A NEW DYNAMICAL CLASSIFICATION OF NEAR EARTH ASTEROIDS BASED ON FUZZY LOGIC

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Abstract This work investigates the problem of NEA (Near Earth Asteroids) dynamics. Due to their many close encounters with the inner planets, their motion is highly chaotic - which leads to problems when one wants to calculate their orbits for long time scales. As of the restrictions of the existing classifications (which can be applied only on short or mid term scales), also a statistical treatment of NEAs leads to ambigious results. We introduce a new classification scheme, based on Fuzzy Logic. With this method, it is possible to derive quantitative and qualitativ results on the dynamics of NEAs even for very long time scales.

Keywords: asteroids – NEAs – classification – chaos – dynamics – fuzzy logic – collisions

1. Introduction

Our Solar System is populated with a large number of bodies orbiting the Sun in more or less eccentric orbits. Near circular orbits – like that of the planets – do not cross the orbits of other bodies while smaller bodies are known to suffer from close approaches and even collisions, as we know from many craters on the surfaces of the Solar System bodies. In this work we are investigating the the so called "Near Earth Asteroids (NEAs)" whose orbits bring them close to the Earth. We want to show the problems that arise when one wants to deal with the longterm evolution of this asteroids and how they can be solved by introducing a new classification scheme.

2. Why a new classification of NEAs?

Why is there a need to classify Near Earth Asteroids (NEAs)? And why are the existent classifications no longer suitable for some aspects of scientific research? In order to answer this questions, one has to understand why the existing classifications where created at all. Up tow, there are two different models of NEA classifications.

2.1 Shoemaker's Model

The members of G4, the **N**ear **E**arth **A**steroids (=NEAs), are usually divided into three subgroups[1]:

- the ATENS, with a semimajor axis smaller than the one of the Earth and an aphelion distance $Q = a(1 + e) > 0.983$ AU (mean perihelion distance of Earth)
- the APOLLOS, with a semimajor axis larger than or equal to the one of the Earth and a perihelion distance $q = a(1 - e) \leq 1.017$ AU (mean aphelion distance of Earth)
- the AMORS, with a semimajor axis larger than the one of the Earth and a perihelion distance 1.017 AU $< q < 1.3$ AU

Today (March 2006) the total number of discovered NEAs is 2787 (324 Atens, 1923 Apollos, 1672 Amors). NEAs larger than 1 km in diameter is about 2000 and that of the asteroids larger than 0.1 km in diameter is about 320 000 [8]. A new estimation with slightly different values can be found in Bottke et al. [1].

2.2 Milani's Model

Milanis Model of asteroid classification was derived by the data of the SPACEGUARD project. This project includes data from the integration of 410 asteroids for 200 000 years. The classification of the fore mentioned 89 asteroids was performed by observing their long term behavior. There are four main criteria :

- Values and changes of the orbital elements (a, e, i, q, Q)
- Number and changes of node crossings (NC)
- Number and depth of the close approaches (CA)
- Resonances[2]

According to this main criteria, one distinguishes between the following classes (for details see Milani et al. [7] or Sec. 4): Geographos Class, Toro Class, Kozai Class, Alinda Class, Oljato Class and Comet Class

2.3 Problems with the classification

The need for a new classification is easily explained if one considers the dynamical evolution of NEAs: a large amount of NEAs suffers under continuous close encounters with the inner planets of our solar system. Such a close encounter changes drastically the orbital elements of the asteroid, especially the semimajor axis. The strength of the change depends on the depth of the encounter and the masses involved. This can be seen in Fig. 1, where every close encounter of the asteroid (10563) Izhdubar is reflected in a jump of the semimajor axis. These close encounters and the resulting changes of orbital elements make the orbits of the NEAs highly chaotic (see also [4] and [3] for details on the chaotic behaviour of NEAs)

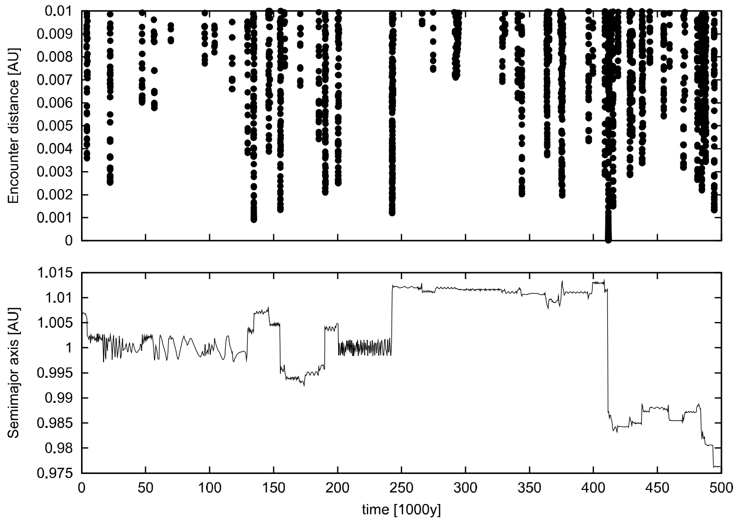

Figure 1. The semimajor axis of the NEA (10563) Izhdubar for 5×10^5 years (lower graph) and the encounters with the inner planets (upper graph); every jump in a reflects a close encounter.

