EXCHANGE ORBITS IN PLANETARY SYSTEMS

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Abstract In this study we show the results of a numerical determination of the stability of planets in exchange orbits. These kinds of orbits are defined such that two small but massive bodies with almost the same semimajor axes on nearly circular orbits are moving around a much more massive host. Because the planet on the inner orbit is faster it approaches the outer body from behind. Before they meet, the inner body is shifted to the orbit of the outer and vice-versa the former outer body moves to an orbit with a smaller semimajor axis. We did our numerical experiments for different masses of the two planets involved and different initial separation of the semimajor axis. It turned out that for stable exchange orbits the sum of the mass of the two planets can only slightly exceed the one of Saturn.

Keywords: Extrasolar planetary systems, terrestrial planets, exchange orbits

1. Introduction

The search for extrasolar planets led up to now to the knowledge of 185 planets in 149 extrasolar planetary systems (EPS)[1]. Almost all these planets are giants with a few exceptions; the planet with the lowest mass found has 5.5 masses of the Earth. One primary goal of searching for EPS is to find terrestrial planets in so-called habitable zones (HZ) ([10]). There are different possibilities for terrestrial planets (TP) to move on stable orbits even when a large planet is present: when the giant is outside the HZ a TP may move inside (like in our Solar System), when a hot Jupiter is moving close to the host star a TP may move on a stable orbit in the HZ. A lot of effort has been undertaken to define such stability zones in existing EPS (e.g. [1], [4], [5], [6], [12], [11], [13], [14], [16], [18], [19], [20], [21]). Additionally we can imagine TPs as satellites of giant planets and also in 1:1 mean motion resonances (MMR) with a Jupiter like planet.

These 1:1 resonant orbits are of special interest for asteroids in our Solar System. It is due to the fact that in a region 60° before Jupiter and 60° behind the largest planet a large number of asteroids are populating this region. Many analytical and numerical work has been devoted to the stability of these two 'clouds' of asteroids, which are named after the warriors of the Trojan war. The Trojans librate about these two stable equilibrium points[2] in the so-called tadpole orbits having orbits with two well distinct periods (almost 12 years and 149.6 years) which are visible in Fig. 1 (upper graph).

When the libration around the one Lagangian point grows and reaches a point which is opposite to the location of Jupiter with respect to the Sun the orbits merge with the orbits around the other equilibrium point. These kind of orbits – because of their appearance in a rotating frame in which Jupiter and the Lagrangian points have fixed positions – are called horseshoe orbits (see Fig. 1, lower panel)

In the case of an asteroid and Jupiter in the 1:1 resonance there is not any 'measurable' effect on Jupiters orbit, because of the smallness of the mass of the asteroid compared to the one of Jupiter. The situation is quite different when the two celestial bodies involved have comparable masses and are both small compared to the central mass. Surprisingly enough some years ago two satellites in the Saturn system were discovered which have exactly these kind of orbits which we call now *exchange orbits*. The exchange orbits (e-orbits) of the general three body problem can be described as follows:

Two small but massive bodies are moving on nearly circular orbits with almost the same semimajor axes around a much more massive host. Because of the 3^{rd} *Keplerian law the one with the inner orbit is faster and approaches the outer body from behind. Before they meet, the inner body is shifted to the orbit of the outer and vice-versa the former outer body moves to an orbit with a smaller semimajor axis: they have changed their orbits and their semimajor axis!*

This interesting interplay may be stable for millions of encounters as we will see in the next chapters. In the satellite system of Saturn the two moons Janus and Epimetheus (the orbits of these two moons differ only by 50 km[3] and have themselves diameters of more than 100 km) have exactly these kinds of orbits; so we postulate that this may apply to extrasolar planets too. Early work concerning exchange orbits was accomplished by [22] who described the u-shaped orbits during the close encounter (in a rotating frame!) and also [2] who established stability regions depending on the masses involved. Recent numerical integrations and analytical estimations show that e-orbits are stable up to a mass ratio where a TP is in exchange with a Saturn like planet (e.g. [14]).

