# TERRESTRIAL TROJAN PLANETS IN EXTRASOLAR SYSTEMS

#### Richard Schwarz

Institute of Astronomy
University of Vienna
Türkenschanzstrasse 17
A-1180 Vienna, Austria
schwarz@astro.univie.ac.at

#### **Abstract**

In this article we examine, that terrestrial planets in extrasolar planetary systems can have stable orbits in the 1:1 mean-motion-resonance (MMR) with a Jovian like planet. In our stability study of the so-called Trojan planets in the habitable zone, we used the restricted three-body problem with different mass ratios of the primary bodies. The application of the three-body problem showed that even very massive Trojan planets can be stable in the 1:1 MMR. From the approximately 145 extrasolar planetary systems with about 170 planets only 15 systems were found where a giant planet is in the habitable zone. In our numerical studies we examine the orbital behaviour and the size of the stable zone respectively of extrasolar systems where the initial orbit of the gas giant lies fully in the habitable zone. The investigation of either the initial mean anomaly (M) or the initial argument of perihel  $(\omega)$ , showed, that the variation of  $\omega$  yield more stable orbits than the variation of M.

**Keywords:** trojan planets – exoplanets – habitable zone

## 1. Habitable planets

Today we have only observational evidence of extrasolar planets of 7 earth masses (Gliese 876 d) and larger. The size of such planets is to large for formation of life and only a few of these planets lie in the 'Habitable Zone' (=HZ¹). That's the reason why a study of dynamical stability of possible additional terrestrial planets (planets with a size comparable to Earth) is a hypothetical one. But what we can do is to ask, which dynamical configurations are possible to host a habitable planet in the HZ of an extrasolar planetary system? From the dynamical point of view, there are four possible configurations for terrestrial like planets in the HZ (shown in Fig. 1).

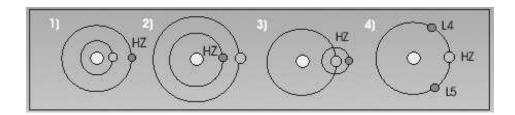


Figure 1. Four different classes of orbits where possible terrestrial planets may exist.

- 1 The HZ is outside the giant planet (=OHZ): Most of the discovered gas giant (=GG) planets are located very close to their star. From the dynamical point of view, there may exist terrestrial planets with stable orbits in the HZ and sufficiently small eccentricity over time scales long, enough to develop a biosphere.
- 2 The solar configuration (=SOL): When a Jupiter like planet moves far enough from its central star to allow additional planets moving on stable low eccentric orbits closer to the star inside the HZ.
- 3 The satellite configuration (=SAT): A terrestrial planet that orbits a GG in the HZ (as the ones orbiting Jupiter, e.g. Europa) could have the right conditions to develop a biosphere.
- 4 The Trojan configuration (=TROJ): When the GG moves in the habitable region a terrestrial Trojan planet may move in a stable orbit around the Lagrangian equilibrium points  $L_4$  or  $L_5$ .

Menou and Tabachnik (2003) quantified the dynamical habitability of extrasolar planetary systems in general via simulations of their orbital dynamics in the presence of potentially habitable terrestrial plantes. The OHZ and the SOL configurations have been the subject of a number of investigations (e.g. Sándor(2006), Érdi and Pál (2003), Pál and Sándor (2003), Dvorak et al. (2003a, 2003b and 2004)). If the gravitational zone of a GG overlaps with that of a terrestrial planet in the HZ, gravitational perturbation can push the terrestrial planet out of the HZ. For this reason, we focus our work on the dynamical stability of the TROJ configuration, in which possible terrestrial planets have a 1:1 MMR with a GG. Nauenberg (2002) found a stable configuration for motions in the 1:1 MMR, where the more massive planet has an almost circular orbit, while the smaller body has a high eccentric orbit. Further investigations of the TROJ configuration focused on Trojan planets in the HZ (Érdi and Sándor (2005)). We are mainly interested in Trojan planets in 1:1 MMR with a GG that moves fully in the HZ. The main goal was to see how many orbits (of the Trojan planets) of the stable region are fully in the HZ after the calculation. These stable orbits are a main requirement for a possible formation of life. Laughlin and Chambers (2002) considered the possibility of two planets in a 1:1 MMR as a result of an interaction with the protoplanetary accretion disc. We emphasize that the discussion of habitable regions around a host star is an interdisciplinary one: astrophysics is involved, because the spectral type and the age of the host star define the HZ (e.g., Lammer et. al. (2003)), atmospheric chemistriy is fundamental when we considering planetary habitability (e.g., Kasting et. al. (1993)), and astrodynamics is important with regard to the determination of the orbital stability.

