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ON ANTHROPIC PRINCIPLES FINE TUNING AND CHAOS

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Abstract
Anthropic principles were grown from the problem of fine tuning. Although anthropic principles have been discussed in cosmology for years there are no exact definitions for fine tuning. To define the fine tuning quantitatively we investigate how one can use Lyapunov indicator in the definition of fine tuning. Our result is an alternative Lyapunov indicator, which shows how fine tuned is a possible Universe with physical constants different from ours.
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The Universe and life are connected. Man exists because the Universe is governed by delicately balanced laws that have led to a very high level of organization of matter called life. There are apparent coicidences in the dimensionless basic constants of nature. These basic numbers of nature not only allow the existence of stable atoms from which matter is built but also lead to the formation of galaxies and stars and even more complex stuctures all are necessary for the existence of life. A small change in the basic constants would result a universe without life-supporting conditions. We can say that the Universe is finely tuned for life.

Anthropic principles address the question why has the Universe the fine-tuning. Is this a fortunate condition, inevitable or it is expected. The weak anthropic principle stresses that we, intelligent observers, may observe only very special properties being compatible with our existence. The strong anthropic principle claims that the Universe must have those features which are necessary for life to develop at some stage of its history. The immediate consequence of SAP is that the physical laws and constants must be such as to allow the emergence of life. There exist several different versions or supplements of SAP. The most known is the design argument which states that life can occur because of some purposive design. That is the values of physical constants were selected purposively. For more specific anthropic definitions see Barrow-Tipler (1996) and Balázs (2005).

To state that the physical constants of the universe derive specific values from an ensemble of different values, we have to suppose the (i) conceptual or (ii) real existence of the numerical ensemble. The design argument has chosen the first solution (i), when the other possibilities exist only as possibilities in the mind of the Designer. The represents the second version (ii), supposing that an ensemble of other different existing universes is necessary for the existence of our Universe (Barrow-Tipler, 1996) There exist some alternative definiton (Müller, 2001), (Smolin, 2004) but their common feature is the lack of using mathematical methods.

A mathematical expression of fine tuning may help discussing anthropic arguments. Our aim is to construct a simple mathematical definiton of fine tuning similar with the one used in the chaos theory. This defintion can be testable therefore arguments of Smolin (2004) about the unfalsificability of anthropic principless are avoidable.

Both design argument and many-words-hypothesis is created to explain the fine tuning of physical constants. Physically say, fine tuning arguments state that there exist such a domain in the n-dimension parameter space of physical constants that if a universe is in this domain it may contain life.(Fig. 1)

The starting point of anthropic arguments that this fine tuning is very strong, that is the probability of life which equals with the fraction of the volume of m nad M domains is small, i.e.

$$p_{life} = \frac{m}{M} = \epsilon \qquad (1)$$

is true. Arguments against the fine tuning concept are formulated (Manson, 2001) and say that (i) there are no *a priori* probabilities, the value of the probability of a phenomenon is a result of our empiric experience. E.g. if we throw a dice and we see that every side is equally probable as a result we will say that our dice is symmetric. To say that every side's probability is 1/6 it is an *a posteriori* statement; (ii) there is not *a priori* gauge and *a priori* topology on

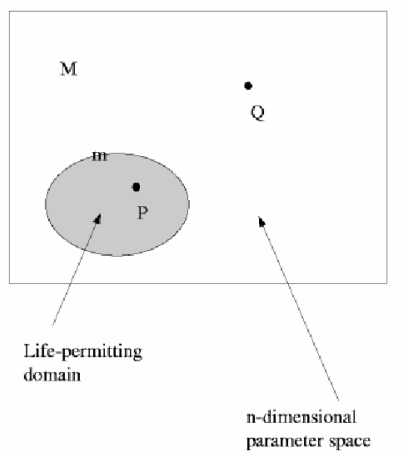

Figure 1: *In the M-multitude of physical parameters there exist such an m domain of phyiscal parameters that if a universe is in this domain it will permit life.* ***P*** *and* ***Q*** *are typical points in and out of the m life-permitting domain.*

M parameter-space (Fig. 1). Therefore it is meaningless to say that Universes signed by **P** and **Q** are near or far from each other on M parameter-space. Hence there is no reason to say that "if gravitation were $(1 + 10^{-40})$ stronger" something would happen with conditions of life.

