DYNAMICAL CONFINEMENT OF THE ECCENTRICITY OF EXOPLANETS FROM TRANSIT PHOTOMETRY

Zsolt Sándor
Department of Astronomy,
Loránd Eötvös University
Pázmány Péter sétány 1/A
H-1117 Budapest, Hungary
Zs.Sandor@astro.elte.hu

Abstract The probability of detection of Earth-like exoplanets may increase after the launch of the space missions using the transit photometry as observation method. By using this technique, however, only the semi-major axis of the detected planet can be determined, and there will be incomplete information regarding its orbital eccentricity. On the other hand, the orbital eccentricity of an Earth-like exoplanet is a very important parameter, since it gives information about its climate and habitability. In this paper a procedure is suggested for confining the eccentricity of an exoplanet discovered by transit photometry if beside the Earth-like planet, an already known giant planet also orbits in the system.

Keywords: exoplanets – planetary transit – restricted three-body problem – stability – chaos detection

1. Introduction

After the discovery of the first extrasolar planet around 51 Pegasi (Mayor & Quéloz, 1995), more than 190 exoplanets have been observed. The detection of exoplanets has a great importance, since they form planetary systems around their hosting stars, and by studying the main properties of these systems the characteristics, the formation and the evolution of the Solar System could be treated as a part of a more general phenomenon. The above picture is unfortunately rather ideal than complete yet, since the exoplanets observed by now are mainly Jupiter-like gas giants. This is the consequence of the fact that by using radial-velocity measurements, which is the most effective ground-based observing technique, there is no chance to detect Earth-like planets yet.

On the other hand, one of the most challenging questions of exoplanetary research is the discovery of the Earth-like planets. Beside their importance in testing and improving the formation theories of the planetary systems, another major question is their habitability. If an Earth-like planet revolves in the habitable zone of the hosting star, there may be chances of developing (a water based) life on its surface. The habitable zone is that region around the star, where liquid water can exist on the surface of a planet (Kasting et al 1993).

In order to find Earth-like planets, there are space missions in construction and planning phase. Such a mission is COROT (sponsored by CNES, ESA and other countries) to be launched in 2006, the Kepler Mission (NASA) with a launch in 2008, Darwin (ESO), and Terrestrial Planet Finder (TPF, NASA) with a launch in the next decade. The first two missions (COROT and Kepler) will use the transit photometry as detection technique, which is based on measuring the periodic dimming of the star's light intensity caused by an unseen transiting planet. Measurements performed by these instruments will provide the semi-major axis a of the transiting planet calculated from Kepler's third law

$$\frac{a^3}{T^2} = \frac{k^2}{4\pi^2}(m_* + m_p),$$

where T is the period of the transits, m_* is the mass of the hosting star, and m_p is the mass of the transiting planet, respectively (k is the Gaussian gravitational constant). In the case of Earth-like planets $m_p << m_*$, so neglecting m_p does not affect significantly the accuracy of a. An uncertainity in the semi-major axis a can appear since the stellar mass is known only with limited accuracy. If this is for example 3%, the inaccuracy in a will be 1%. (We note that the mass of the hosting star can be determined by spectroscopic observations and by stellar model calculations.) However, in this paper we do not investigate the error propagation due to these uncertainities in stellar mass and semi-major axis, we intend to perform these studies in a future research.

In this paper we present a procedure which helps in confining the orbital eccentricity and inclination of the transiting planet if (i) the duration of the transit is known, and (ii) there is another (giant) planet in the system. We derive such an equation, which connects the mass and the radius of the star, the semi-major axis, the eccentricity, the argument of the periastron, the inclination of the transiting planet, and the duration of the transit. In this equation there are three unknowns, namely e, ω, and i. By fixing i, the corresponding (ω, e) pairs can be visualized as curves on the $\omega - e$ parameter plane. Thus the problem is underdetermined and there is no way to confine the orbital eccentricity e of the transiting planet.

On the other hand, as suggested by planetary formation scenarios, we expect that next to the Earth-like planets Jupiter-like giant planets can also be found in the majority of the planetary systems. Having discovered an Earth-like planet around a star, by using complementary techniques (as observations by Space Interferometry Mission and ground-based Doppler spectroscopy) additional more massive planets can be identified in the system, and their orbital parameters can be determined too.