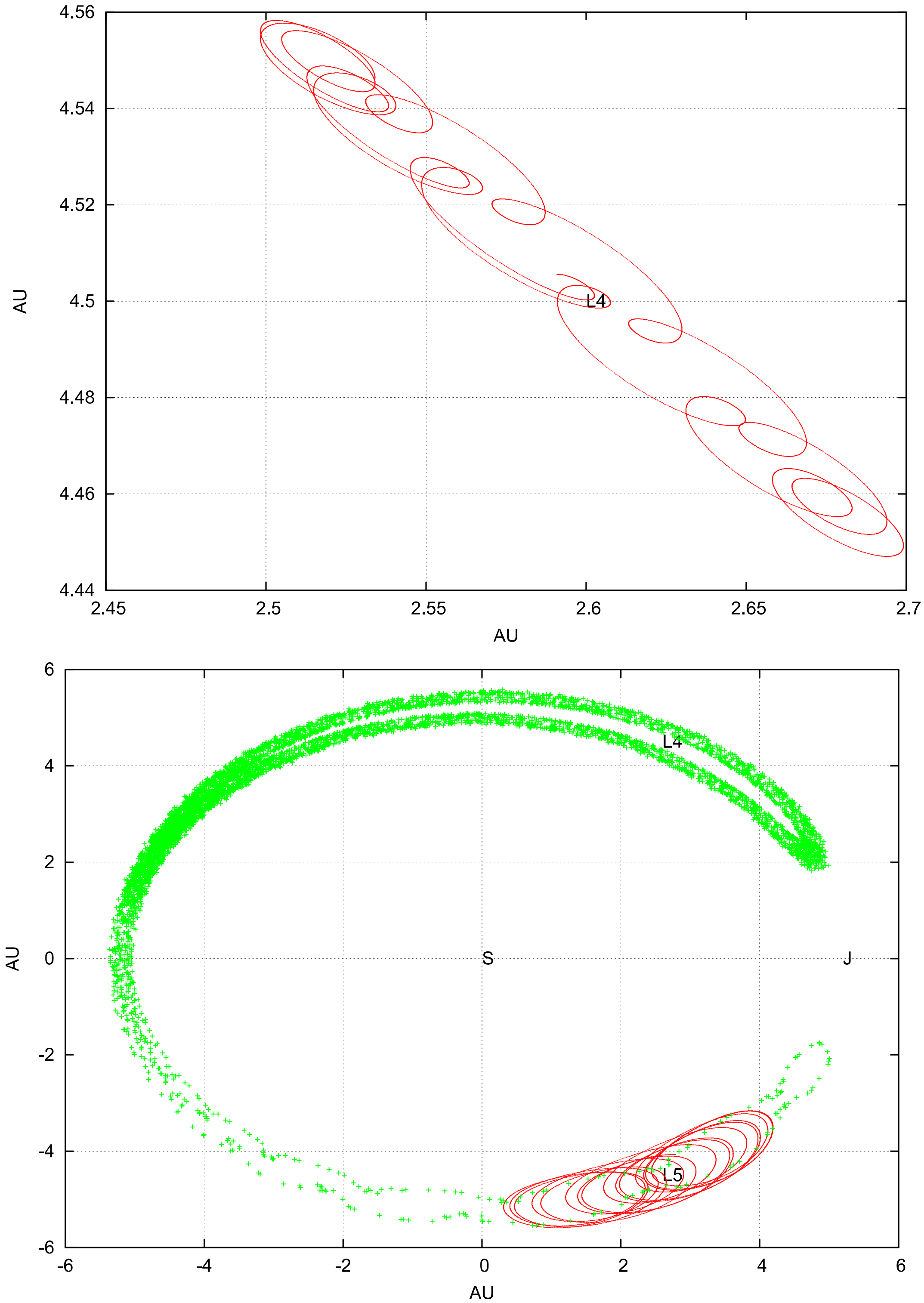

Figure 1. Orbit in the restricted three-body problem around the Lagrangian points in the rotating frame: around L_4 (upper graph), around L_5 and around both equilibrium points (lower graph). Note that the last orbit is in an exchange orbit in the full three body problem; for detailed explanation see in the text.

Another study by [7] was devoted to the problem of the dynamics of systems of two close planets which is in a certain sense similar to the problem we are dealing with. But in contrary to exchange orbits there he studied stability against close encounters which may led to escapes when they come as close as the Hill's radius[4]. [13], [14] studied the different possibilities for two planets in 1:1 resonance and how they would be detectable with the aid of their radial velocity curves. [17] have shown that two planets in 1:1 MMR can be stable for quite different orbital parameters. In a recent paper [8] showed that two ESP with planets in a possible 2:1 MMR could also be in the 1:1 MMR with more or less the same Radial Velocity Curve.

2. The stability limits

To establish stability regions depending on the parameters which determine the exchange orbits (mass ratio of the planets and total mass of the planets compared to the central body and difference in semimajor axis Δa) we did numerical integrations of the equations of motion of the full 3-body problem. We used the Lie-integration (e.g. [9], [15]) with an adaptive step-size to be able to model in a proper way the encounters of the two planets. We always started the two planets on circular orbits on both sides of the central star in 1 AU with an increasing value of the difference in semimajor axes Δa for every single experiment. We checked the maximum eccentricity during the integration time of 10000 years; we emphasize that the number of encounters depend on Δa.