### 2. Numerical setup

More than 170 extrasolar planetary systems were discovered (Extrasolar planets catalogue maintained by Jean Schneider<sup>2</sup>), 14 systems are binaries and 18 are multiplanetary systems. Only 10 single-star systems have a giant planet in the HZ and an initial eccentricity smaller than 0.3, which is important for the stability (see Schwarz, 2005 p.65). We selected in Table 1 six planetary systems, namely HD93083, HD17051, HD28185, HD108874 and HD27442 (the bold written), for which the initial orbit lies fully in the HZ. We studied their size of the stability region by using direct numerical integrations of the equation of motion. The other systems which lies only partly in the HZ were also investigated, see Schwarz et al. (2005a) and Schwarz (2005b). The integration was carried out with the LIE-integration method – which uses an adaptive step size (Hanslmeier and Dvorak, 1984; Lichtenegger, 1984) – in the dynamical model of the elliptic restricted three-body problem consisting of the central star, the GG and a hypothetical (massless) terrestrial planet. The integration time was up to  $10^5$  years.

#### 2.1 Initial conditions

We have taken the following initial conditions for the terrestrial planet: first, the semimajor axis of the massless planet (starting at the fixed semimajor axis of the GG) was computed for a grid with  $\Delta a=0.003AU$ . The argument of pericenter  $\omega$  of the massless planet extends from  $20^{\circ}$  to  $140^{\circ}$  and has a grid-size of  $\Delta \omega=2^{\circ}$ . The extension and the geometry of the stable region for the Trojan planet of several extrasolar systems varies. We change the number of the calculated orbits for each system to reduce the calculation-time (the larger the unstable region the more calculation-time were needed). During the integration time, the largest value of the eccentricity (= $e_{\rm max}$ ) of the hypothetical Trojan planet was determined. The so called maximum eccentricity method (=MEM) shows how much the orbit differs from the circular one. For larger eccentricities it becomes more probable that the asteroids have close encounters and collisions. The stability criterion for a Trojan was, that the eccentricity should not exceed e=0.5; this is good measure which were tested and compared

to other definitions like crossing the line of syzygy (alignment of Sun, Jupiter and the Trojan).

Table 1. List of all single GG moving in the HZ of their host stars, depending on the spectral type the host stars. Main parameters: *1st column*: Name, *2nd column*: spectral type, *3rd column*: mass of the star, *4th column*: the minimum mass of the giant planet  $[=M_{jup}]$ , *5th column*: distance (semimajor axis a[AU]) from the central star, *6th column*: initial eccentricity of the extrasolar planet, *7th column*: extension of the HZ [AU], and *8th column*: partly inside the HZ at the beginning (initial conditions) in [%].

		mass	mass	a		HZ	partly
Name	Spec.	$[M_{sol}]$	$[M_{jup}]$	[AU]	e	[AU]	in HZ
							[%]
HD93083	K3V	0.70	0.37	0.48	0.14	0.40-1.30	100
HD134987	G5V	1.05	1.58	0.78	0.24	0.75-1.40	58
HD17051	G0V	1.03	1.94	0.91	0.24	0.70-1.30	100
HD28185	<b>G5</b>	0.99	5.7	1.03	0.07	0.70-1.30	100
HD108874	<b>G5</b>	1.00	1.65	1.07	0.20	0.70-1.30	100
HD27442	K2IVa	1.20	1.28	1.18	0.07	0.93-1.80	100
HD188015	G5IV	1.08	1.26	1.19	0.15	0.70-1.60	100
HD114783	K0	0.92	0.99	1.20	0.10	0.65-1.25	50
HD20367	G0	1.05	1.07	1.25	0.23	0.75-1.40	76
HD23079	(F8)/G0V	1.10	2.61	1.65	0.10	0.85-1.60	35

