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Irradiated closed Friedmann brane-worlds

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Abstract
We consider the evolution of a closed Friedmann brane irradiated by a bulk black hole. Both absorption on the brane and transmission across the brane are allowed, the latter representing a generalization over a previously studied model. Without transmission, a critical behaviour could be observed, when the acceleration due to radiation pressure and the deceleration introduced by the increasing self-gravity of the brane roughly compensate each other. We show here that increasing transmission leads to the disappearance of the critical behaviour.
Keywords: *cosmology with extra dimensions, brane-worlds, radiating black holes*

1 Introduction

According to the original idea of Kaluza and Klein, new and compact spatial dimensions, commensurable with the Planck scale, can be introduced in the attempt to describe various fundamental interactions. The possibility of a non-compact extra dimension has been advanced only recently [Randall and Sundrum (1999)]. The so-called brane cosmological models contain our physical world as a hypersurface (the brane), embedded into a warped five-dimensional bulk space-time, in which gravity acts. Its dynamics in the bulk is determined by the Einstein-equation, however on the brane we see only a projection of this dynamics. Consequently, on the brane, gravitational dynamics is modified as compared with general relativity. A modified Einstein equation for the most general case, allowing for both exotic energy in the bulk and asymmetric embedding, was given in [Gergely (2003)] (and for a more generic class of models, containing induced gravity contributions in [Gergely and Maartens (2005)]). The predictions of general relativity are recovered at low energies. With cosmological symmetries, the brane represents our observable universe. Branes with various other symmetries were found, like an Einstein static brane [Gergely and Maartens (2002)], a Kantowski-Sachs type homogeneous brane [Gergely (2004)] or a Gödel brane [Barrow and Tsagas (2004)].

Cosmological branes embedded in a bulk with radiation escaping from the brane were studied in [Langlois et al. (2002)], [Gergely et al. (2004)], [Jennings and Vernon (2005)] and [Langlois (2005)]. The scenario with a bulk black hole emitting Hawking radiation (its expression being derived for closed universes with $k = 1$ in [Emparan et al. (2000)], [Hemming and Keski-Vakkuri (2001)] and [Guedens et al. (2002)]) was studied in detail in [Gergely and Keresztes (2006)]. There, only one black hole was considered, with its radiation completely absorbed by the brane. (A somewhat similar model, but with $k = 0$ was considered in [Jennings et al. (2005)]. There a bulk black hole is placed on each side of the brane and the radiation is completely transmitted.)

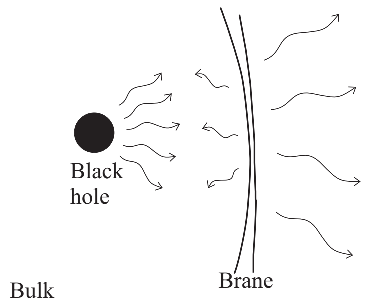

Figure 1: *The bulk contains an evaporating black hole. The radiation escaping this black hole is partially absorbed, partially reflected and partially transmitted across the brane.*

In [Gergely and Keresztes (2006)] the case of total absorption and no transmission was analyzed in detail. Two effects due to the radiation coming from the bulk were identified. First, radiation pressure on the brane accelerates its outward motion, similarly as dark energy. Second, with the increasing amount of radiation absorbed, the self-gravity of the brane increases, contributing towards a recollapse. However both effects represent 10^{-4} order perturbations for the model without radiation. (We note that the asymmetry in the embedding produced by the presence of only one bulk black hole behaves as an other perturbation.)

In this paper we investigate the effect of including transmission into the model developed in [Gergely and Keresztes (2006)], cf. Fig. 1 (where the possibility of reflection is also raised). We keep a (partial) absorption on the brane, but continue to neglect the reflection, as in [Gergely and Keresztes (2006)], basically because there is no known exact solution with cosmological constant describing a cross-flow of radiation streams. Thus the five-dimensional Vaidya-anti de Sitter (VAdS5) spacetime describes both bulk regions, the one with, and the one without the black hole.

In order to distinguish among models with different transmissions, we introduce a transmission rate ε, zero for a total absorption, and one for a total transmission. The brane evolves cf. the energy balance, Friedmann- and Rachaudhuri equations [Gergely (2003)]. These equations, specified for the case $\varepsilon \neq 0$, are given in [Keresztes et al. (2006)]. In this paper we present some of the results of the numerical study on these equations, for different values of the transmission rate ε.

2 Numerical results

We assume that the brane is radiation dominated and its evolution stars at the apparent horizon of the bulk black hole. We choose the same initial data as in [Gergely and Keresztes (2006)].

Allowing the transmission across the brane, we encounter new features. First, the accumulated energy from the absorbed Hawking radiation on the brane will be smaller than in the case of total absorption. Second, the transmitted radiation does not contribute to the radiation pressure on the brane, which is also smaller.

In the case with zero transmission [Gergely and Keresztes (2006)], both the acceleration from the radiation pressure and the deceleration from the increase in the self-gravity of the brane were small perturbations of order 10^{-4}, which roughly cancel for the critical initial energy density. As in the presence of transmission the effects are even smaller, we again will plot only *differences*, taken in the radiating and non-radiating cases.

Figure 2 shows the evolution of the differences in the scale factor when the radiation is switched on and off, for vanishing transmission. The critical-like behaviour appears for $\widehat{\rho}_0 = 520$ (the quantity $\widehat{\rho}_0$ denotes a properly defined [Gergely and Keresztes (2006)] dimensionless initial brane energy density). For lighter branes the radiation pressure is the dominant effect, and the recollapse occurs later in the presence of the radiation. For heavier branes the increase in self-gravity dominates and the recollapse is speeded up.