How does this chaoticity affect the classifications? Fig. 2 shows the evolution of the eccentricity and semimajor axis of an Amor asteroid (1993 BX3) in the $a - e$ plane. One can see that the asteroid (which was initially inside the Amor group) has crossed all group borders and has become a member of the Apollo group, then changed to an Aten; in the end, the asteroid has become a Subaten (such changes of initial group have also been reported in [2]). Here one sees how the restrictions of the Aten/Apollo/Amor classification can cause problems. Initially meant to be only used for observational purposes, these groups are only valid for some 100 years. If one tries to apply the classes for longer time scales, one certainly has to fail – thus the Shoemaker classification can not be used when one deals with the longterm dynamics of asteroids. The SPACEGUARD classification is based on the dynamics of the real NEAs that were obtained from numerical integrations for 200 000 years – thus it can be used to classify the dynamical properties of NEAs for some 100 000 years. What happens now, if one tries to use these existing classifications for other purposes? – the classifications will fail and one will encounter major difficulties when dealing with the dynamics of NEAs. Table 1 shows the *mean membership times* of the classification - that is the time a "mean" asteroids spends inside its initial group. One can see that both classifications are comparable concerning these percentages.

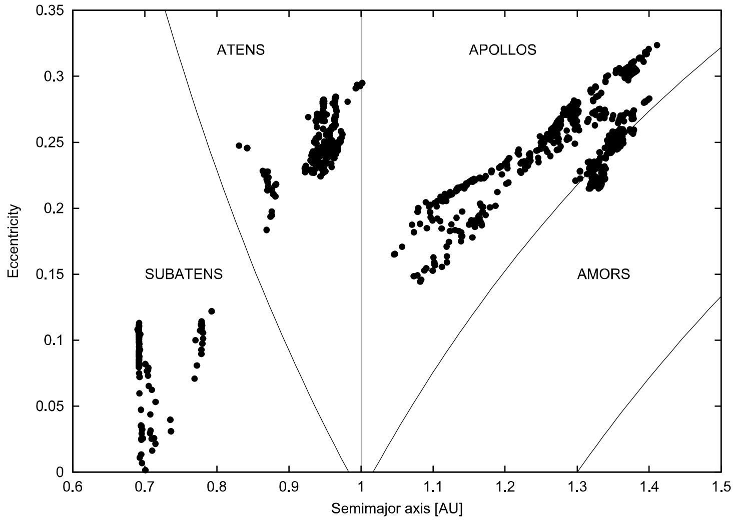

Figure 2. Motion of 1993 BX3 in the $a - e$ plane - the asteroid is a member of all three classes (according to the Shoemaker classification) during the integration time.

Table 1. Mean membership times (in percent of the integration time) for the Shoemaker and the Spaceguard classification (the numbers for the Spaceguard classification where taken from Milani et al. (1989)).

Shoemaker classification		
Atens	Apollos	Amors
76.81	80.98	51.87

Spaceguard classification					
Geographos	Toro	Kozai	Alinda	Eros	Oljato
75.86	22.90	91.70	55.05	83.72	65.13

2.4 Single objects versus groups

If the classifications can not be applied anymore when dealing with longer time scales, why not investigate only single objects? Single objects on chaotic trajectories can not be investigated independently – one has to work with groups of asteroids. If the properties one wants to investigate are the ones the existing classifications were made for (observational properties, short/mid term dynamics), there are no difficulties. But as shown in the last subsection, there are certain problems were the chaoticity of NEAs makes things complicated. If, for example, one wants to calculate the collision probability of 1993 BX3, Fig. 2 shows the problems that arise: to derive the collision probability, one has to take into account the data of the whole time-series – that means the values of the orbital elements for all time steps to derive a single value. It was shown [4] that it makes no sense to use this number – the collision probability – as a property of 1993 BX3 itself: any other integration on another machine would result in a different number. Thus one has to use statistics and to interpret the collision probability of 1993 BX3 as one contribution to the common collision probability of a certain group, whereof 1993 BX3 is a member. But which group would be the right for 1993 BX3? Fig. 2 shows that there is no evident choice – the asteroid is a member of all three groups during the integration time.

2.5 Mixing

The behavior described above is called *mixing*[3]. Because of the *chaoticity* it is difficult to investigate single objects. Because of the *mixing* it is difficult to investigate groups of objects. Thus, for certain problems, a new way of grouping asteroids is needed! For this purpose it is useful to perform a detailed investigation on the mixing behaviour of NEAs. To obtain a new classification, two properties of NEAs were especially important: the collision probability with the inner planets Venus, Earth and Mars and the *BCN* (Border Crossing Number) - a quantity defined as the number of times an asteroid crosses any group border in the Aten/Apollo/Amor classification. These parameters were calculated for all known NEAs (in 2003) - for details see [4]. The next section will explain how they can be used to construct a new classification.

3. Fuzzy classification of NEAs

This section will give some general comments on fuzzy classification and then it will propose new fuzzy classes for NEAs. For an introduction on fuzzy sets see the appendix.

3.1 General remarks

In general, the process of fuzzy classification will proceed as described in the following:

- **Definition:** To obtain a fuzzy set, first of all, a valid definition has to be found. Although fuzzy sets are mathematically exact constructions, their *definition remains relative*. That means, it is possible to translate every "vague" linguistic definition into mathematical notation. Thus one has to be careful, *how* one defines a new class: e.g. although, in spoken language, the words "large" and "tall" can often be interchanged, there exists a difference in their meaning. So a fuzzy class of "large people" should not be identically with a fuzzy class of "tall people". When defining fuzzy classes, one has always to be aware of the meaning of the definition.