In Fig. 2 we can see the limits of stability for e-orbits where two equally massive TPs are involved. We plotted these differences Δa versus the maximum of the eccentricity . There it is visible (upper graph, shaded region in Fig. 2) that up to the distance $\Delta a = 0.02$ AU ($a = 1.01$ AU for the outer and $a = 0.99$ AU for the inner planet) the eccentricity stays very small; it means that the orbits were stable even after thousands of encounters. Then we see large maximum eccentricities between 0.96 AU and 0.99 AU (1.01 AU and 10.4 AU) which are the sign that after an encounter the planets had quite different orbits and left the exchange orbits. Then they are again stable because they are too far from each other and may pass without major perturbations (the eccentricities are again small). The small 'hills' on both sides are due to high order resonances which cause slightly larger perturbations, but the two orbits are well separated and almost circular. We have undertaken these numerical experiments for four different pairs of planets with equal masses are the following ones: Earth – Earth, S-Earth – S-Earth[5], Uranus – Uranus and Saturn – Saturn.

In Fig. 3 we depicted a zoom of the results shown in Fig. 2: here it can be seen that for large masses involved the differences in semimajor axis can be larger and the e-orbits are still stable: the respective results for two equally massive planets in exchange orbits with a mean semimajor axis $a = 1$ AU are shown in Table 1.

Table 1. The extension of the stable regions for exchange orbits for two equally massive planets around a Sun-like star with a small difference Δa in semimajor axes

2 planets	**lower limit**	**upper limit**	Δa
Earth – Earth	0.994 AU	1.006 AU	0.012 AU
S-Earth – S-Earth	0.990 AU	1.010 AU	0.020 AU
Uranus – Uranus	0.988 AU	1.012 AU	0.024 AU
Saturn – Saturn	0.982 AU	1.018 AU	0.036 AU

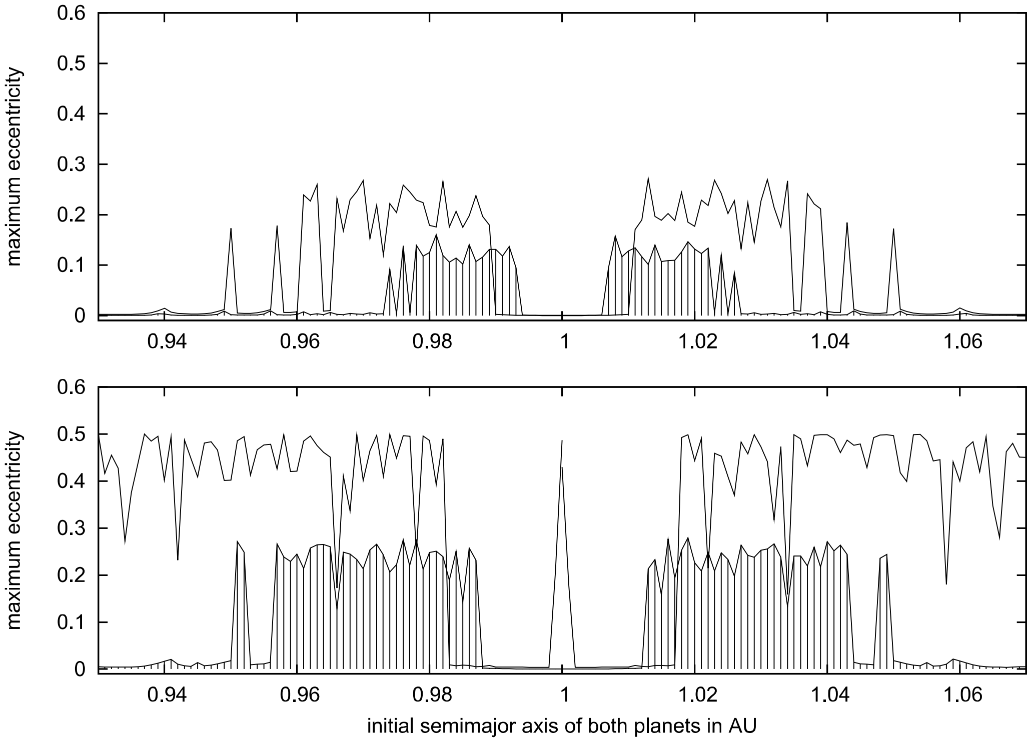

Figure 2. Stable regions for exchange orbits for 4 examples of two equally massive TPs: Earth, Super-Earth, Uranus, Saturn; the initial semimajor axis of the two planets is plotted versus the maximum eccentricity.

