STABILITY INVESTIGATIONS OF HIGHLY INCLINED PLANETARY ORBITS IN BINARY SYSTEMS

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Abstract The stability of P–type orbits in a binary system (mass–ratio equal to 0.5) was studied on the semi–major axis vs. inclination plane, similar to [10]. In the present work we investigate a larger part of the phase space, by calculating the relative Lyapunov Indicators and maximal eccentricities.

Keywords: exoplanets, binaries, stability of planetary systems

1. Introduction

Observations show that 60% of the main sequence stars are in binary or multiple systems (see [3]). Moreover, pre-main sequence stars may indicate that almost all of the stars are born in multiple systems (see [4], [5]). On the other hand, until now more than 160 exoplanets have been discovered, and some of them belong to binary systems. These facts show, that the investigation of the stability of planetary orbits in binaries is very important.

The discovered planets in binaries move on satellite orbits i.e. the planet revolves around one stellar component (S–type orbit; see Fig. 1). Theoretically there is another possible type of motion, the so called planetary orbit (P–type; see Fig. 1), whereas the planet moves around both stars. The S–type orbits were studied for some known systems by [6], [7], [8] and [9].

The stability of P–type orbits was also studied by [10] on the semi–major axis vs. inclination plane for a binary's mass–ratio ($\mu = m_2/(m_1 + m_2)$) equal to 0.5 by calculating the Fast Lyapunov Indicators (FLI) and escape times. They concluded that the stability limit varies between 2.1 and 3.85 binary separation (bs) depending on the eccentricity of the binary, and found a finger-like unstable island at inclinations $i = 15°$ to $i = 45°$.

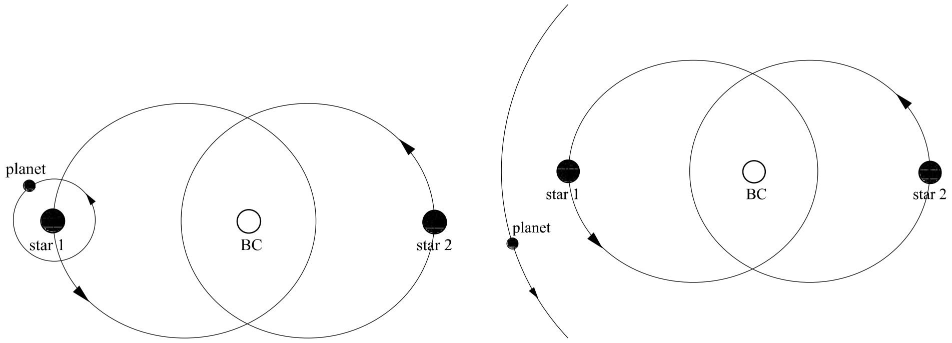

Figure 1. In the left panel: satellite–type or S–type motion: the planet revolves around one of the stars; right panel: planetary-type or P–type motion: the planet revolves around both star, i.e. it moves around the barycenter (BC).

In this paper we also study the stability of P–type orbits in a larger part of the phase space by using the methods of the Relative Lyapunov Indicators (RLI) and the maximum eccentricity. In the next section we give a description of the investigated system, the initial conditions and the applied numerical methods. After that we delineate and summarize our results.

2. Numerical setup

2.1 Initial conditions

For the integration of the equations of motion of the 3D restricted three-body problem we used the Bulirsch-Stoer integrator with adaptive stepsize control in the case of the RLI, and the Runge-Kutta-Neystrom-Felhberg RKN7(8) integrator with adaptive stepsize control for calculating the maximum eccentrity. The orbit of the primaries, and initially the massless planet's orbit is also circular, i.e. the eccentricity of the planet $e = 0$. The semi–major axis of the planet a is measured in the unit of the distance between the primaries and the initial value a_0 varies from 0.55 to 4 with stepsize $\Delta a = 0.005$. We use four starting mean anomaly (M_0) values for the planet: $0°$, $45°$, $90°$ and $135°$. These angles are measured from the connecting line of the primaries (see Fig. 2). (The resulting maps are the average of the four M_0. See later.) The inclination i is the angle between the orbital plane of the planet and the reference plane (xy–plane), which is the orbital plane of the binaries; initial value i_0 varies from $0°$ to $180°$ with stepsize $\Delta i = 1.25°$. The x–axis is the line connecting the primaries at $t = 0$. We note, that this line coincides with the line of node if $i \neq 0$, $t = 0$, i.e. the node of the planet is $\Omega_0 = 0°$. Initially the argument of the pericenter of the planet is $\omega_0 = 0°$.