- **Membership Functions:** If the definition of the fuzzy class is set up, one will have to obtain the membership function. These functions should represent reality and describe the properties of objects in the basic set according to the definition. Thus one needs a certain parameter that is connected with the definition and according to the distribution of this parameter among the members of the basic set, construct a valid membership function.

- **Classification:** The objects of the basic set can now be classified according to the membership functions. That means, one calculates their grade of membership to all defined fuzzy classes.

- **Analysis:** After all objects were classified, they have to be analyzed. This can be done by using α-cuts (see Equ. (A.6)). As now, in contrary to classical sets, objects can simultaneously be members in different fuzzy sets, they are an adequate tool to obtain a deeper understanding of the new groups: if an α-cut is applied on a fuzzy class, one obtains a classical set, whose members have special properties. E.g. one could apply an α-cut with $\alpha = 0.95$ on the fuzzy set of "large people" (and obtains a set containing only people that belong to this group with a grade of membership larger than 0.95). The important advantage lies therefore in the cross relations of the members of an α-cut and the remaining other fuzzy classes. The α-cut represents an important feature of the objects in the basic set (e.g. "being large") – but every object has also a certain grade of membership to the other groups that were defined. Investigating the distribution of these grades of membership henceforth delivers information on the additional "tendencies" that the objects have besides their dominant features. This makes a fuzzy classification especially interesting for the investigation of the long term dynamics of asteroids!

3.2 Fuzzy NEA classes

The most interesting (and important) feature of NEAs is the possibility that they can collide with the planets of the inner Solar System. Thus, the proposed new classification will describe the collisional properties of NEAs. As of the chaoticity of NEAs an *exact* prediction of collisions (that is, forecasting the date, the time and the place of a collision) is only possible for very short time scales (some hundred years). In this work, the focus lies on the long term behavior and the collision *probabilities* of the asteroids. The fuzzy classification shall now be used to investigate the *tendency of a collision* (which is a slightly different feature). An asteroid, that e.g., due to its orbits has many close encounters with Venus will of course also have a high collision probability with Venus – the "Venus-crossing" orbit is the dominant feature in its dynamics. But deep close encounters with Venus can cause a and e to change drastically and bring the asteroid also close to Earth – so a Venus-crossing NEA can also have a certain tendency for a collision with Earth (and also Mars). These interactions and connections between the planet crossing NEAs can be investigated quantitatively and qualitatively by using fuzzy classes and α-cuts.

3.2.1 Definition. As said before, defining fuzzy classes needs to be done carefully. The purpose of the proposed new classification is to investigate the connections between planet crossing asteroids. So the definition of the new NEA classes will be the following:

- The class of NEAs that can collide with Venus.
- The class of NEAs that can collide with Earth.
- The class of NEAs that can collide with Mars.

Note that the classes are defined by using the words "can collide": an asteroid, that "can collide" with Earth not necessarily has to collide with Earth! As said before, the proposed new classification will be used to investigate the interactions between the planet crossing asteroids – thus a too strict definition would not give the desired results. To get also some information on the variations of the orbital elements, an additional class is introduced:

- The class of NEAs that show almost no mixing.

"Mixing" is defined as above (asteroids that cross group borders during integration time). The underlying classification will be the one according to Shoemaker. The mixing in the Aten/Apollo/Amor classification gives (for long time scales) information on the variations of a and e. If they are very large, the asteroid will cross many group borders and have a larger amount of mixing. As the group borders are "centered" on Earth and (more or less) marked-off by the influence of Venus and Mars, the mixing also gives information, if the motion of the asteroid is “bounded” or not: as described in Sec. 2.3, if the asteroid moves in the right region, larger variations in a and e do not necessarily lead to the crossing of group borders – e.g. an asteroid with moderate variations in a and e can still be in the region between the orbits of Venus and Earth (and thus an Aten) for a very long time and is not related to the population of the group of Mars-crossing asteroids – but *if* it develops tendencies to encounter also Mars, this will be reflected by an increasing amount of mixing[4].

3.2.2 Membership Functions. After having defined the new fuzzy classes

- **G1:** The class of NEAs that show almost no mixing.
- **G2:** The class of NEAs that can collide with Venus.
- **G3:** The class of NEAs that can collide with Earth.
- **G4:** The class of NEAs that can collide with Mars.

now the membership functions have to be derived. One starts with an investigation of the distribution of the basic parameters that are most important for these groups: for G1 this is the BCN, for G2, G3 and G4 these are the close encounters with Venus, Earth and Mars. The distributions for these four quantities are shown in Fig. 3.

These distributions can now be used to obtain a fuzzy membership function:

- **1:** Fit a function through the data (this can be e.g. a linear interpolation).
- **2:** Normalize this function to have only values between 0 and 1.
- **3:** Adjust this function to make sure that it really describes the properties of the desired group.