The presence of a giant planet (beside the transiting one) results in that both ordered and chaotic regions can be found in the phase space of the system. If the trajectory of the Earth-like planet is in the ordered region of the phase space, the motion of the planet is stable for arbitrary long times. If the initial conditions of its orbit are in a chaotic region of the phase space, the motion of the planet can be unstable after a certain time. In this paper we exclude those orbital parameters of the transiting planet, which result in chaotic motion. We shall demonstrate that in some cases it is possible to determine an upper limit for the eccentricity and a lower limit for the inclination of the transiting planet. We stress again that the eccentricity is a very important orbital parameter not only from dynamical point of view but also in studying the habitability and climatic variations of the Earth-like planet.

The paper is organized as following: first we derive a connecting equation between the duration of the transit and some important parameters of the star and the transiting body, then we solve this equation numerically. After examining the solutions of this equation, we map the stability structure of the system assuming the presence of a known giant planet. Then we can determine lower limits for the inclination and an upper bounds for the eccentricity of the transiting planet depending on the eccentricity and the semi-major axis of the known giant planet.

2. A connecting equation between the orbital parameters of the transiting planet

In this section we shall derive an equation between the orbital parameters of the transiting planet, the star's mass, and the duration of the transit from the geometry of the transit.

Let us suppose that the star's disc is a circle with a radius R, and a planet is moving in a front of this disc with an average velocity $v_{\rm tr}$. If the duration of the transit is denoted by τ and the lenght of the path of the transiting planet is d (see Fig. 1), the following approximation holds:

$$v_{\rm tr} \approx \frac{d}{\tau}. \tag{1}$$

We note that according to Kepler's second law, the velocity of the planet is changing during the transit (except in the case of circular orbits), however this change is negligible, if the planet orbits far enough the star. Since the triangle in Fig. 1 is a pythagorean one, it can be written

$$R^2 = m^2 + \left(\frac{d}{2}\right)^2 . \tag{2}$$

From Equation (2) the lenght of the transit's path d can be expressed as

$$d = 2\sqrt{R^2 - r^2 \cos^2 i}, \tag{3}$$

where, according to Fig. 2,

$$m = r \cos i, \tag{4}$$

where i is the inclination (e.g. the angle between the orbital plane and the tangent plane to the celestial sphere), and r is the distance between the center of the star and the planet.

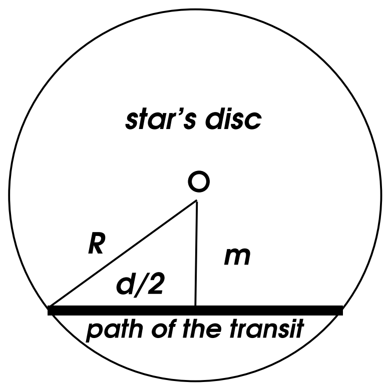

Figure 1. The transit of a planet in the front of the stellar disc. The straight sections denoted by R, m, and $d/2$ form a pythagorean triangle.

By using the well known formula for r:

$$r = \frac{a(1 - e^2)}{1 + e \cos v} , \tag{5}$$

(where a is the semi-major axis, e is the eccentricity, and v is the true anomaly of the transiting planet), and Equations (1) and (3), the average orbital velocity of the transiting planet ($v_{\rm tr}$) can be written as

$$v_{\rm tr} = \frac{2}{\tau}\sqrt{R^2 - \left[\frac{a(1 - e^2)}{1 + e \cos v}\right]^2 \cos^2 i} . \tag{6}$$

On the other hand, $v_{\rm tr}$ can also be approximated on the basis of the two-body problem. In the coordinate system (ξ, η), in which the axes of the orbital