The above defined orbital elements are referred to a barycentric reference frame, where the mass of the barycenter is $M = M_1 + M_2$. Using the usual procedure, the barycentric co-ordinates and velocities were calculated. After

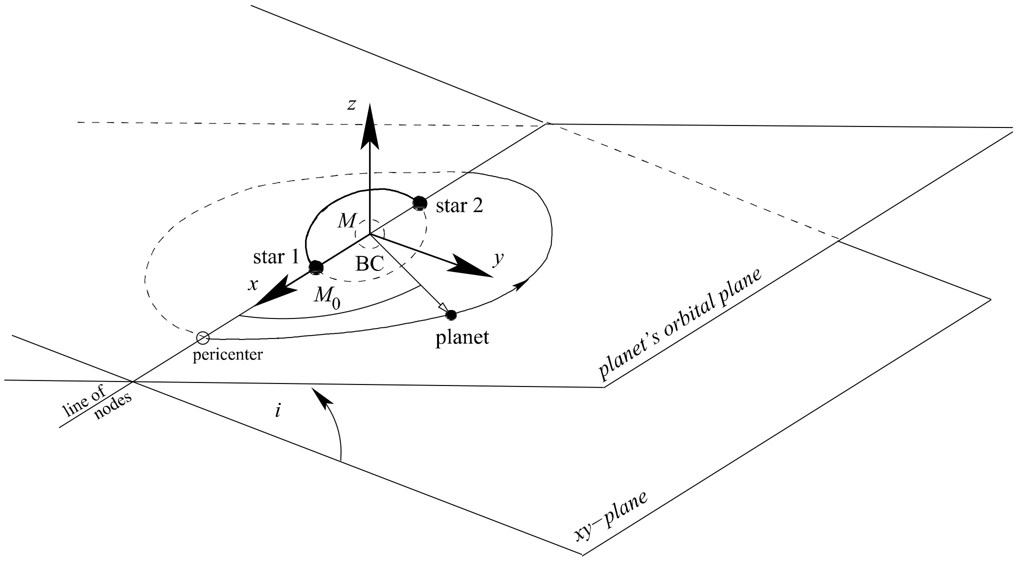

Figure 2. Configuration of the system: BC is the barycenter, the separation of the stars is the unit of distance, i is the planet's inclination with respect to the reference plane and M_0 is the initial mean anomaly of the planet.

that we transformed our co-ordinate and velocity vectors to a frame of reference with S_2 in the origin.

2.2 The maximum eccentricity method (MEM)

For an indication of stability a straightforward check based on the eccentricity was used. This osculating orbital element shows the probability of orbital crossing and close encounter of two planets, and therefore its value provides information on the stability of the orbit. We examined the behaviour of the eccentricity of the planet along the integration, and used the largest value as a stability indicator; in the following we call it the maximum eccentricity method (hereafter MEM). This is a reliable indicator of chaos, because the overlap of two or more resonances induce chaos and large excursions in the eccentricity. We know from experience, that instability comes from a chaotic growth of the eccentricity. This simple check has already been used in other stability studies, and was found to be a powerful indicator of the stability character of an orbit (see [2], [1]).

Calculating the maximum eccentricity an upper threshold was used. Whenever the eccentricity reached 0.8, the orbit was considered unstable, and the integration was stopped.