Fig. 4 shows now the membership functions for the four classes (the necessary numerical integrations of the asteroids were done with initial orbital elements of the JPL Horizons system using the Lie-Series integration technique (see [6, 5])

It can be seen that the shape of the membership functions for G2-G4 are quite similar - which is not very surprising because of the definition of the groups: they were meant to describe classes of asteroids "that can collide" with a planet. Having in mind the chaotic motion of NEAs, it is very unlikely that an asteroid has no close encounters with any planet during its evolution for long times. Thus, also the grade of membership in these classes will be high for a large amount of asteroids. If one would construct a membership

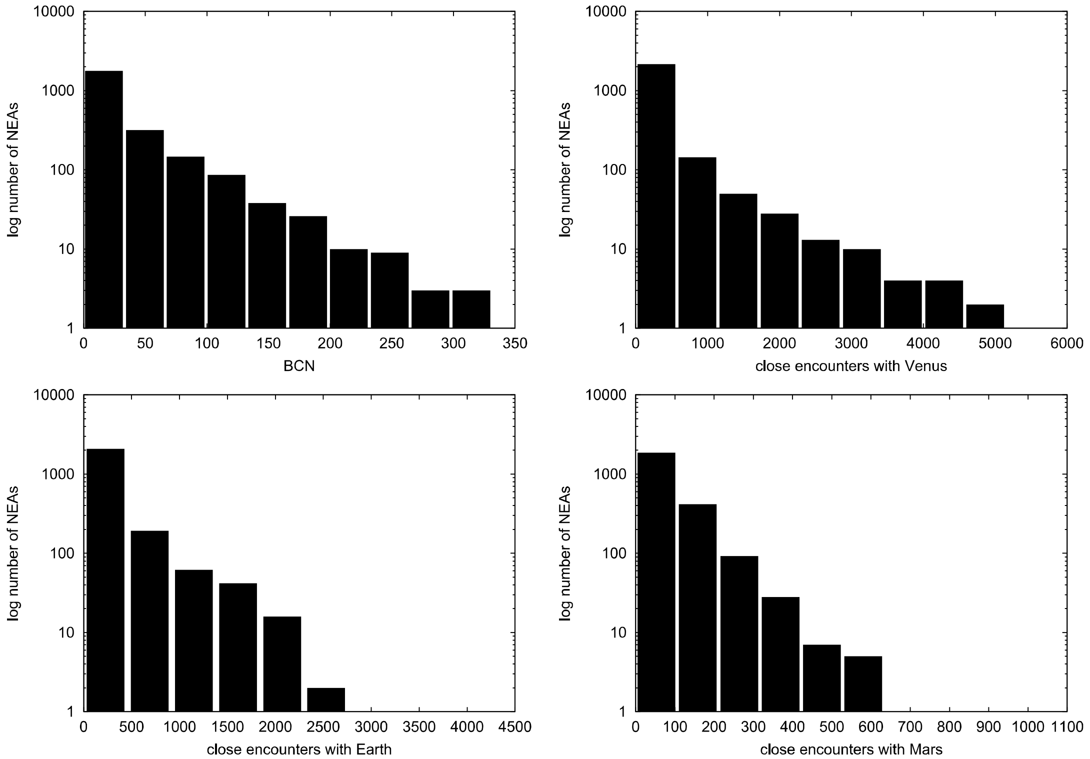

Figure 3. Distribution of BCN (top left) and close encounters with Venus (top right), Earth (bottom left), Mars (bottom right) for the real NEAs (y scale is logarithmic).

function for the group of *"asteroids that are very likely to collide with Earth"* or *"asteroids that could be really dangerous for Earth"*, one would obtain a membership function which is much less steep than the ones presented here. The largest increase shows the membership function for G4 - the class of NEAs that are probable to collide with Mars. As Mars has a very small mass, it is also not so likely for an asteroid to collide with it. Thus *almost any* asteroid that shows at least some encounters with Mars should belong to the group with a higher grade of membership – because due to the chaotic motion every close encounter gives rise to a probable collision in the following evolution.

3.2.3 Classification. With the membership functions derived in the last section, it is now possible to calculate the grade of membership of every real NEA to G1-G4.

Fig. 5 shows the distribution of all real NEAs according to their grade of membership to G1-G4. It can be seen, that for G1, most asteroids have a grade of membership of ~ 1 and thus seem to show only small or moderate changes in a and e. The local maximum of the distribution sets near 0; for the intermediate values, there is a slight increase of the number of asteroids from values for the grade of membership of 0.2 up to 0.9. For G2, G3 and G4, most asteroids have a grade of membership of ~ 0. This maximum is well defined for G2 and