In Fig. 4 we plotted the stability regions for three different pairs of planets in e-orbits: upper graph the Earth (as inner planet for the beginning of the integration) with S-Earth (as outer planet for the beginning of the integration); middle graph the Earth with Uranus and lower graph the Earth with Saturn. The large e_{max} values (y-axis on left part of the plot) are the ones of the Earth, the smaller ones (y-axis on right part of the plot) are the e_{max} values for the more massive planet. The region in the middle, with e_{max} close to zero, is the

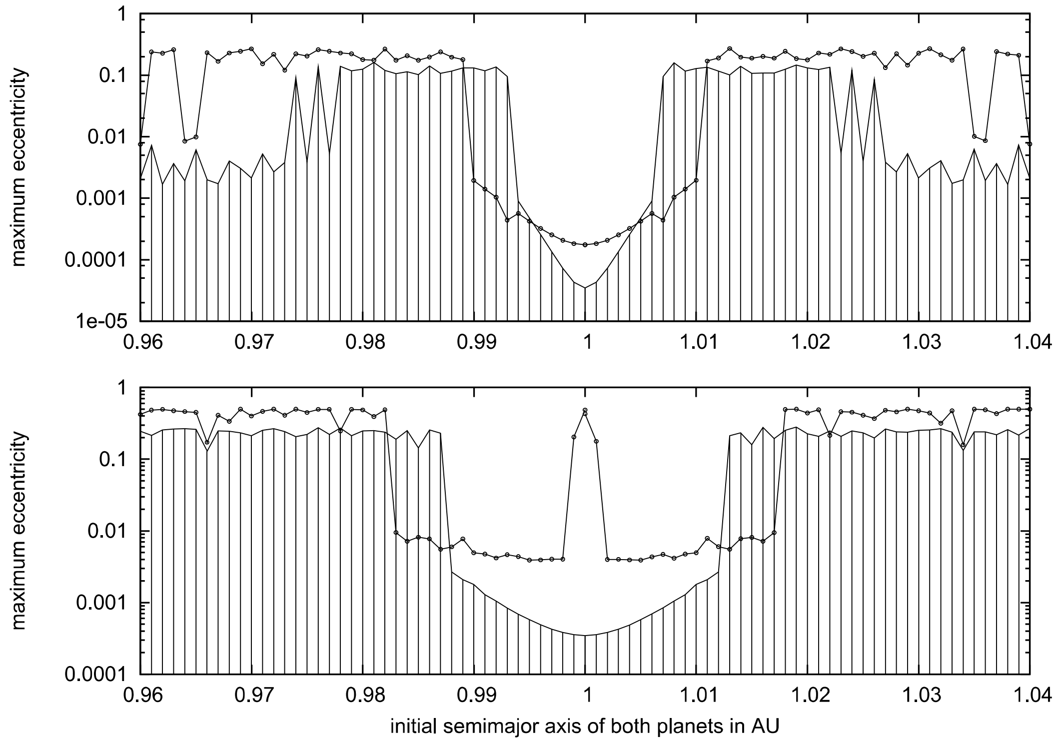

Figure 3. Zoom of Fig. 2 but for logscale in y.

domain of stable e-orbits. One can also see that the larger the total mass, the larger is the stable region, a fact which is also visible from Tab. 1.

In Fig. 5 we show the results of numerical integrations of different pairs of planets in e-orbits: S-Earth with Uranus (upper graph); S-Earth with Saturn (middle graph) and as Uranus with Saturn. The trend is the same as already shown in the last figures: the stable region increases with the masses involved. Nevertheless there is a limit for the total mass of the planets: the numerical experiments for e-orbits have shown that approximately 2 Saturnmasses are this limit (which also agrees with the analytical model by [3]).

In Fig. 6 we show for three examples of equally massive planets how the semimjaor axes evolves during 2000 years. We can see a butterfly - like diagramm: the 2 planets start on opposite sides of the planet($\Delta\lambda = 180°$) with a small difference in semimajor axes (Δa). During the integration the two planets approach, Δa increases and just before their encounter Δa has its largest value. Now the two planets change their orbits: the inner one, which is moving faster, is shifted outwards and the outer one is shifted innwards and therefore it is on a 'faster track'. Consequently now this planet approaches the other planet from the inside; when they are on the opposite sides of the host star the Δa is again as small as at the beginning of the integration and the procedure repeats. The upper graph is for an Earth-Earth pair, the middle graph for S-Earth –