2.3 The relative Lyapunov indicator (RLI)

The method of the relative Lyapunov indicator (RLI) has been introduced by [11] for a particular problem, but its efficiency was demonstrated in a later paper [12] for 2D and 4D symplectic mappings and for Hamiltonian systems.

This method based on the idea that two initially nearby orbits are integrated simultaneously and also the evolution of their tangent vectors are followed. For both orbits the Lyapunov characteristic indicator (LCI) is calculated and the absolute value of their difference averaged over time is defined as RLI:

$$RLI(t) = \frac{1}{t}|LCI(x_0) - LCI(x_0 + \Delta x)|, \tag{1}$$

where Δx is the distance in phase space between the two orbits. The definition of RLI contains an arbitrary parameter Δx, which may affect the result. The authors have tested the sensitivity of RLI versus the norm of this parameter and found that the RLI depends almost linearly on Δx in the regular domain, while it is practically independent of it in the chaotic domain. Nevertheless, the value of the RLI is characteristically always several order of magnitudes smaller in a regular domain than in a chaotic region.

3. Results

The resulting figures were obtained as follows: we started the integration at mean anomaly $M_0 = 0°, 45°, 90°, 135°$ so we got $RLI^{(0)}$, $RLI^{(45)}$, $RLI^{(90)}$, $RLI^{(135)}$ and maximum eccentricity $ME^{(0)}$, $ME^{(45)}$, $ME^{(90)}$, $ME^{(135)}$ also. The plotted value is an average:

$$\frac{\overline{RLI}(a,i)}{\overline{ME}(a,i)} = \frac{1}{4} \sum_{M_0=0,45,90,135} \frac{RLI^{(M_0)}(a,i)}{ME^{(M_0)}(a,i)}. \tag{2}$$

We note, that this averaging in the case of the RLI stress the chaotic behaviour of an orbit, whereas in the case of the maximum eccentricity it is not so drastic.

At first we calculated the same part of the phase space as in [10], which is $a_0 = 1.8-2.5$ and $i_0 = 0-50°$. We performed the calculations on a finer grid: $\Delta a_0 = 0.005$ *bs* and $\Delta i_0 = 1.25°$ (see Fig. 3). Our maps are very similar to [10], except that our figures are more detailed, especially the second RLI map, where the system was integrated up to 1000 binary periods (*bp*). In Fig. 3 one can see some resonant formations, which appear at lower inclinations and are deviated at higher inclinations.

In Fig. 4 we show two maps for a larger domain of the phase space, which corresponds to $a_0 = 0.55-4$ *bs* and $i_0 = 0-180°$, with stepsizes $\Delta a_0 = 0.005$ *bs* $\Delta i_0 = 1.25°$. Both maps contain 691×145 points, resulting more than 10^5 orbits, if we take into account the averaging detailed above this number rises to