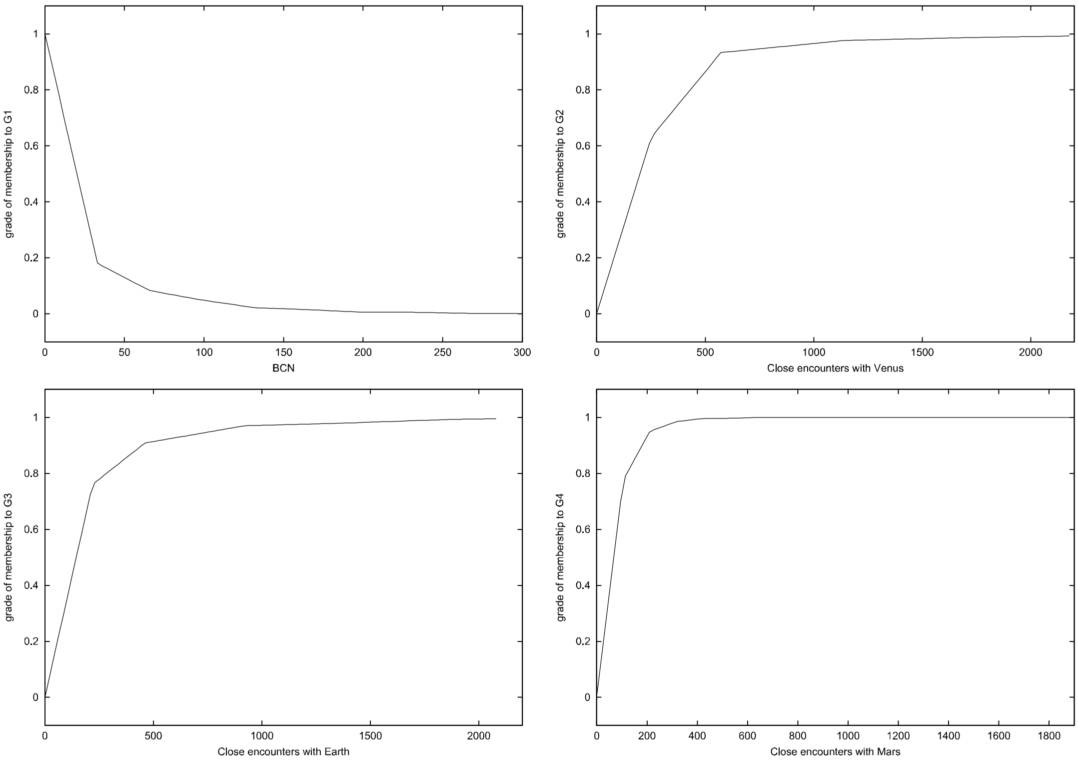

Figure 4. Membership function for the group G1 (top left), G2 (top right), G3 (bottom left), G4 (bottom right).

G3 but for G4, asteroids with a zero grade of membership have only a slight majority. Again, for G2 and G3 the second maximum sets near values of 1; for G4 at slightly smaller values. Also the intermediate values show the same characteristic: the number of asteroids decreases slightly up to grades of membership of ~ 0.6, then increases again. The grades of membership for all NEAs[5] can be found online under `http://www.astro.uni-jena.de/~florian`.

4. Results

Before starting with analyzing the new classes by means of α-cuts, the validity of the classification can also be checked by a comparison with the existing SPACEGUARD classification. As Milani et al. [7] have also classified the asteroids i.a. according to their collision probabilities and close encounters the results should be consistent – at least there, where the two classifications are comparable. The new fuzzy classification will now also include the long term behavior; additionally the basic set of asteroids was much bigger (Milani et al. could only use 410 asteroids – here 2442 NEAs were included). For comparison, we can look at the namesakes of the seven SPACEGUARD classes:

- **(1620) Geographos:** according to the SPACEGU'Dynamical evolution and collisions of asteroids with EarthARD classification, an asteroid of

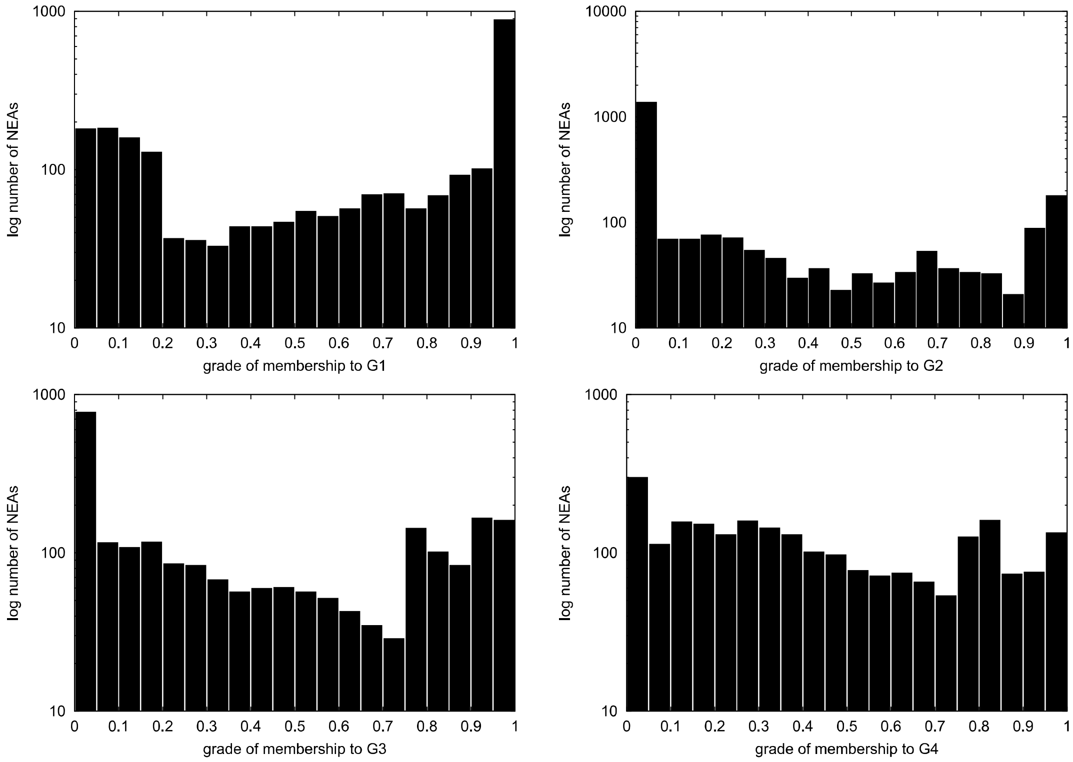

Figure 5. Distribution of real NEAs according to their grade of membership to G1 (top left), G2 (top right), G3 (bottom left) and G4 (bottom right) (y scale is logarithmic).