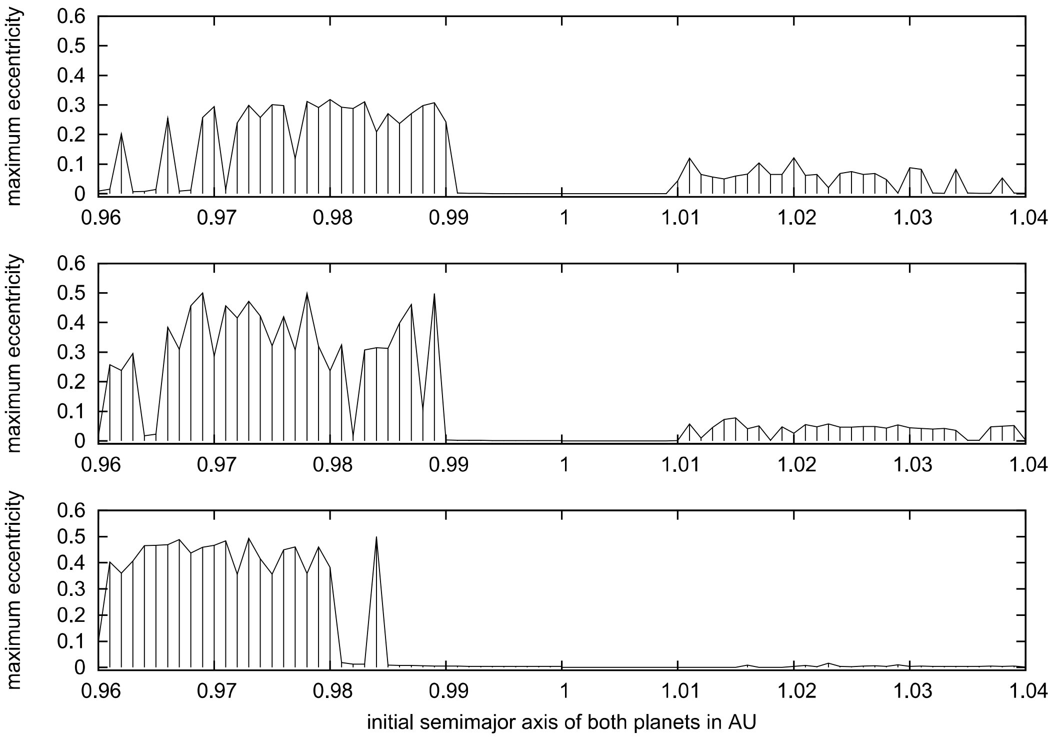

Figure 4. Stable regions for exchange orbits for 3 examples of pairs of planets: in all three examples the inner planet is the Earth; the outer planet is a S-Earth (upper graph), Uranus (middle graph) and Saturn (lower graph). The initial semimajor axis of the planets is plotted versus the maximum eccentricity of their orbits.

S-Earth and the lower graph for Uranus – Uranus. The encounter frequency depends on the separation of the semimajor axes Δa.

In Fig. 7 we depicted the change of the semimajor axes after every close encounter of a pair of Earth and Saturn. It is clearly visible that the Earth suffers from bigger jumps in semimajor axis during the encounter than Saturn, a consequence of the smaller mass of the Earth.

An important point is the long-term stability of such orbits. Is this a transit configuration, or, can these kind of orbits survive for millions of encounters? To answer this question severeal tests were undertaken and we could show that these kind of orbits are very stable. In the respective Fig. 8 we depicted the semimajor axes for the first million years and for the time interval from 9 to 10 million years; in Fig. 9 we show the development of the eccentricity for the same periods of time. The results of two Earth-like planets with an initial $\Delta a = 0.08$ show the regularity of the orbits even after almost ten thousands of encounters (exchanges).