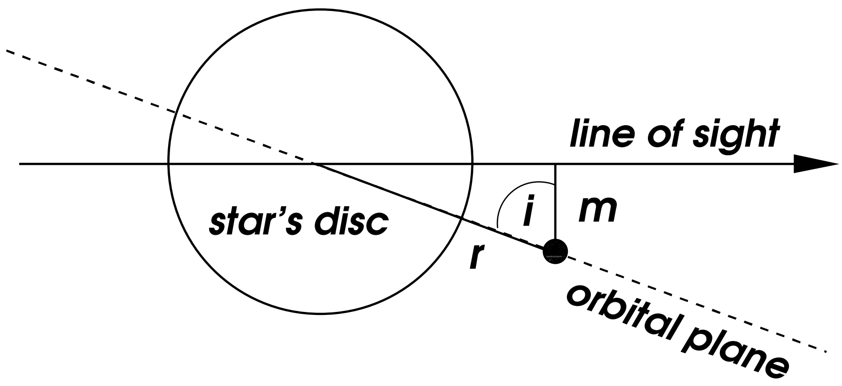

Figure 2. Side-view of the transit, where r is the distance of the planet from the star's center and i is the inclination of its orbital plane.

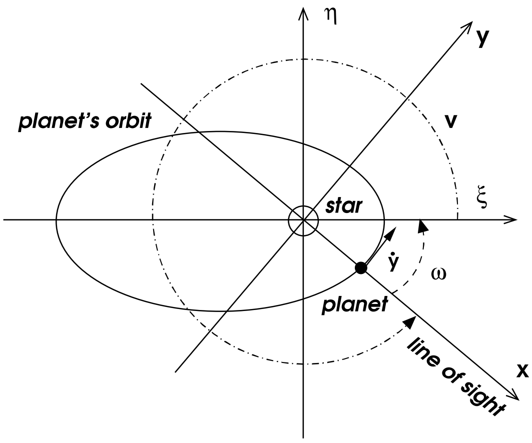

Figure 3. The transit as viewed from above. At the mid of the transit $v_{\rm tr}$ is nearly equal to $\dot{y}$. The coordinate system (ξ, η) is the rotation of the coordinate system (x, y) by ω.

ellipse are on the axes ξ and η, the components of an orbital velocity vector are (see Murray and Dermott, 1999):

$$\dot{\xi} = -\sqrt{\frac{\mu}{p}} \sin v \,, \tag{7}$$

$$\dot{\eta} = \sqrt{\frac{\mu}{p}} (e + \cos v) \,,$$

where $p = a(1 - e^2)$ is the parameter of the ellipse and $\mu = k^2(m_* + m_p)$, m_* and m_p are the stellar and planetary masses, respectively. Let (x, y) denote a cartesian coordinate system where the x-axis is parallel to the line of sight (e.g. the line connecting the center of the star to the observer). From Figure 3 it can be seen that the system (ξ, η) is the rotation of the system (x, y) by

ω, which is the argument of the periastron of the transiting planet. Thus in the coordinate system (x, y) formulae (7) transform as

$$\dot{x} = \dot{\xi}\cos\omega - \dot{\eta}\sin\omega \ , \tag{8}$$

$$\dot{y} = \dot{\xi}\sin\omega + \dot{\eta}\cos\omega \ .$$

From Fig. 3 it is clearly visible that the average velocity of the transiting planet $v_{\rm tr}$ can be approximated with $\dot{y}$, which is the velociy of the planet at the mid of the transit. (We note that this approximation fails for large eccentricity of the transiting planet.) Then by using the above approximation and Equations (7) and (8) we find

$$v_{\rm tr} \approx \dot{y} = -\sqrt{\frac{\mu}{p}}\sin v \sin\omega + \sqrt{\frac{\mu}{p}}(e + \cos v)\cos\omega. \tag{9}$$

Studying again Figure 3, it is also true that at the mid of the transit

$$v + \omega = 360^\circ \ , \tag{10}$$

thus the average orbital velocity of the transiting planet is

$$v_{\rm tr} = \sqrt{\frac{\mu}{p}}(1 + e\cos\omega) \ . \tag{11}$$

Combining Equations (6), (10), and (11) we obtain the following equation:

$$\sqrt{\frac{\mu}{p}}(1 + e\cos\omega) - \frac{2}{\tau}\sqrt{R^2 - \left[\frac{a(1-e^2)}{1+e\cos\omega}\right]^2\cos^2 i} = 0 \ , \tag{12}$$

where the unknown quantities are the eccentricity e, the inclination i, and the argument of the periastron ω. The other quantities, such as the semi-major axis a, the mass parameter (μ), the radius of the star (R), and the duration of the transit (τ) are known with certain accuracies already discussed in the Introduction.