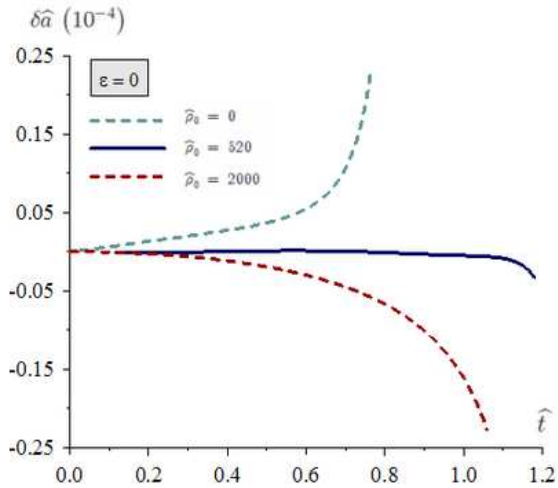

Figure 2: *The difference between the scale factors in the radiating and non-radiating cases for a fully absorbing brane. The critical-like behaviour is observed at* $\widehat{\rho}_0 = 520$ *(solid line). For small initial energy densities (e.g.* $\widehat{\rho}_0 = 0$*, upper dotted line), the pressure of the Hawking radiation is dominant. For high initial energy densities (*$\widehat{\rho}_0 = 2000$*, lower dotted line) the increase of self-gravity due to absorption overtakes the radiation pressure.*

On Figure 3, plotted for the transmission rate 0.2 we see similar behaviours. There are however two major differences, both due to the appearance of the transmission. First, the critical value of the initial brane energy density is decreased, as compared with the case of total absorption. Second, the sinusoidal-like pattern of the critical curve is much more accentuated, its amplitude is increased considerably.

Eventually, with increasing transmission rate the critical-like behaviour com-

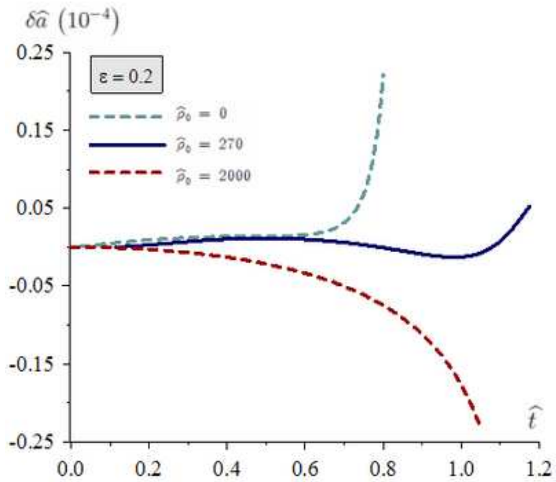

Figure 3: *For a higher transmission rate of $\varepsilon = 0.2$, the critical brane initial energy density is less ($\widehat{\rho}_0^{crit} = 270$) than for the case of total absorption. The amplitude of the sinusoidal evolution of $\delta\widehat{a}$ is higher.*

pletely disappears. This is illustrated on Figure 4 for $\varepsilon = 0.4$.

3 Concluding remarks

We have considered a brane-world scenario with a brane embedded into a bulk impregnated by radiation. The radiation is emitted by an evaporating bulk black hole. When this radiation hits the brane, it is partially absorbed and partially transmitted across the brane.

Since there is one black hole in the bulk, the embedding of the brane is asymmetric. It was shown in [Gergely and Keresztes (2006)], that this asymmetry is not able to change the recollapsing fate of the closed Friedmann brane. Neither does the completely absorbed Hawking radiation.

In this paper, we have generalized the above statements for branes allowing transmission. As was the case with total absorption, with a nonvanishing transmission rate the absorbed radiation keeps to increase the self-gravity of the brane, contributing towards deceleration, while the radiation pressure contributes towards the acceleration of the brane.

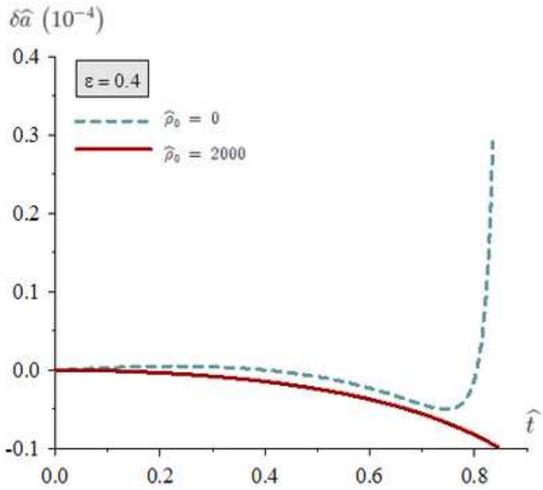

Figure 4: *For the transmission rate* $\varepsilon = 0.4$ *no critical-like brane evolution can be observed. For all values of the initial energy density the radiation drives the brane towards a faster recollapse.*

Our main result is that there are different behaviours depending on the actual value of ε. Whether the Hawking radiation contributes toward the recollapse of the brane-world universe or it speeds it up, depends on both the transmission rate and the brane energy density.

For small transmission rates and properly chosen initial brane energy density, the two effects can compensate each other. We have shown that by increasing the transmission rate, the critical-like behaviour disappears. In conclusion, a high transmission rate speeds up the recollapse, regardless of the value of the initial brane energy density.

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