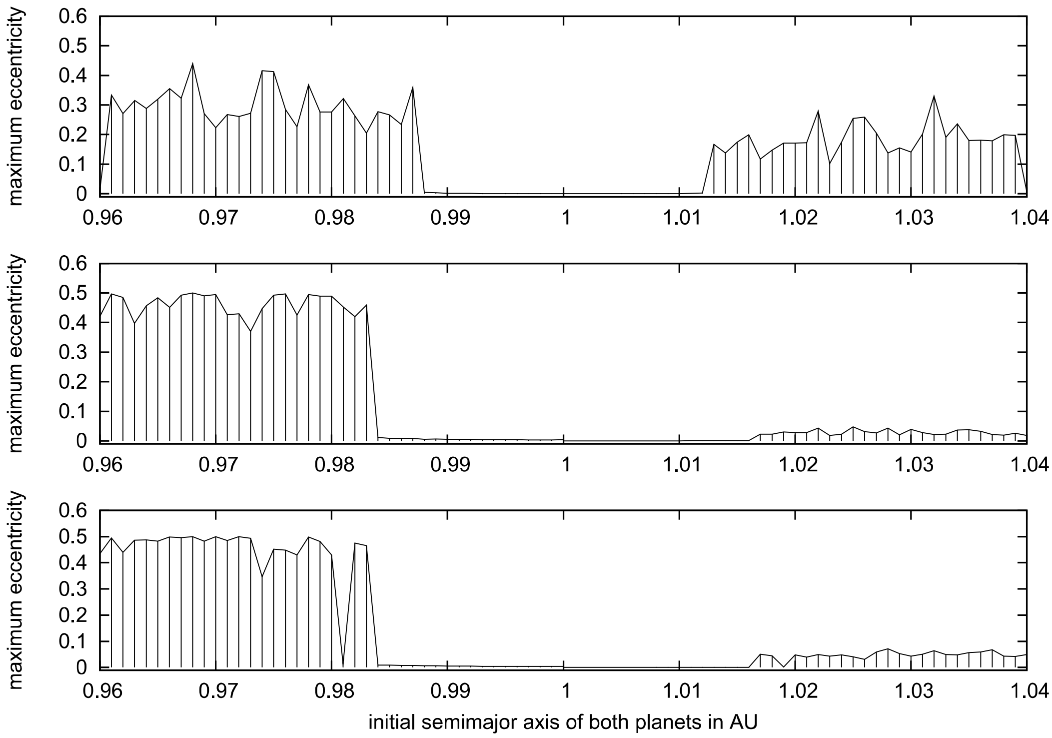

Figure 5. Stable regions for exchange orbits for 3 examples of pairs of planets: S-Earth and Uranus (upper graph), S-Earth and Saturn (middle graph) and Uranus and Saturn (lower graph). The initial semimajor axis of the planets is plotted versus the maximum eccentricity of their orbits.

3. Conclusions

The search for terrestrial planets in EPSs is a hot topic for observing astronomers nowadays. To establish regions where stable orbits of TPs in extrasolar planetary systems may survive is a challenge for astronomners working in Astrodynamics. Besides inside or outside the orbit of a giant planety one possibility for TPs is to move in 1:1 MMR like Trojans and satellites of Jupiter-like planets. Numerical estimations led to the conclusion that for a Solar type host star the mass limit for exchange orbits in the distance of 1 AU (thought as the habitable zone) is just below two Saturns ($0.0003 M_{\text{sun}}$). These means that even a Saturn like giant may exchange orbits with an Earth-like planet; unfortunately most of the gasplanets discovered up to now are in the size of Jupiter or even larger. Nevertheless we expect for planetary systems to host also Neptune and Uranus like planets – and also smaller planets, namely the terrestrial ones – and consequently we cannot exclude the realisation for planets in this type of orbit. Although it seems to be a very unlikely configuration the fact, that Janus and Epimetheus in the Saturn satellite system have such orbits, teaches us that the probability of a realisation of planets in e-orbits is not zero. To summarize we can see that e-orbits are possible only for almost

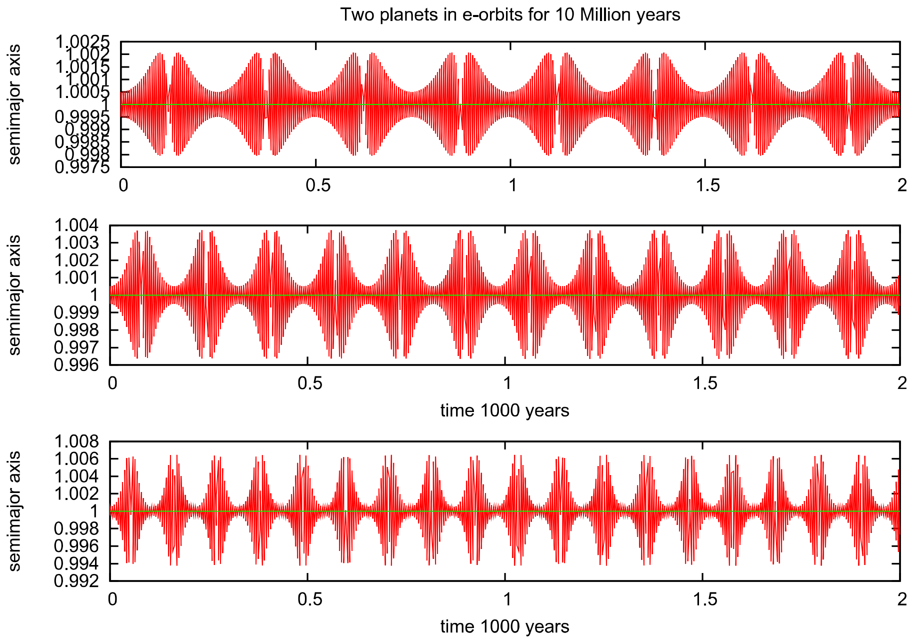

Figure 6. Time development of semimajor axis for 200 years of 3 different pairs of planets: Erath – Earth (upper graph), S-Earth – S-Earth (middle graph) and Uranus – Uranus (lower graph).