the Geographos group should show many close approaches to Earth and some to Venus. This means that physical collisions with Earth can occur, if the time scale is long enough. The semimajor axis of a Geographos is almost constant; the eccentricity shows secular trends on small scales – thus it is not expected to move very much in the $a - e$ plane. (1620) Geographos has a membership grade to G1 of 1 – so it is indeed a full member of the group of asteroids that show almost no mixing and its semimajor axis and eccentricity are not expected to change very much. Also the membership grade to G2 (0.03) and G3 (0.76) reflect the behavior described above. The membership grade to G4 (0.86) shows now the influence of the long time scales: during the integration time, some deep close encounters can change drastically the semimajor axis of an asteroid inside the Geographos class (see e.g. figure 5 in [7] for the asteroid (1862) Apollo) and thus force the asteroid to leave the group. Depending on the "direction" of the close encounter such an asteroid can now have also many close encounters with Mars (like for (1620) Geographos) or with Venus (like for (1862) Apollo, which has a grade of membership to G2=0.91). This is a good example how the problems of mixing are bypassed by the new fuzzy classification: in the SPACEGUARD classification, after the close encounter with Venus, (1862) Apollo was no longer a member of the Geographos group – in the fuzzy classification, however, all the dynamical properties that belong both, to the Geographos group and the one, (1862) Apollo would enter afterwards, are known simultaneously.

- **(1685) Toro:** according to the SPACEGUARD classification an asteroid of the Toro group shows close approaches with Earth. Toros are the most unstable group of the SPACEGUARD classification – they tend to leave the group after very short times (In [7], the group of Toro asteroids has only 9 members – thus the statistics are very bad in that case)[6]. Although Toros have close encounters with Earth, they are very shallow; Toros are also protected against collision with Earth by mean motion resonances. *If* they are Venus crossers (which happens not very often), the close approaches with Venus could result in very large changes of the semimajor axis (and thus the resonance with Earth is disrupted). In general, the semimajor axis and eccentricity show only small variations. (1685) Toro indeed has a membership grade to G1 of 1 – so its semimajor axis and eccentricity are not expected to change very much. The membership grade to G2 (asteroids that can collide with Venus) is 0 – also in [7] (1685) Toro is in resonance with Venus and thus protected from close encounters. Membership grades to G3 (0.77) and G4 (0.73) show, that for longtime integration the fore mentioned resonant protection against close encounters with Earth ceases to exist – due to deep close approaches, also encounters with Mars are possible.

- **(1863) Antinous:** it was not possible to compare the results for (3040) Kozai (the most prominent member of the class of Kozai asteroids) because although it belongs to the Mars-crossing asteroids, its perihelion distance is larger than 1.3 and thus is not a NEA in strict sense (Milani et al. did not just use NEAs but all planet crossing asteroids for their classification). (1863) Antinous is, according to the SPACEGUARD classification an asteroid of the Kozai group. Kozai asteroids are, due to Kozai resonances of type I, protected against close encounters and collisions. The evolution of the semimajor axis is very regular and shows only small oscillations. The group of Kozai asteroids is the most stable class in the SPACEGUARD classification. (1863) Antinous indeed shows the described behavior: the grade of membership to G1 is 1, that to G2 (0.23), G3 (0.16) and G4 (0.14) is considerably smaller than that of the fore mentioned asteroids. Nevertheless, although Kozai asteroids should be protected from collisions, the time scales, that the fuzzy classification is based on, are longer than the protection time scale – thus the grades of membership to G2-G4 are not zero.

- **(887) Alinda:** according to SPACEGUARD classification, asteroids of the Alinda group are in (low order) mean motion resonances with Jupiter. The Alinda class is the one, for which the most difficult boundary problems existed – thus it was often difficult to decide, if an asteroid was an Alinda or not. Their eccentricities can undergo large changes, the semimajor axes oscillate around the resonant value. As of their probable large changes asteroids can encounter all inner planets but are often protected against collision by resonances.(887) Alinda has a smaller grade of membership to G1 (0.93) than the asteroids mentioned before; also the grades of membership to G2 (0.12), G3 (0.08) and G4 (0.04) are considerably smaller – showing the resonant protection.
- **(433) Eros:** according to SPACEGUARD classification, asteroids of the Eros group are those, which do not cross the orbit of Earth because their perihelion is always higher than 1 AU. All Eros asteroids are Mars crossers and have close approaches with Mars. The eccentricities of Eros asteroids can show very large changes. The grade of membership of (433) Eros to G2 and G3 is 0, it is only a member of G4 (0.72). Also the membership to G1 (0.9) is smaller than 1, indicating the larger changes of a and e. Another good example for the behavior of Eros asteroids is (719) Albert. Its membership to G1 (0.08) is very low (indicating very large changes in a and e), again the grade of membership to G2 and G3 is zero and that to G4 is 0.32.
- **(2201) Oljato:** according to the SPACEGUARD classification, asteroids of the Oljato group have orbits that show large-scale chaotic effects. They have very high eccentricities and can have close approaches to all inner planets. (2201) Oljato indeed has a grade of membership to G1 of 0.04, indicating the chaotic changes in a and e; it also shows a medium grade of membership to G2 (0.38), G3 (0.36) and G4 (0.28) – thus it encounters all inner planets.
- **Comet class:** a comparison with the class of Comet asteroids of the SPACEGUARD classification is not possible in this work. This class consists of all asteroids that spent a sufficient part of integration time in the outer Solar System. In this work, asteroids with that behavior were excluded from the fuzzy classification – first because of reasons of comparison: some of these asteroids escaped the inner Solar System that fast that not enough data would be left to calculate a valid membership grade. Second, to compare the data with the SPACEGUARD classification, it would have been also necessary to investigate the entry/exit path of the asteroid in the inner Solar System, which would have resulted in the calculation of hyperbolic/parabolic orbits which lies outside the framework of this study

The new fuzzy classification is indeed capabale to describe the dynamics of NEAs. In contrary to the SPACEGUARD classification, now the effects that take only place on long time scales are included; also the problem of mixing has been bypassed!