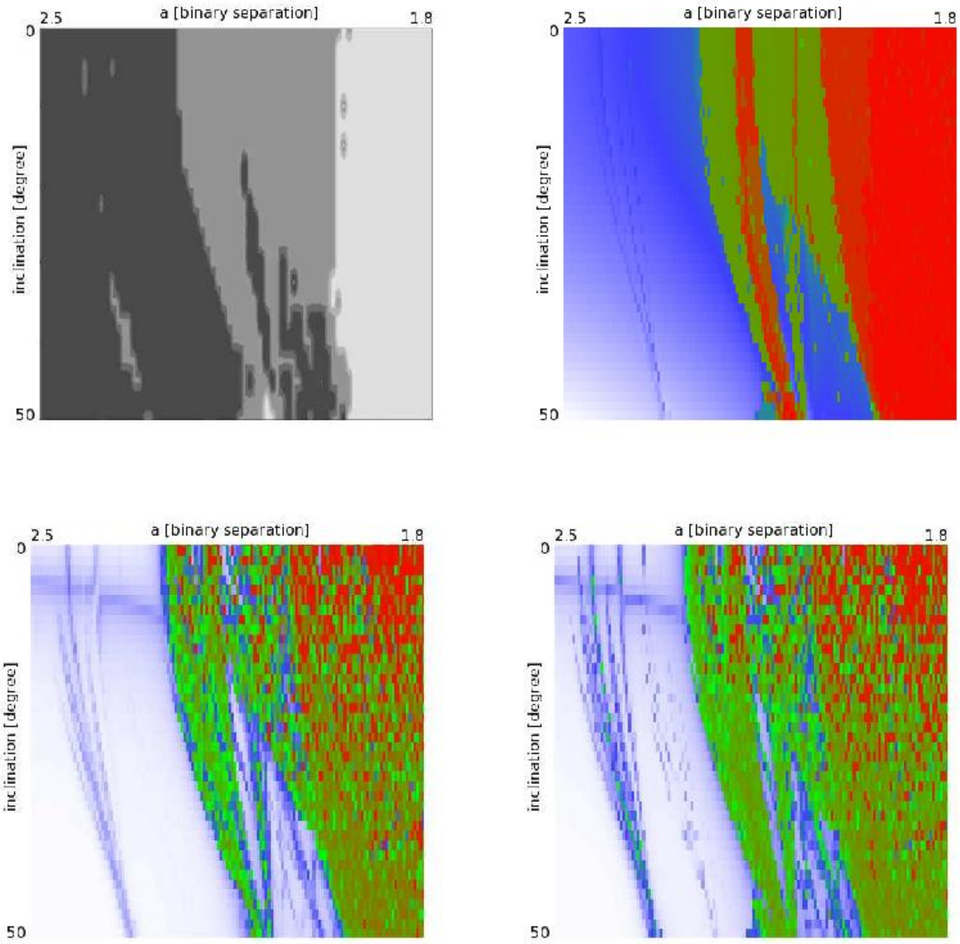

Figure 3. *Upper right*: FLI for 10000 *bp* in [10] Black indicates the stable zone, white the unstable. *Upper left:* Maximum eccentricity for 1000 *bp*. White shows the stable zone, black the unstable. *Lower left:* RLI for 200 *bp*. Colors like in max. ecc. *Lower right:* RLI for 1000 *bp*. Colors like in max. ecc.

4×10^5 orbits. Each orbit was integrated for 1000 *bp* in the case of the MEM and for 500 *bp* in the case of the RLI.

It is interesting, that the stable regions are wider in the case of retrograde orbits ($i_0 > 90°$) than for direct ones ($i_0 < 90°$). In the RLI map we can see several resonant formations. A resonant curve splits into three stronger and some fainter branches which makes it similar to a fork. The shape is generated by the applied averaging. For example in the case of the 3:1 resonance: when $M_0 = 0°$, we can see a sharp vertical line at $a = 2.085$ *bs*, at $M_0 = 45°$, the centre of the line is shifted to $a = 2.175$ *bs* and at $M_0 = 90°$ the centre is at $a = 2.23$ *bs*. The case of $M_0 = 135°$ is similar to $M_0 = 45°$. The width of the line grows with the distance from the 3:1 resonance ($a = 2.08$ *bs*). The averaging shows simultaneously the three cases, producing the fork shape. The