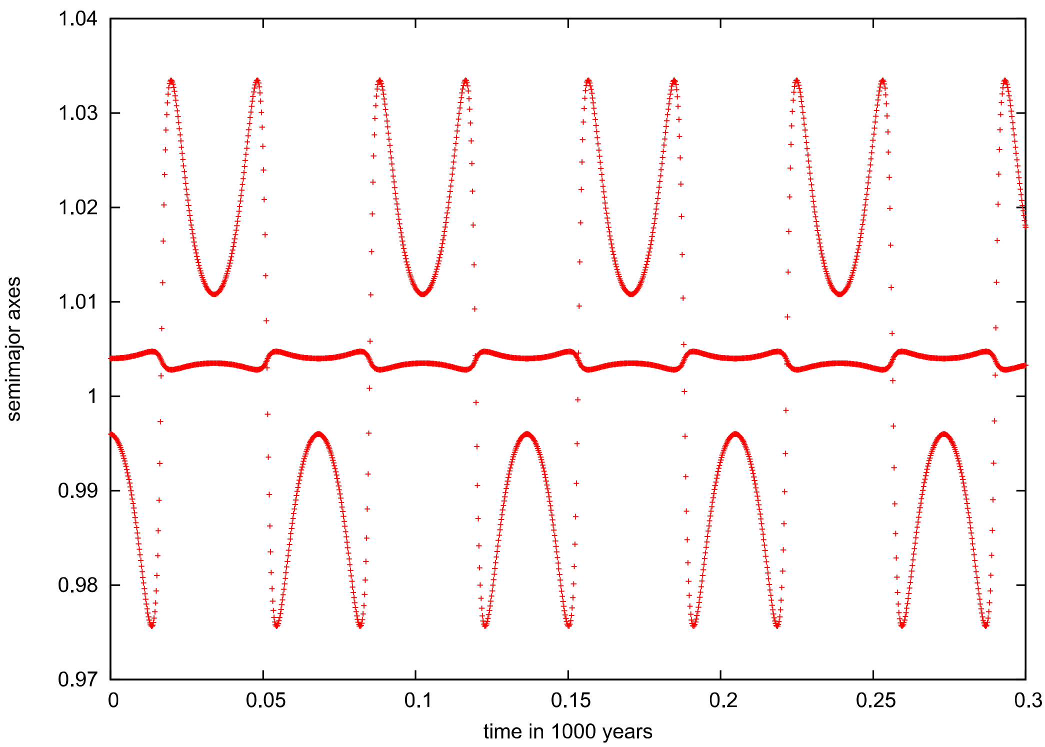

Figure 7. Exchange orbits for Earth and Saturn for 300 years in a distance of 1 AU; every 40 years the close encounter leads to an exchange of the orbits. The semimajor axes (y-axis) are plotted with respect to the time; the thick line close to $a = 1$ AU is a_{Saturn}.