4.1 α-cut analysis

In contrary to classical sets, the asteroids can simultaneously be members in different fuzzy sets. Thus an adequate tool to investigate the fuzzy classes has to be used. As shown before, α-cuts are a proper way to investigate fuzzy classes. By applying an α-cut to a certain fuzzy group, one obtains classical sets and can now investigate the properties of its members. For this purpose, out of the fuzzy classes of *"asteroids that can collide"* with a planet, classical sets containing that bodies, that are *very likely to collide* are extracted by means of α-cuts. Then the members of the classical sets can be examined according to their grade of membership to the remaining groups. This type of investigation is the greatest advantage of the new fuzzy classification. In contrary to existing theories, where asteroids can inhabit only one class at time and transitions between the classes can only be investigated as time passes by, the fuzzy classes allow one to examine the membership to the different classes *simultaneously*. We will show the details of the α-cut analysis only in one case; additional studies can be found in [4].

4.1.1 $G3^{>0.9}$. The set $G3^{>0.9}$ contains all NEAs with μ_{G3} larger than 0.9 – these are "*asteroids that are likely to collide with Earth*". This group contains 329 bodies. Fig. 6 shows the distribution of these asteroids according to their grade of membershipe[7] to G1, G2 and G4.

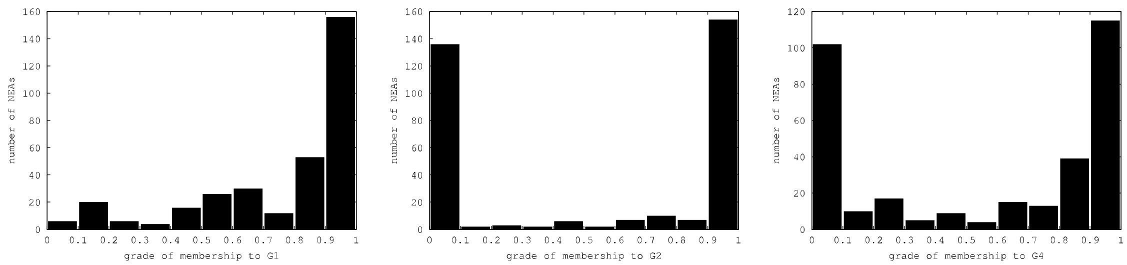

Figure 6. Distribution of asteroids in the group $G3^{>0.9}$ according to their grade of membership to G1, G2 and G4.

Unlike $G2^{>0.9}$, $G3^{>0.9}$ has more members with a medium grade of membership (50.76%), only 47.52% have high grades. The fact, that NEAs that are likely to collide with Earth show more mixing (and thus larger variations in a and e) is first due to the type of basic groups that is used to derive the mixing: the Shoemaker classification is "centered" on Earth – so asteroids that often come close to Earth also have a higher probability to cross the border between Atens and Apollos. But Fig. 6 (middle and right) shows, that asteroids from $G3^{>0.9}$ also have larger variations of a and e in general and thus come (very) close to Venus and Mars too: 46.81% of them have high grades of membership to G2, 34.95% to G4. An important property of $G3^{>0.9}$ asteroids can be seen in the difference of low and intermediate values to G3 and G4: only 11.85% of them have medium grades to G2, whereas three times more of them (34.04%) have intermediates grades to G4. Thus, the "connection" between Earth and Mars-crossing asteroids is more fluent: NEAs that are likely to collide with Earth in the majority are also likely to collide with Mars – and, also in the majority, are likely to collide with Venus; but the lack of intermediate grades of membership to G2 shows that the interaction between Earth and Venus is much stronger. *If* deep close encounters bring an asteroid near Venus (which is the case for slightly more than half of asteroids), it is very probable that they have very much close encounters (and thus also a higher collision probability) with Venus. On the other hand, if they come close to Mars (which is also the case for slighlty more than half of asteroids) the probability that they have a high or intermediate number of close encounters is almost equal (34.04% of $G3^{>0.9}$ have medium grades of membership, 34.95% have high grades). Earth is able to "protect" its crossing asteroids much more easier from the influence of Mars than that of Venus.

5. Conclusions

Concerning the question of the danger of Earth by NEAs, Fig. 7 shows in detail, how the different groups consist of members of the other groups. One should stress again the fact, that due to the combination of fuzzy set theory and dynamical studies of NEAs, it was possible to obtain a *quantitatively* description of the planet-crossing behaviour on long time scales – that was not possible in the past because of the chaoticity of the NEAs and the problems that were due to the fixed, not flexible existing classifications! The group of NEAs that are likely to collide with Earth not only itself has the largest number of members, also the asteroids in the other groups are more often members in the group of NEAs that are likely to collide with Earth than vice versa. This leads to the following conclusion: NEAs move on orbits with semimajor axes from ~ 0.6 to ~ 3 AU (depending on their eccentricity). They can come close (and also collide) with all large inner planets. For long time scales, the NEA population is of course not constant: their number can be reduced due to collisions with the planets or the sun ("sun grazers"); it can be increased by asteroids that are thrown out of the main belt. But *as long* as they are NEAs, independent from their position in the $a - e$ plane, it was shown by introducing fuzzy classes, that they have the tendency to evolve Earth-crossing orbits. Thus, for very long

