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Stability of (exo)planetary systems

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Abstract
Dynamical stability of the habitable zone of exoplanetary systems are investigated by a recently introduced chaos indicator (RLI). Study of individual systems, and the idea of a stability catalogue of habitable zones for exoplanetary systems are presented.
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1 Introduction

The existence of extrasolar planets was a long-standing open issue of astronomy till 1995, when the first exoplanet has been discovered around the star 51 Pegasi by Mayor & Queloz (1995). Since this discovery we know more than 133 extrasolar planets, which form around their hosting star 117 exoplanetary systems. The majority of these systems are single systems in the sense that they are consisting of only one planet, and 13 are multiple systems consisting of two or even more planets.

Exoplanetary research focusses mainly on the observation of exoplanets. However, there are other very important questions related to this field: one of these is the investigation of formation scenarios of planetary systems, and another one, which is addressed in this paper, is the dynamics and stability of such systems. Also a very important question arising in the context of exoplanetary research is the problem of habitability. Beside the very important atmospheric and geophysical characteristics of an exoplanet, its orbital (dynamical) properties are also essential in studying its habitability. From a human point of view, a planet is habitable, if the temperature on its surface is high enough to keep water in liquid phase, see Kasting et al. (1993) for a more detailed definition. We note however, that the question of habitability defined above, has sense only in the case of terrestrial planets.

Unfortunately, the exoplanets observed by now are gas-giants. This is the consequence of the fact that by using radial-velocity measurements, which is the most effective ground-based observing technique, there is no chance to detect Earth-sized planets. Thus there are space missions aiming at the detection of Earth-like exoplanets in planning phase, which hopefully will be launched in the near future.

The paper is organized as following: first we review shortly the basic characteristics of exoplanets and exoplanetary systems discovered by now, then we describe briefly the ground-based observing techniques, and the above mentioned space missions. After discussing our method of stability investigation, we present our results obtained by the stability investigation of hypothetical terrestrial extrasolar planets.

2 Characterization of exoplanets and their methods of detection

2.1 Properties of exoplanetary systems

Studying the orbital characteristics of exoplanets (see: http://www.exoplanets.org) one can conclude that the exoplanetary systems discovered until now differ very much from our planetary system, where the Jupiter-like gas giants are at larger distances (from the Sun), and between the gas giants and the Sun there are the Earth-like planets. In the case of the Solar System the orbital eccentricities of giant planets are rather small, or moderate.

It is expected however, that by using other observing techniques Earth-like exoplanets will be detected, and such planetary systems that are more similar to the Solar System. In what follows we present the most important observing methods and the space missions planned for observation of Earth-like exoplanets.

2.2 Methods of investigation

There are a couple of methods to observe extrasolar planets, the most efficient is based on the Doppler-shift of the hosting star's spectral lines. Another very promising method is the transit photometry. Although there are some results provided by the OGLE program, a breakthrough is expected after launching space instruments using transit photometry. Finally, we mention the interferometric methods. There are grandious space missions as Terrestrial Planet Finder (NASA) or Darwin (ESA) planned to launch in the future, which may use interferometric measurements. These instruments will be able to detect even some signs (if there are any) of possible extraterrestrial life. In what follows, we summarize briefly the Doppler-method and the transit photometry.

2.2.1 Radial velocity measurements

By now, this is the most efficient method to observe Jupiter-like exoplanets around a star. The basic principle of this method is that the star and an unseen planet move around their common barycenter, which results in the periodic displacement of the star's spectral lines. From this effect the radial velocity curve of the star can be calculated, and various physical properties of the unseen planet can be deduced: mass ($m \sin i$), semi-major axis, eccentricity, etc. This method can be applied to detect 3-10 m/s change in radial velocity. We note that Jupiter causes a 12 m/s radial velocity change in Sun's motion.