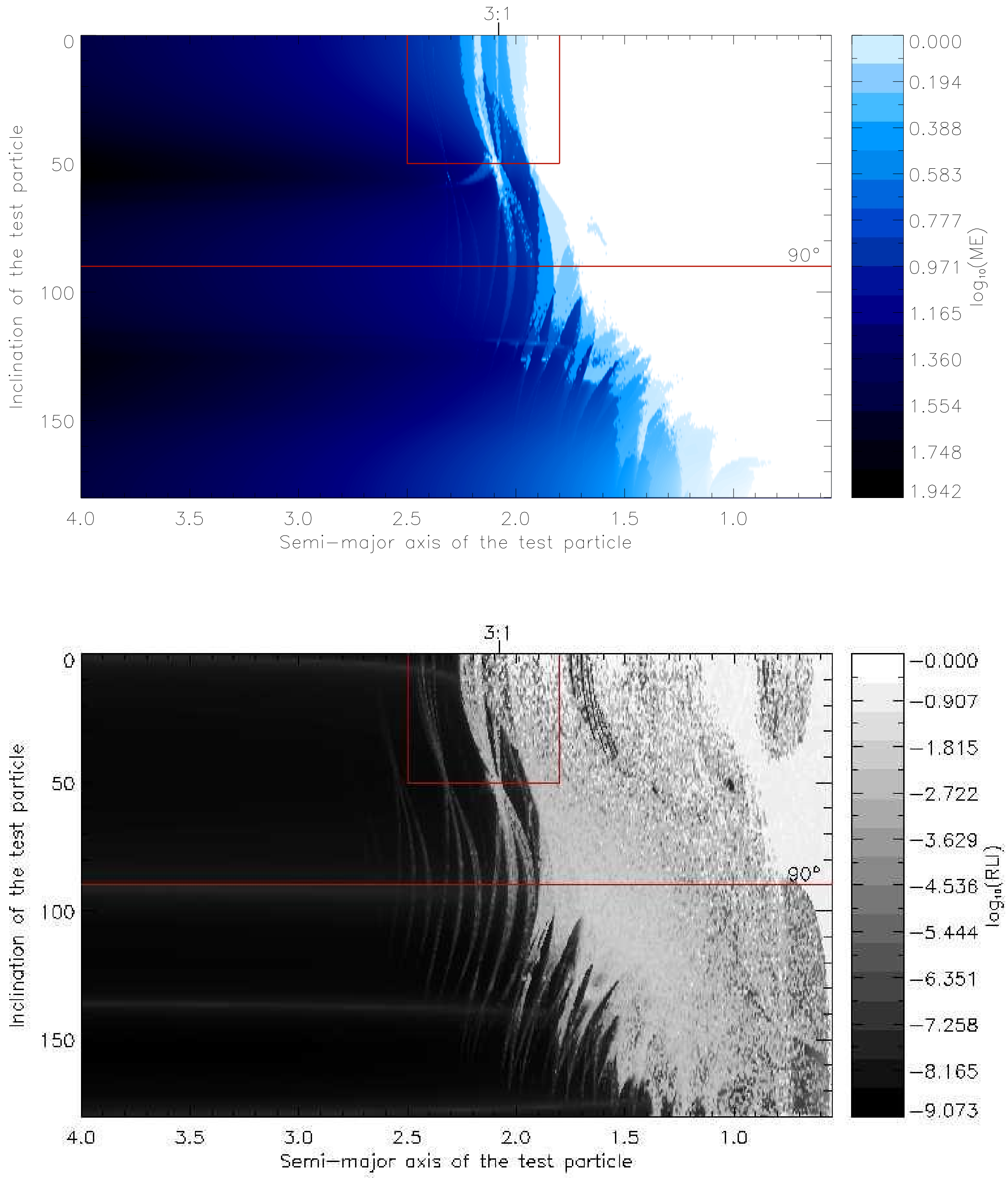

Figure 4. *Upper:* MEM for 1000 *bp Lower:* RLI for 500 *bp* White shows the unstable zone, black the stable.

fork belonging to the 3:1 resonance iduces Pilat-Lohinger's finger-like unstable island (see Fig. 4).

4. Summary

We investigated the stability region around a binary on the $a - i$ plane by calculating the RLI and the ME. Our results are in very good agreement with the results of [10], on the other hand they give information about a more extended part of the phase space. The maps obtained by the RLI show very fine resonant structures. The stable regions are wider when $i_0 > 90°$ (retrograde orbits). The resonant curves have a fork-like shape which is caused by the averaging. We demonstrated that Pilat-Lohinger's unstable island is created by a triple fork-like resonant shape.

Acknowledgments

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