#### 3. Global results

The stability region around the Lagrangian points was studied in the model of the elliptic restricted three-body problem by many investigations (e.g. Rabe, 1967, Lohinger and Dvorak, 1993 etc.). Furthermore, a study by Marchal (1991) was undertaken in the framework of the general three-body problem (where  $m_3 > 0^3$ ). These results were used to show the positions – in the stable zone (see Fig. 2) – of all extrasolar systems where the gas giant is near the HZ. This is given in Table 1, where  $m_3$  is equal to one earth mass. Therefore it is necessary to define the mass parameter  $\mu$  through the equation

$$\mu = \frac{m_2 + m_3}{M} + m_2 \cdot m_3 + O\left(\frac{m_2^3 \cdot m_3}{m_1^4}\right) \tag{1}$$

which is used instead of the mass ratio in the elliptic restricted three-body problem. The stability zone (Fig. 2 depending on the mass parameter  $\mu$  and the eccentricity show that all selected extrasolar systems of Table 1 lie in the zone of stable motion. Only HD141937 (partly in the HZ) which has a planet with 9.7 Jupiter masses is close to the border (see Fig. 2) of unstable motion. Consequently all planetary systems with one planet in the HZ can have stable Lagrangian points ( $L_4$  and  $L_5$ ).

We conclude that orbits of hypothetical Trojan planets with a small initial  $\mu$  and e are stable. The stability analysis does not give any information about the extension of the stable region around the equilibrium points. A more detailed answer can be given with the results of numerical simulations of each extrasolar systems under consideration shown in the next paragraph.

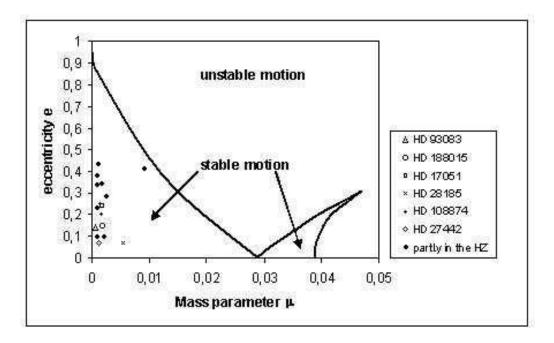


Figure 2. Stability zone depending on the mass parameter  $\mu$  and the eccentricity e.

#### 4. Results

Table 1 shows the parameters of all studied single<sup>4</sup> extrasolar systems. The six selected extrasolar planetary systems – printed in bold in Table 1 – have one GG lying at the starting positions fully in the HZ. Note that from the dynamical point of view there is no difference to the other systems.

#### 4.1 HD17051

HD17051 is a G0V star with one solar mass ( $M_{sun}$ =1.03) which hosts a GG of 1.94 Jupiter masses (= $M_{jup}$ ) on an eccentric orbit (e=0.24) with a semimajor axis of a=0.91 AU. This system was calculated for 0.1 Myrs, to see how the stability region shrinks – this is shown by the number of stable orbits – (see Table 2). To get the number of stable orbits it was necessary to determine the value of  $e_{max}$  after 0.1 Myrs (this new  $e_{max}$  of the stable region ranges from 0.06 to 0.32), as it is shown in Table 2. New  $e_{max}$  means that we set the upper limit for the Trojans eccentricity so that they are still in the region of stable motion (more details about the MEM are shown in Sec. 2.1).

The results are shown in Fig. 3 and Fig. 4, where we can see a convex structure which extends from  $\omega=25^\circ$  to  $35^\circ$ . The convex structure is getting flatter, if the initial eccentricity is very small. After this convex region (well visible in Fig. 4) the value of  $e_{\rm max}$  rises up to 0.32. Our calculations also revealed that the  $e_{\rm max}$  of the stable region was twice as large as that of the  $e_{\rm ini}$  (shown in Table 2), a result that illustrates how the size of the stable region and the value of  $e_{\rm max}$  depends on  $e_{\rm ini}$ . The numerical simulation shows that the stable region extends from  $\omega=20^\circ$  to  $65^\circ$  and the semimajor axis from a=0.89 [AU] to 0.94 [AU]. We can conclude that 17% or 286 orbits of the 1680 calculated ones are stable.

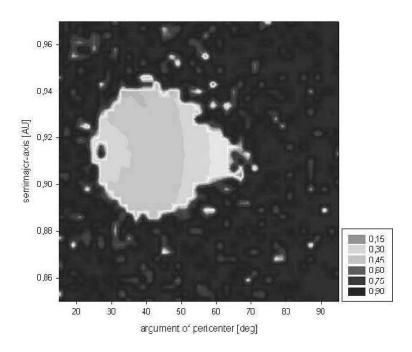


Figure 3. This figure shows system HD17051 for a computation time of 0.1 Myrs. The light region is the most stable whereas the dark region indicates chaotic motion.