Neither (i) nor (ii) arguments are indisputable. They may mirror rather an opinion than strict fact. A possible explanation of argument (i) will not be detailed here, but argument (ii) is investigated.

A mathematical expression of fine tuning may help the discussion of anthropic arguments. Our aim is to construct a simple mathematical definiton of fine tuning analogue to the Lyapunov-indicator in the chaos theory. Such a definition can lead to testable statements therefore arguments of Smolin (Smolin, 2004) about the unfalsificability of anthropic principless are avoidable.

Life depends on the presence of the proper chemical elements as building blocks and the existence of stars which can radiate enough energy for long time for evolution of life. If the abundance of carbon and other essential elements are lower or the number of properly radiating stars are smaller then we have a smaller probability for the evolution of intelligent life in a universe. To define fine tuning in a more quantitative way we can study the probability function for intelligent life of a universe. The form of this probability function can be regarded as an anthropic adaptation of the Drake equation (Ellis et al., 2004). Physical constants are regarded here as variables. Therefore we can present a probability distribution of life-bearing potentiality for universes as a function of the basic constans of the physics.

In order to characterize fine-tuning we introduce a quantity like to the one used to describe the chaotic behavior. Chaos in a dynamical system can be characterized by the Lyapunov indicator γ. It gives a number as result if the motion is chaotic and gives zero if not. Let us consider two trajectories not far from each other. The initial distance is d_0 and after Δt the final distance is d. If the distance is groving exponentially, i.e. $d(t) = d_0 exp(\alpha t)$ is true, then the Lyapunov indicator γ tends to α:

$$\gamma = \frac{\ln d/d_0}{\Delta t} \rightarrow \alpha \tag{2}$$

For the case $\gamma > 0$ the system is chaotic. If the increase of distance is smaller, e.g. $d(t) = d_0(\beta t)$ then γ tends to zero.

In the extreme limit of fine tuning, the probability function can be represented by a Dirac-delta function taken at the point of the parameter space of physical constants which corresponds to the single life-bearing universe. The Dirac-delta function $\delta(Q - Q_0)$ can be represented as the $n \rightarrow \infty$ limit of the Gaussian function

$$\delta(Q - Q_0) = \lim_{n\rightarrow\infty} \frac{n}{\sqrt{\pi}} exp\left(-n^2(Q - Q_0)^2\right) \tag{3}$$

Now we assume that if there is a fine tuning in the parameter Q around the maximum probability at Q_0 then the probability function for the life-bearing of universes can be approximated by the Gaussian form:

$$p_{\text{life}}(q) = \frac{n}{\sqrt{\pi}} exp(-n^2 q^2) \tag{4}$$

where $q = |Q - Q_0|$. The measure of fine tuning can be defined as

$$\gamma_{\text{fine-tuning}} = -\frac{(\ln p/p_0)^{1/2}}{\Delta Q} \rightarrow n \tag{5}$$

According to this definition, we have fine tuning, if $\gamma_{\text{fine-tuning}} > 0$. Fine tuning is stronger if $\gamma_{\text{fine-tuning}}$ is larger.

Discussing the production of the carbon and oxigen in the Universe, the position of the famous resonance in C_{12} nuclei plays a crucial role. According to (Oberhummer et al., 2000) outside a narrow window of 0.5 and 4 % of the values of the strong and nuclear forces, respectively, the stellar production of carbon or oxigen is reduced by factors of 30 to 1000. These production functions express Gaussian-like form. We can guess that the probability to find planets with a proper life-bearing mass would have even a higher decrease. Now the $\gamma_{\text{fine-tuning}}$ indicator of Eq. (5) has a definite nonzero value.

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