3. Solution and analysis of Equation (12)

According to the last paragraph of the previous section, the unknown quantities in Equation (12) are the inclination i, the argument of periastron ω, and the eccentricity e of the transiting planet. Thus by fixed values of i, Equation (12) can be solved numerically, and the (ω, e) pairs of the solutions can be represented as curves on the $\omega - e$ parameter plane.

In order to study the solutions of Equation (12), we give specific values for the parameters in Equation (12). Let us assume that the mass of the transiting planet is 1 Earth-mass, and it revolves around a 1 Solar-mass star with radius $R = 6.96 \times 10^8$ m, in an elliptic orbit characterized by $a = 1$ AU, $e = 0.1$ being its inclination $i = 89.95°$. Then we suppose that the direction of the observation of the planetary transit is $\omega = 30°$. It can be calculated easily that in this case the duration of the transit ise $\tau = 0.488029$ day.

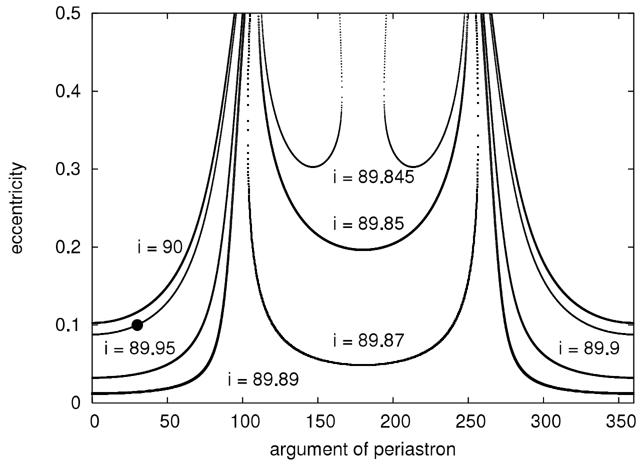

Figure 4. Solutions of Equation (12) for different inclinations when $\tau = 0.488029$ day. The original solution, which results in the above τ, is marked with a filled circle at $\omega = 30°$, $e = 0.1$, and $i = 89.95°$.

By observing transits caused by the above planet, we can measure their duration τ and period T, from which the semi-major axis a can be calculated. In our case $\tau = 0.488029$ day, and for different values of i the corresponding $\omega - e$ curves are plotted in Fig. 4. We show these curves only for $e < 0.5$ since we think that larger values of e are unrealistic for Earth-like planets. We also mark the real (ω, e) solution by a filled circle on the curve corresponding to $i = 89.95°$, but as we can see, there is no way to restrict efficiently the infinite set of solutions. The only restriction is that the solutions can not be chosen from the region above the $\omega - e$ curve corresponding to $i = 90°$.

Equation (12) has an infinite set of solutions formed by pairs of (ω, e) values. If only the duration of the transit is known, it is not possible to choose which (ω, e) pair represents the real parameters of the transiting planet.

4. A possible confinement of the eccentricity of the transiting planet

In this section we shall investigate the case when, beside the newly discovered planet, an already known giant planet orbits around the hosting star. The presence of such a planet makes the problem non-integrable and both ordered and chaotic regions can be found in the phase space of the system. We suppose that the most probable orbital solutions of the transiting planets are those, which emanate from the ordered regions of the phase space. The orbital parameters of the transiting planet, which would result in chaotic behaviour are unlikely, since in long terms the orbit of the planet could be unstable, therefore these solutions might be avoided. We expect that the presence of a second (giant) planet r epresents a dynamical constraint reducing the infinit set of solutions of Equation (12) by giving an upper limit for the maximum eccentricity of the transiting planet. We shall also demonstrate that by studying the solution-curves of Equation (12) together with the stability structure of the $\omega - e$ plane, a lower bound for the inclination can also be determined. In what follows we shall investigate the stability in the $(\omega - e)$ plane within the framework of the planar restricted three-body problem.