# 4.2 Stability regions of HD93083 and HD27442

Both extrasolar systems have main sequence stars, but no sun like spectra. HD27442 has a large stable region, because the new  $e_{\rm max}$  (shown in Table 2) of the Trojan planet is very small and lies fully in the HZ after 0.1 Myrs (see Fig. 6). The stable region of HD93083 which is smaller has an elongated shape (see Fig. 4). That's the reason, because the GG is very close to the star (a=0.48 AU) and has a relatively large initial eccentricity (e=0.14) shown in Table 1. The new  $e_{\rm max}$  (see Table 2) of the stable region go up to 0.26, but nevertheless the orbits lie 96 percent in the HZ.

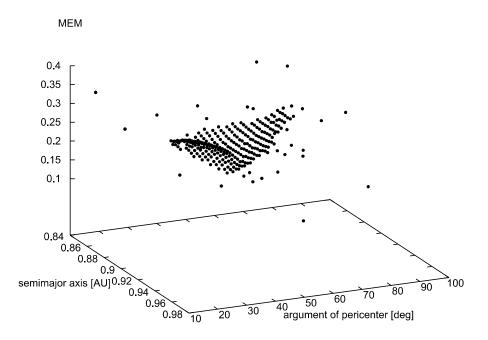


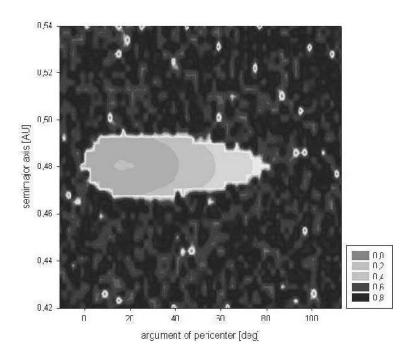
Figure 4. Shows a 3D depiction for the system HD17051. A large MEM indicates unstable motion.

Table 2. List of all results for four of the systems listed in Table 1, which illustrates the extension of the stable region of the Trojan planets after 0.1 Myrs. *1st column:* name of the investigated system, *2nd column:* inital eccentricity of the GG, *3rd column:* new  $e_{\text{max}}$  of the stable region, *4th column:* number of the stable orbits vs. the calculated one, *5th column:* minimum of the perihel with the new  $e_{\text{max}}$ , *6th column:* maximum of the aphel with the new  $e_{\text{max}}$ , *7th column:* partly in the HZ [%] after 0.1 Myrs. The number of the calculated orbits were changed, because of the different geometry of the stable regions.

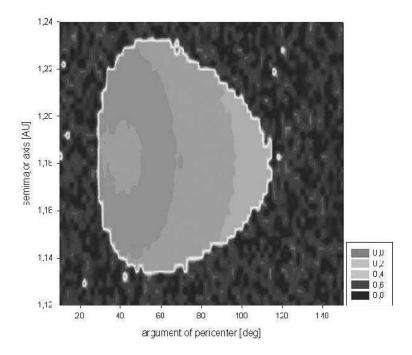
System	$e_{ini}$	new	new Number of min. of		max. of	partly in
-		$e_{max}$	stable orbits	the perihel	the aphel	the HZ
			/calc. orbits	[AU]	[AU]	[%]
HD93083	0.14	0.00 - 0.26	318 / 2580	0.36	0.61	96
HD17051	0.24	0.06 - 0.32	286 / 1800	0.62	1.20	87
HD28185	0.07	0.02 - 0.19	555 / 1800	0.83	1.23	100
HD108874	0.20	0.11 - 0.30	421 / 2000	0.76	1.38	87
HD27442	0.07	0.00 - 0.19	360 / 2000	0.96	1.40	100
HD188015	0.15	0.00 - 0.25	684 / 2250	0.89	1.49	100

# 4.3 Stability regions of HD108874 and HD188015

From the examination of HD108874 and HD188015 - both are main sequence stars (G5) - followed that the Trojan planets of the GGs are mainly



*Figure 5.* System HD93083 for a computation time of 0.1 Myrs. The light region is the most stable whereas the dark region inidcates chaotic motion.