2.2.2 Transit photometry and space instruments

Transit photometry can be applied if the unseen planet, its hosting star, and the observer are approximatelly in the same plane. Planetary transit then results in the periodic dimming of the star's light intensity, however, a terrestrial planet causes only a $\Delta m = 10^{-5}$ change in the light intensity! Clearly, this method can be used to detect Earth-like planets only from space, thus there are space missions for observing Earth-like planets in planning phase. Such a mission is COROT (COnvection, ROtation and planetary Transit) sponsored mainly by CNES, partly by ESA and other countries. Its planned launching date is 2006. COROT is a space telescope with a 30 cm diameter mirror and an array of CCD's as detectors. It has two scientific aims: stellar seismology and the detection of few times larger terrestrial planets than Earth. The minimal expectations of the program after observing 3×10^4 stars, and supposing that 5% of them have Earth-like planets, are the detection of 10 exoplanets having radius $R = 2R_\oplus$ and 6 exoplanets with radius $R = 1.58R_\oplus$.

Another space instrument, which is devoted entirely to observe Earth-like exoplanets by using the transit photometry is KEPLER (NASA). Practically this is a large Schmidt-telescope with 95 cm aperture, its mirror diameter is 1.4 m, and it has 42 CCD chips. The planned life-time of this instrument is 4 years, thus during this (in ideal case) three transits with period 1.33 year can be detected. KEPLER is calibrated to detect a transit of an Earth-sized planet with semi-major axis $a = 1AU$ around a star with $m_v = 12^m$ apparent brightness.

Analysing the light curve and the period of the transit the orbital parameters of an exoplanet can not be derived uniquely. More accurate orbital parameters can be obtained if dynamical constraints are present. Such a dynamical constraint could be a Jupiter-like gas giant, which results in appearing dynamically unstable regions in the system, which are similar to the Kirkwood-gaps in the Solar System. If a hypothetical object is placed in an unstable region its orbit would be chaotic. Thus by using a chaos-detection method, orbital solutions which result in chaotic orbits can be avoided. This can help in chosing the right orbital parameters of the observed exoplanet by using the transit photometry.

3 Method of stability investigation

A traditional way to detect chaotic behaviour of orbits in dynamical systems is the calculation of the maximum Lyapunov characteristic exponent (LCE). The LCE of a trajectory emanating from an initial point $\mathbf{x}^*$ of the phase space is defined as the limit

$$L^1(\mathbf{x}) = \lim_{t\to\infty} \frac{1}{t} \log \frac{||\xi_t||}{||\xi_0||}, \tag{1}$$

where ξ_t is the image of an initial infinitesimal small deviation vector ξ_0 after time t. The evolution of ξ_t can be calculated by numerical integration of the equations of motion together with their linearized equations:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}), \\ \dot{\xi} &= D\mathbf{f}(\mathbf{x})\xi, \end{aligned} \tag{2}$$

where $D\mathbf{f}(\mathbf{x})$ is the Jacobian matrix evaluated at $\mathbf{x}$. In the case of Hamiltonian systems (dynamical systems describing the behaviour of planetary systems are Hamiltonian systems) if $L^1(\mathbf{x}) = 0$, the orbit evolving from $\mathbf{x}$ is regular, if $L^1(\mathbf{x}) > 0$ it is chaotic. A serious disadvantage of the calculation of the LCE is that it can not be obtained after finite integration time, thus its value can only be extrapolated, which makes the identification of weakly chaotic orbits very uncertain. Furthermore, weak chaos plays an essential rôle in understanding the long-term stability of exoplanetary systems (including the Solar System as well). Thus in recent years there has been a growing interest in the development and application of fast chaos detection methods. Being aware of the incompleteness of the references below, we refer to the FLI method of Froeschlé et al. (1997), to the method of spectral distance by Voglis et al. (1998, 1999), and to the SALI method of Skokos (2001). In Sándor et al. (2000) we introduced and in Sándor et al. (2004) we refined the concept of the Relative Lyapunov Indicator (RLI), which has also been proved in our investigations an efficient tool of chaos detection. Before the definition of the RLI, we recall the definition of the finite-t Lyapunov indicator originating from an initial point $\mathbf{x}_0$:

$$L(\mathbf{x}_0, t) = \frac{1}{t} \log \frac{||\xi_t||}{||\xi_0||}. \tag{3}$$

The RLI method is based on the fact, that the finite-time approximation of two neighbouring orbits, $L(\mathbf{x}, t)$ and $L(\mathbf{x}+\delta\mathbf{x}, t)$ evolves similarly (as a function of t) for regular orbits, and differently for chaotic orbits. In order to quantify the time-evolution of $L(\mathbf{x}, t)$ for neighbouring regular and chaotic orbits, we introduced the idea of the Relative Lyapunov Indicator (Sándor et al., 2000)

$$\Delta L(\mathbf{x}, t) = |L(\mathbf{x} + \delta\mathbf{x}, t) - L(\mathbf{x}, t)|, \tag{4}$$

where $\delta\mathbf{x} << 1$. Our investigations on different dynamical systems such as the restricted three-body problem (Sándor et al., 2000); the 2D, 4D symplectic mappings, and the stability of certain exoplanetary systems Sándor et al. (2004); Érdi et al. (2004); and Érdi & Sándor (2005) show that the curve $\Delta L(\mathbf{x}, t)$ exhibits typical behaviour for regular and chaotic orbits, which differ essentially from each other.

Since the chaotic behaviour of orbits can be detected after a relatively short time numerical integration, the method of the Relative Lyapunov Indicators enables us to study a large set of initial conditions, thus to discover the regular and chaotic regions of the phase space. In order to separate regular and chaotic regions of the phase space we calculate the average value of $\Delta L(\mathbf{x}, t)$ for a given integration time t^*:

$$\langle \Delta L(\mathbf{x}^*) \rangle_{t^*} = \frac{1}{t^*} \sum_{j=1}^{t^*/\Delta t} \Delta L(\mathbf{x}, j\Delta t). \tag{5}$$

If $\mathbf{x}$ is in a regular region $\langle \Delta L(\mathbf{x}) \rangle_{t^*}$ is small, otherwise, if $\mathbf{x}$ is in a chaotic region $\langle \Delta L(\mathbf{x}) \rangle_{t^*}$ will be larger.

4 Dynamical stability of the habitable zones of exoplanetary systems

In this section we present our results obtained in studying the dynamical habitability of extrasolar planetary systems. After discussing the stability of the habitable zones of individual systems, we present our concept on a stability catalogue for hypothetical Earth-like planets. Finally, we discuss the case of Earth-like exotrojans. (The habitable zone is that region around a star where water can exist in fluid phase on a surface of a planet.)

4.1 Stability of individual systems

The investigation of the stability of the habitable zones of exoplanetary systems began with the work of Jones et al. (2001). They investigated four systems by long-time numerical integration, and found that two of these systems could have stable Earth-like orbits in their habitable zone. More recently, Menou & Tabachnik (2003) have studied the stability of the habitable zones for 85 exoplanetary systems. They found that one fourth of these systems might have stable habitable zones. In the above cases the method of investigation was the numerical integration of the equations of motion of fictitious Earth-like planets.

Instead of numerical integration of individual orbits, we have explored the semi-major axis – eccentricity plane of some exoplanetary systems having two giant objects. In our research (Sándor et al., 2004; Érdi et al., 2004) we studied the systems HD 38529, HD 169830, and HD 168443. In these cases the habitable zones are between the two giant planets.