In order to map the stability properties of the $(\omega - e)$ plane we used the Relative Lyapunov Indicator (RLI) (Sándor et al. 2000, 2004). The initial ω and e values are chosen from the intervals $e \in [0, 0.5]$ and $\omega \in [0^\circ, 360^\circ]$ with $\Delta e = 0.025$ and $\Delta\omega = 2^\circ$. The initial value of the semi-major axis of the transiting planet is always $a = 1$ AU, while its true anomaly is calculated from Equation (10) as $v = 360^\circ - \omega$ (see also Figure 3).

For each pair of the initial (ω, e) values we assign the RLI of the corresponding orbit calculated for 500 periods of the transiting planet. If the RLI is small ($\sim 10^{-12} - 10^{-13}$), the corresponding orbit is ordered and stable. If the RLI $\sim 10^{-11} - 10^{-9}$ the orbit is weakly chaotic. In practical sense this orbit could be (Nekhoroshev) stable for very long terms as well, however, it can not be stable for arbitrary long time. Thus the regions characterized by these RLI values can already be the birth places of unstable orbits. Orbits having larger RLI $\sim 10^{-8} - 10^{-5}$, are strongly chaotic orbits, and they will be unstable after certain time. In our stability maps the ordered regions are denoted by light, the weakly chaotic regions by grey, and the strongly chaotic regions by dark shades.

In what follows we consider the cases where the parameters of the known giant planet having 1 Jupiter mass are the following: $a_1 = 2.0$ AU, $e_1 = 0.1$, 0.2, and 0.3 respectively. We fix the angular elements of the giant planet to $\lambda = \omega = 0^\circ$. In Fig. 5, Fig. 6, and Fig. 7 we show the dynamical structure of the $\omega - e$ parameter planes for increasing values of the eccentricity of the giant planet. In these figures we also plot the solution curves of Equation (12) by using $\tau = 0.488029$ day.

From Fig. 5 it can be seen that there are two upper bounds for the eccentricity of the transiting planet depending on whether the transit occurs near the periastron, or near the apoastron. If the transit is near the periastron $\omega < 80^\circ$, the upper limit of the eccentricity is $e < 0.3$, since the $\omega - e$ curves cross the chaotic region around this value. If the transit would happen at the apoastron $\omega \in [180^\circ, 200^\circ]$, the upper limit of the transiting planet's eccentricity is higher, $e < 0.4$. The real solution is marked (as a filled circle) on the curve corresponding to $i = 89.95^\circ$.

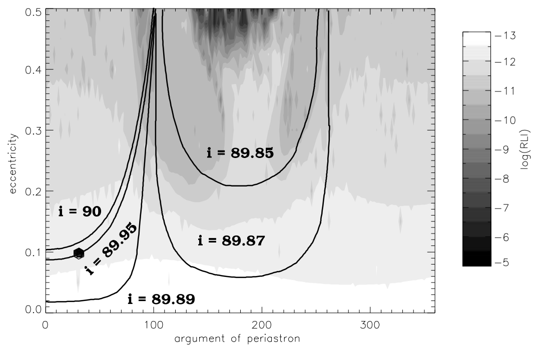

Figure 5. The stability map of the $\omega - e$ parameter plane, when $a_1 = 2.0$ AU and $e_1 = 0.1$. The $\omega - e$ curves for different i are also plotted when $\tau = 0.488029$ day.

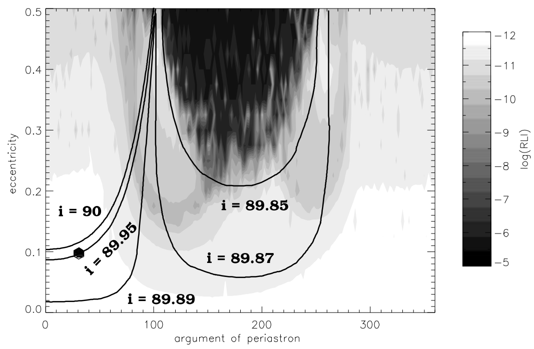

Figure 6. The stability map of the $\omega - e$ parameter plane, when $a_1 = 2.0$ AU and $e_1 = 0.2$. The $\omega - e$ curves for different i are also plotted when $\tau = 0.488029$ day.