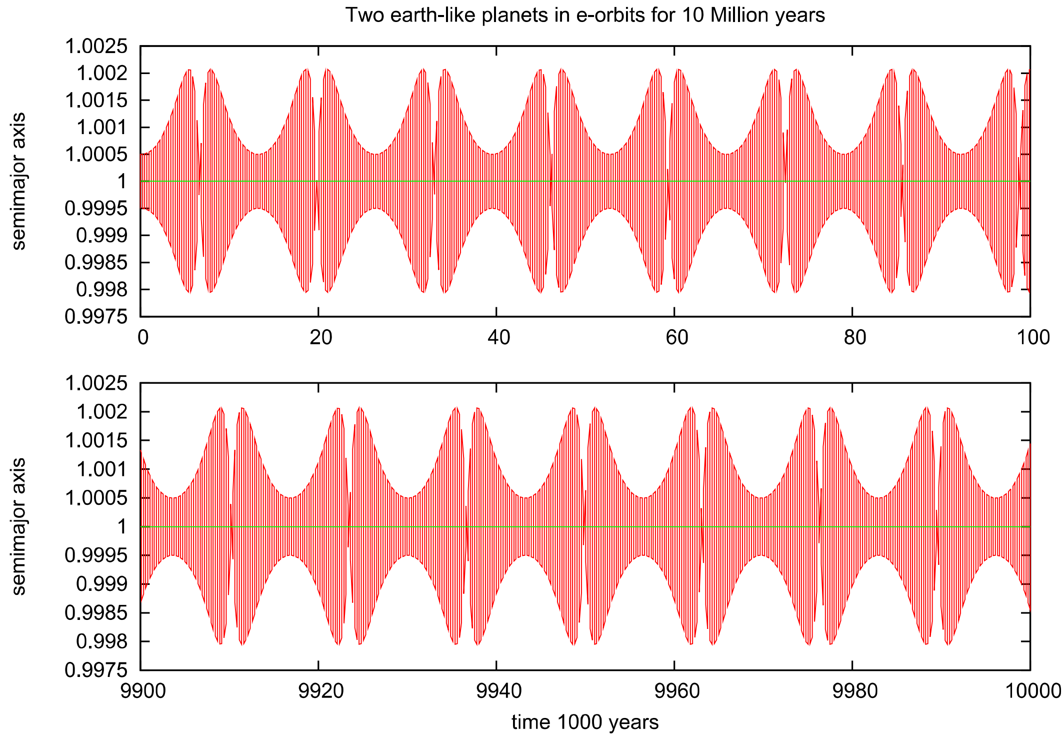

Figure 8. Semimajor axes of the exchange orbits for 10 million years; upper graph: first million years, lower graph last 1 million years.

circular orbits ($e < 0.001$) and almost coplanar orbits. These limits for eccentricity and inclination depend on the mass and also on the seperation of the two planets involved: a numerical results (which fit well to analytical estimations) give for the values of the separation in semimajor axis Δa in a distance of 1 AU to a sunlike planet: 0.012 AU (Earth), 0.020 AU (S-Earth), 0.024 AU (Uranus) and 0.034 AU (just below Saturn). In a next step we will investigate how perturbations of an inner or outer perturbing gas giant may destroy these limits.

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Notes

1. The homepage for the catalogue of extrasolar planets is maintained by J. Schneider: http://vo.obspm.fr/exoplanetes/encyclo/catalog.php

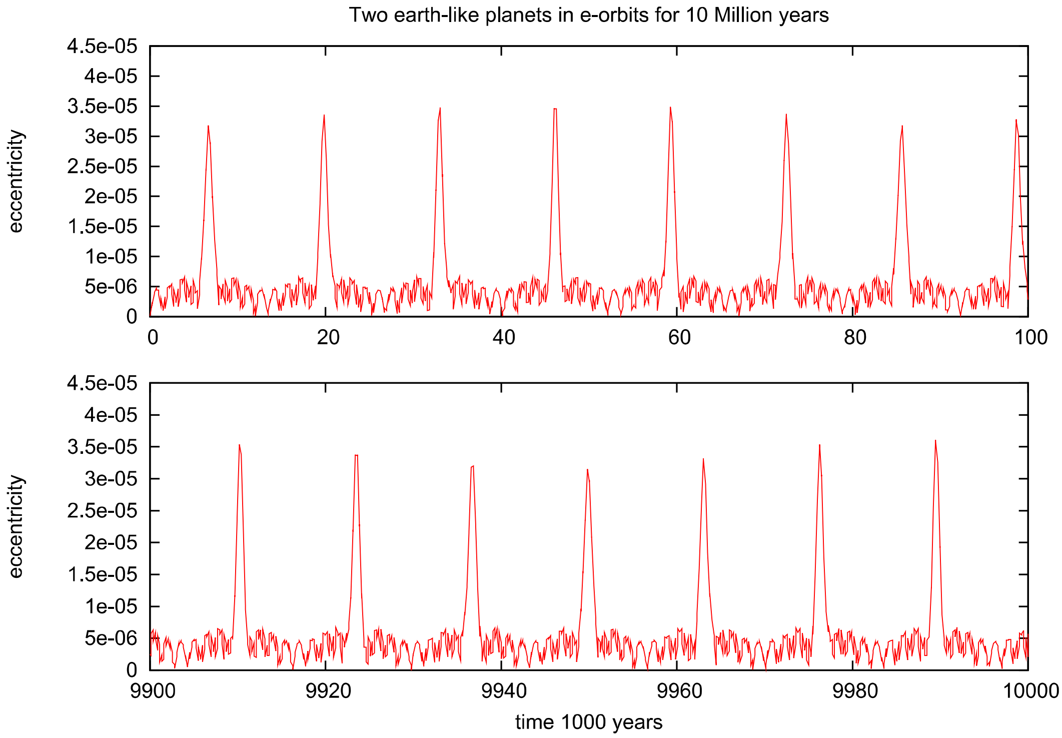

Figure 9. Eccentricities of the exchange orbits for 10 million years; upper graph: first million years, lower graph last 1 million years.

2. named after Joseph Louis, comte de Lagrange (Giuseppe Lodovico Lagrangia, 1736, Turin - 1813, Paris)
3. 151472 km and 151422 km
4. $r = a_{\text{planet}}(m/3M)^{1/3}$; M the larger mass and m the smaller one
5. With S-Earth we mean a TP with the mass $m_{\text{S-Earth}} = 5m_{\text{Earth}}$

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