4.1.1 HD38529

The habitable zone of this system is between 1.4 - 3.0 AU. In Figure 1, left panel we displayed the dynamical structure of the $a - e$ plane between $a = 0.6 - 2.0$ AU. Figure 1 shows how a small Earth-like planet would behave if it was started with initial conditions corresponding to the points of the $a - e$ plane of the plot. The dark region on the right hand side indicates strongly chaotic motion, the light region on the left ordered, thus stable motion. The grey strips correspond to the different mean motion resonances. Studying Figure 1 one can see that near the inner edge of the habitable zone a third planet with negligible mass can exist.

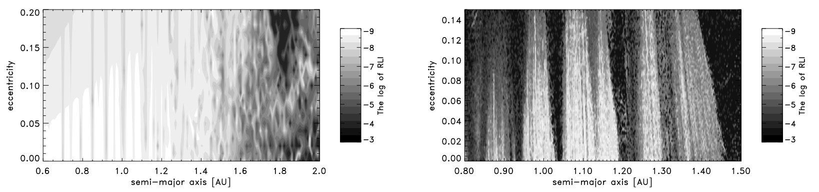

Figure 1: *The $a - e$ planes of HD38529 (left panel) and HD168443 (right panel)*

4.1.2 HD 168443 and HD 169830

The $a-e$ plane of HD 168443 systems are shown in Figure 1, right panel. The habitable zone for HD 168443 is between $0.7 - 1.3$ AU, while for HD 169830 is between $1.4 - 3.0$ AU. In Figure 1 (right panel) one can see that the $a - e$ plane is very chaotic. There are some lighter regions, which seem to allow stable motion for the Earth-like planet, but our careful analysis showed that all orbits are unstable.

The $a - e$ plane of HD 169830 is very similar to HD 168443 therefore we do not show it. The habitable zone of this system is very chaotic, thus it is very unlikely to host Earth-like planets with stable orbits.

4.2 A stability catalogue of Earth-like exoplanets in exoplanetary systems

As we mentioned before, by using data from transit photometry, the orbital parameters of an Earth-like planet can be calculated only with some error limits. The presence of another giant planet results in appearing chaotic (and therefore unstable) regions in the system, thus it means a dynamical constraint, which can help in deriving more accurate orbital elements. Our idea is to compile a catalogue of dynamical stability for exoplanetary systems consisting of a giant Jupiter-like planet and a small Earth-like planet. This stability catalogue can be used to establish immediatelly the stability properties of the habitable zones of the known exoplanetary systems too.

The planned catalogue uses as dynamical modell the elliptic restricted three-body problem (ERTBP), assuming that the giant planet moves around the star in an elliptic orbit with semi-major axis normalized to unity: $a_p = 1$, eccentricity e_p, periastron argument ω_p, and mean anomaly at the epoch M_p. ERTBP depends on two parameters: the eccentricity of the massive planet, and the mass parameter $\mu = m_p/(m_s + m_p)$, where m_p is the mass of the giant planet and m_s is the mass of the star. By using the RLI we shall calculate the structure of the $a - e_p$ plane for several values of the mass parameter μ, where a is the semi-major axis of the hypothetical Earth-like planet and e_p is the eccentricity of the giant planet. First we shall fix $e = 0$, $\omega = 0$ $M = 0$, $\omega_p = 0$, and $M_p = 0$. Later on we shall change M_p between $0° - 360°$.

The compilation of the above described catalogue has begun, in what follows we present one "page" of it. Figure 2 (left panel) shows the structure of the $a - e_p$ plane for $\mu = 0.001$ (the case of the Sun–Jupiter system), when the (normalized) semi-major axis of the hypothetical Earth-like planet is larger than a_p. The light regions correspond to ordered, therefore stable orbits. There are grey "V"–shaped strips, which correspond to different mean-motion resonances. Increasing e_p these resonances overlap each other and a strongly chaotic, thus unstable behaviour appears.