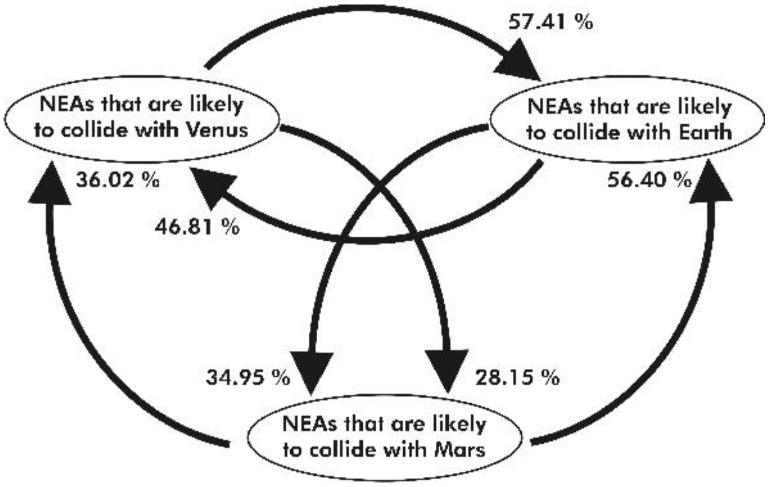

Figure 7. Groups of asteroids that are likely to collide with a planet. The arrows show, how many NEAs of one group, are also members of an other group.

time scales, the major reason for the decrease of NEA population will be due to collision with Earth! Encounter and collision frequencies of asteroids with Earth (and the other planets) will therefore differ from the current values when including the evolution of NEAs for very long times. Future studies, that have to include the flux from main belt asteroids to NEAs and also the effect of sun-grazing bodies, should confirm these statistical results numerically.

Appendix: Fuzzy sets

Fuzzy set theory or *Fuzzy Logic* was developed in 1965 by L.A. Zadeh [10]. Fuzzy sets are an extension of classical sets. A classical set is *two-valued*: for every set A there exists a function f_A that has either the value 1 or 0 with:

$$f_A(x) = 1 \Leftrightarrow x \in A \text{ and } f_A(x) = 0 \Leftrightarrow x \notin A. \quad \text{(A.1)}$$

This function is called *characteristic function* of A. Fuzzy sets, in contrary, have a characteristic function μ_A defined for *all values between (0,1)*, describing the degree to which an element x is included in the set A. Fig. A.1 shows an example of the membership functions describing the degree of membership to the groups of "short", "huge" and "average" sized people.

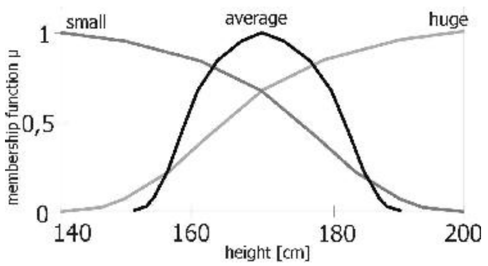

Figure A.1. Fuzzy membership functions for the classes "short", "huge" and "average".

Fuzzy sets have the following properties:

- Classical sets can be interpreted as fuzzy sets with membership grades of only 0 and 1
- Two fuzzy sets A and B are equal over a whole set X if

$$A = B \Leftrightarrow \mu_A(x) = \mu_B(x) \qquad \forall x \in X \tag{A.2}$$

- The union of fuzzy sets A and B is the fuzzy set defined by the following membership function:

$$\mu_{A\cup B}(x) = \mu_A(x) \vee \mu_B(x) \tag{A.3}$$

- The intersection of fuzzy sets A and B is the fuzzy set defined by the following membership function:

$$\mu_{A\cap B}(x) = \mu_A(x) \wedge \mu_B(x) \tag{A.4}$$

- The complement $\bar{A}$ of a fuzzy set A is defined by the following membership function:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \tag{A.5}$$

- For a fuzzy set A

$$A^{>\alpha} = \{x \in X \mid \mu_A(x) > \alpha\}, \qquad \alpha \in [0,1] \tag{A.6}$$

$$A^{\geq\alpha} = \{x \in X \mid \mu_A(x) \geq \alpha\}, \qquad \alpha \in [0,1] \tag{A.7}$$

  are called the **weak α-cut** and the **strong α-cut**, respectively.
- The α-cuts of fuzzy sets are classical sets.

Notes

1. This definition follows [9]
2. The typical periods of resonances are longer than 200 years, therefore the analysis of resonances is not affected by the filtering process.
3. In this work, the word *mixing* is only used to describe the fact that an asteroid changes from one class to another – it is not meant to be confused with other definitions of mixing (like in statistics or chaos theory).
4. Note that the mixing itself does not depend on the type of classification (see Sec. 2.3) – the Aten/Apollo/Amor classes were chosen because they are much simpler to handle than the SPACEGUARD classification.
5. Note that here some asteroids were excluded because they escaped the Solar System during integration time
6. In this context, the word “unstable” is not related to orbital stability! It only means, that asteroids do not fulfill the requirements to be a Toro for long time scales and change to other groups often.
7. Here and in the following, *high* grades of membership are defined by $\mu_{Gi} \geq 0.9$, *medium* grades by $0.1 \leq \mu_{Gi} < 0.9$ and *low* grades by $\mu_{Gi} < 0.1$.

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