for a much longer time span, for 10^5 T_C.

We found that the maps obtained from simulations for a time span of 10^5 T_C and 10^3 T_C are in a close agreement. Thus we can conclude that the phase space of the Pluto–Charon system can be surveyed in a reliable way by using a time span of 10^3 T_C.

4.1 The phase space of the Pluto–Charon system

We investigated the behaviour of P-type orbits systematically by changing the initial orbital elements of the test particle as described in Section 2 (see also Table 1). Beside direct orbits ($i < 90°$) we studied also retrograd P-type motion ($i > 90°$) of the test particle. All the integrations were made for 10^3 T_C. The

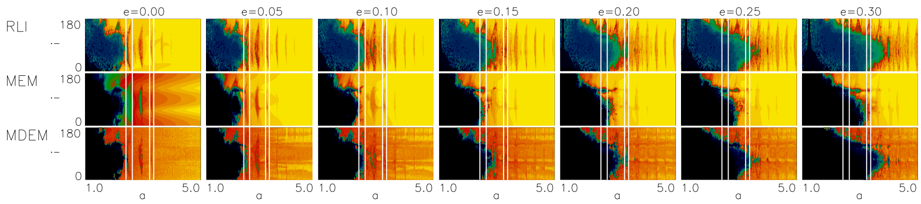

Figure 1: *The results of the* 10^3 T_C *simulations for* $e = 0, \ldots, 0.3$ *in the* a, i *plane. The dark area is unstable, the grey regions are stable. See text for details.*

results are shown in Fig. 1, where the indicators $\bar{I}$ are plotted on the a, i plane for different values of e.

In general, the results show an increase of the chaotic area for higher eccentricities: for $i < 160°$ the chaotic region grows with e. However, the rate of the increase strongly varies with the inclination. It can also be seen that the resonant islands (4:1 at $a = 2.52$ A, 5:1 at $a = 2.92$ A etc.) merge with the growing chaotic zone. The most striking feature is that the stability of the retrograd P-type motion practically does not depend on e. Inspecting Fig. 1 it is evident that the border of the chaotic zone for $i > 160°$ stay almost constantly at $a \approx 1.7$ A.

4.2 Stability of the satellites P1 and P2

We have addressed the problem of stability of the recently discovered satellites of Pluto. The results are shown in Fig. 2, where the values of the MDE (computed for 10^3 T_C and averaged over for the mean anomalies) are plotted on the a, e plane for the planar case ($i = 0°$). Below $a = 2.15$ A the system is unstable for all e, above $a = 2.15$ A there is a stable region depending on e. The two satellites are situated here, in the small rectangles, indicating their dynamically possible most probable places of occurance. These rectangles are defined by the ME, computed in the vicinity of each satellite. This means that we took a grid around the present value of a of each satellite with a stepsize $\Delta a = 0.005$ A, $\Delta i = 1.25°$ in the interval $i = 0 - 180°$, and with initial $e = 0$ we computed the largest ME_{max} during 10^5 T_C (including averaging over the five values of M). We obtained that $\mathrm{ME}_{max} = 0.045$ for P2 and $\mathrm{ME}_{max} = 0.02$ for P1. In Fig. 2 these values give the height of the rectangles. We computed the possible minimal $r_p = a(1 - \mathtt{ME}_{max})$ pericenter and maximal $r_a = a(1 + \mathtt{ME}_{max})$ apocenter

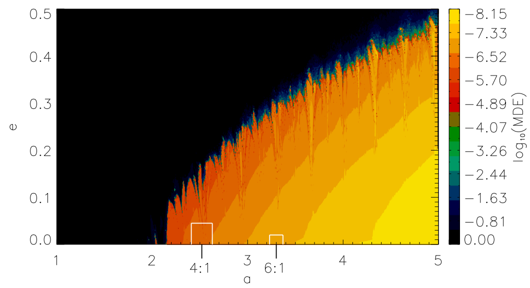

Figure 2: *Stability map in the a, e plane.*

distances of the satellites, these are 2.41 and 2.63 A for P2 and 3.23 and 3.37 A for P1. These values define the horizontal limits of the rectangles in Fig. 2, and also the places of the vertical lines in Figs. 1.

From Fig. 2 it can be seen that the determined orbital elements of the two satellites are well in the stable domain of the phase space. If P2 and P1 move in the orbital plane of Charon, their eccentricities cannot be larger than 0.17 and 0.31, respectively. The present semimajor axis a of P2 is very close to the 4:1 resonance, whereas that of P1 is close to 6:1. The locations of the exact resonances are well inside the small rectangles. We presume that these satellites probably move in resonant orbits. This could be confirmed by new observations.

5 Conclusions

Up to now the Pluto–Charon system is the only known binary system which has a relatively large mass-ratio and celestial bodies revolve around it in P-type orbits. This circumstance and the high ratio of binary stellar systems among stars make it important to study the stability properties of P-type orbits in binaries and particularly in the Pluto–Charon system. Our investigations show that the stable region is wider for retrograd than for direct P-type orbits. With the increase of the eccentricity the chaotic region becomes larger, and because of it the eccentricities of the two satellites, at their present semi-major axis, cannot be higher than 0.17 for P2 and 0.31 for P1. Below $a = 2.15$ A orbits are unstable for all eccentricities, thus no satellite could exist here.

Stern et al. (2005) has shown that P1 and P2 were very likely formed together with Charon, due to a collision of a large body with Pluto, from material ejected from Pluto and/or the Charon progenitor. This is based on the facts that P1 and P2 move close to Pluto and Charon in nearly circular orbits in the same orbital plane as Charon, and they are also in or close to higher-order mean motion resonances.

Our results are also against the capture origin of these satellites. Firstly, since the stability region for retrograd orbits is wider, it would have been more probable for the satellites to be captured into retrograd than for direct orbits. Secondly, capture into orbits close to the Pluto–Charon binary cannot be with high eccentricity ($e > 0.17$ and 0.31 at the present semimajor axis of P2 and P1), since these orbits become unstable on a timescale of $10^3\ T_C$. On the other hand, for eccentric capture orbits the tidal circularisation time for P1 and P2 is much longer than the age of the Solar System as Stern et al. (2005) pointed out.

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