By using this catalogue one shall easily decide whether the habitable zone (HZ) of a given system is dynamically stable, or not. If the physical properties of the central star, the eccentricity and the mass of the giant planet are known, it is easy to place the system's habitable zone on the $a - e_p$ plane. In Figure 2 (left panel) the HZ of four systems are shown. Two of them is entirely in the chaotic region, but the HZ of HD121504 is almost ordered, thus stable. The HZ of HD52265 is mainly in chaotic region, but it contains stable regions too.

Finally we note that the dynamical properties of the $a - e_p$ planes are governed by the different mean motion resonances between the giant planet and the Earth-like planet. The most important resonances are marked in Figure 2 (left panel). These resonances can represent both ordered (stable) or weakly chaotic (becoming unstable after very long time) orbits, depending on the initial angular positions of the planets.

4.3 Stability of co-orbital Earth-like exoplanets

We speak of co-orbital motion when two planets move in nearly the same orbits. In this case they are in a 1/1 mean-motion resonance. In the Solar System there are many examples for this type of motion: the best known representatives are the Jupiter's Trojan asteroids. Thus it can be expected that co-orbital objects exist also in exoplanetary systems. In our investigation (Érdi & Sándor, 2005) we studied that case when the giant planet moves in the habitable zone of the system. Then habitable Earth-like planet could exist only in the vicinity of the stable Lagrangian points of the giant planet – star system since around a stable Lagrangian point there is a region

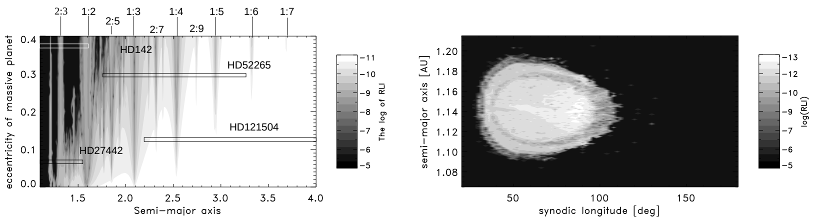

Figure 2: *One page of the stability catalogue (left picture). HD177830: the stability around L_4 (right picture).*

of non-linear stability. Test particles starting from this region librate around the corresponding Lagrangian point in stable orbits. We investigated 5 systems, in which the giant planet is always in the habitable zone, and 4 systems, in which the giant planet, due to its larger eccentricity, can leave temporarly the habitable zone. For our computations we used the modell of the elliptic restricted three-body problem. We found that in all systems co-orbital Earth-like planets can exist.

Figure 2 (right panel) shows such a stability region around the Lagrangian point L_4 of the system HD177830. On the $x-$axis there is the synodic longitude τ, which is the difference between the mean longitudes of the hypothetical Trojan and the giant planet; on the $y-$axis there is the semi-major axis of the hypothetical Trojan planet. We take initial τ and a values from a regular grid on this plane, and assuming that the remaining orbital elements (eccentricity, argument of periastron) are zero, we calculate the RLI for each orbit originating from these initial conditions. It can be seen that around L_4 there is a stability region, which may host stable Earth-like exotrojan planets for very long time in a very stable orbit.

5 Summary

The most efficient ground-based observation method of exoplanets is based on the radial velocity measurements of a star. Unfortunately, this method is unable to detect Earth-like planets. The existence of Earth-like planets, however, is a very important question regarding the formation theories of the Solar System, where both rocky planets and gaseous giant planets have been formed. Thus there are space missions aiming at the detection of Earth-like exoplanets in planning phase, which will be launched in the near future.

In this paper we have investigated the dynamical stability of Earth-like planets in exoplanetary systems. We have found that there are exoplanetary systems, which can host Earth-like exoplanets even in such an exotic case that the Earth-like planets are co-orbital companions of the gas giants. We have begun the compilation of a stability catalogue of Earth-like planets in exoplanetary systems, which can be used to establish the stability properties of the habitable zones of the known exoplanetary sytems. This catalogue will help in determining more accurate orbital elements of Earth-like planets detected by transit photometry too.

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