In Fig. 6, corresponding to $e_1 = 0.2$, there are two upper limits of the eccentricity of the transiting planet as well. For $\omega < 80°$ the eccentricity is $e < 0.27$, for $\omega \in [150°, 220°]$ the eccentricity is $e < 0.22$. In this case a lower limit can be given for the inclination too, $i > 89.°85$.

If the eccentricity of the giant planet is $e_1 = 0.3$, see Fig. 7 the maximum upper limit of the transiting planet's eccentricity is $e < 0.18$. However, in this case there exists a lower limit $e > 0.05$ as well. If the transit would take place around the periastron the corresponding ω and e values would result in weakly chaotic orbits. A lower bound of the inclination in this case is $i > 89.89°$. Among the three possible values of the giant planet, this latter would represent the most effective dynamical constraint for the orbital parameters of the transiting planet, which are $a = 1.0$ AU, $e = 0.1$, $\omega = 30°$, and $i = 89.95°$.

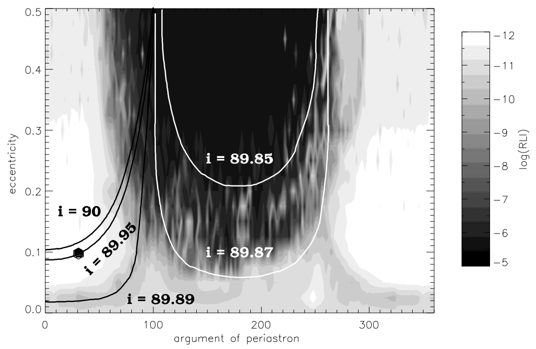

Figure 7. The stability map of the $\omega - e$ parameter plane, when $a_1 = 2.0$ AU and $e_1 = 0.3$. The $\omega - e$ curves for different i are also plotted when $\tau = 0.488029$ day.

We have also investigated the cases when the semi-major axis of the known giant planet were smaller and larger than 2 AU. If a_1 is smaller, a smaller e_1 is enough to result in an effective dynamical constraint. If a_1 is larger, the eccentricity of the giant planet should be larger as well for an efficient dynamical constraint.

5. Conclusions

The detection of Earth-like extrasolar planets by using ground based spectroscopic methods is beyond the present capabilities of observational astronomy. In the near future there will be launched space instruments such as COROT and KEPLER which are devoted to observe such planets by using transit photometry.

In this paper we addressed the question whether it is possible to determine the orbital elements of Earth-like planets discovered by transit photometry if, apart from the period, the duration of the transit can be measured too. We supposed that the mass and the radius of the hosting star is known. We derived an equation, which connects the stellar and planetary masses, the duration of the transit, the semi-major axis, the eccentricity, the argument of periastron and the inclination of the transiting planet. By fixing the inclination, this equation contains two unknown variables, the argument of periastron ω and eccentricity e of the transiting planet. Thus the solutions for different inclinations can be represented as curves on the $\omega - e$ parameter plane.

In the last section of the paper we assumed that beside the transiting Earth-like planet a giant planet orbits around the star as well. This assumption is quite reasonable if we accept the formation theories of planetary systems supporting the simultaneous presence of both rocky, Earth-like and gaseous, Jupiter-like planets. Since the detection of giant planets is possible by measuring their radial velocity by Doppler-effect, we assumed their orbital parameters to be known. By using the framework of the restricted three-body problem, we investigated the influence of the known giant planet to the $\omega - e$ parameter plane of the transiting planet. We found that on the $\omega - e$ parameter plane there appeared chaotic regions as well, which in long terms may result in unstable motion for the transiting planet. Assuming that chaotic behaviour for the transiting planet are unlikely, we could determine an upper limit for the eccentricity, and a lower limit for the orbital inclination of the transiting planet.

In a future work we plan to extend our studies by investigating systematically the stability structure of the $(\omega - e)$ parameter plane for various values of the giant planet's semi-major axis and eccentricity. Since the mass of the hosting star is known only with a limited accuracy, we also plan to follow the propagation of this error throughout the method presented in this paper. In our future investigations we intend to consider the cases of more massive transiting planets as well.

Acknowledgments

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