*Figure 6.* System HD27442 for a computation time of 0.1 Myrs. The light region is the most stable whereas the dark region indicates chaotic motion.

in the HZ with a new  $e_{\rm max}$  (see Table 2) not higher than 0.29. The results are shown in Table 2 or Fig. 7 for HD108874 and Fig. 8 for HD188015. The results

show that the system HD108874 has a large stable region, but lies only partly in the HZ, because the new  $e_{\rm max}$  is to large. Whereas HD188015 has a large stable region which lies fully in the HZ. We investigated Trojan like motion in 10 single planetary systems where the initial eccentricity is not larger than 0.3 and the gas giant lies partly or fully in the HZ. Than we selected 6 systems, where the gas giant lies also mainly in the HZ. Numerical simulations show, how much orbits of the Trojan planet lie in the HZ after an integration time of 0.1 Myrs. That happens if the new  $e_{\rm max}$  continues (during the integration) very small so that the stable region in the HZ becomes very large. We found out that from the six selected extrasolar systems only three extrasolar systems are completely inside the HZ (see Table 2), but only two of them have Sun like spectra.

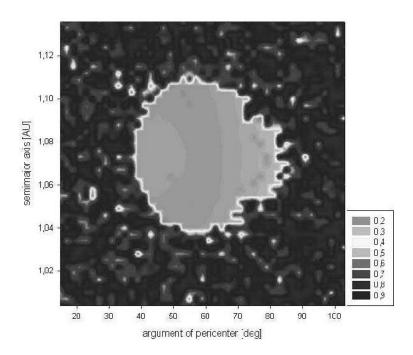
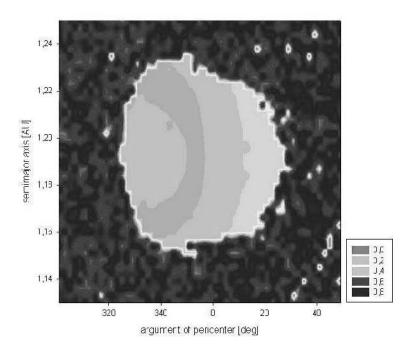


Figure 7. Stability region for the system HD108874 for a computation time of 0.1 Myrs. The light region is the most stable whereas the dark region indicates chaotic motion.

### 5. Influence of the orbital elements M and $\omega$

In the last chapter the size and structure of the stable zones were investigated. This was done by the variation of  $\omega$ , but former investigations (see Schwarz 2005a and Schwarz et al. 2005b) used the variation of M. Now we are able to compare the variation of this two parameters and show if there is any difference. A variation of of M changes the location of the Trojans whereas, if we use the orbital element  $\omega$  we change the location of the Trojans ellipse. Table 3 shows the calculation of the four extrasolar systems. In this table we compare



*Figure 8.* Stability region for the system HD188015 for a computation time of 0.1 Myrs. The light region is the most stable whereas the dark region indicates chaotic motion.

the number of stable orbits for M and  $\omega$ . It is well visible, that the number of stable orbits is larger for  $\omega$  than for M. This can also be seen in the new  $e_{\text{max}}$  of the stable region. New  $e_{\text{max}}$  has two values, because the eccentricity of the stable region is not homogeneous (Table 3 shows two values the upper and the lower limit of the new  $e_{\text{max}}$ ). Therefore we have an example HD17051, were the Fig. 3 and Fig. 4 shows how the new  $e_{\text{max}}$  is distributed.

Now I want to present the interaction of the initial M vs. the initial  $\omega$  (shown in Fig. 8). The comparison of both orbital elements was done for the extrasolar system HD17051 for an integration time of  $10^4$  years (initial conditions see Table 1). The  $\omega$  and M extends from  $0^\circ$  to  $360^\circ$  and have a gridsize of  $\Delta\omega=4^\circ$  and  $\Delta M=4^\circ$ . The first thing to notice is that in Fig. 9 we have two stable diagonal regions. The left region (goes from  $\omega=275^\circ$  to M=275°, the width of the stable region is approximately between +-25°) shows the  $L_5$  region and the right one ( $\omega=50^\circ$  to M=50° the width is also +-25°) that of  $L_4$ . There are also two small stable regions in the left lower corner and in the right uper corner, which belongs to the  $L_4$  and  $L_5$  regions. Another investigation of the extrasolar systems (HD 28185) shows that the stable region (of  $\omega$  vs. M) depends on the mass of the gas giant (HD28185 has a very massive gas giant  $M_{jup}=5.7$ ) and the eccentricity of both (Trojan planet and gas giant). This investigation (shown in Fig. 10) was undertaken for a smaller gridsize of  $\Delta\omega=2^\circ$  and M=2° and also for an integration time of  $10^4$  years. For higher eccentricities

the continous stable region will splitted in to two islands (see Fig. 10, lower panel). We can conclude that there exist for a mass of the gas giant of about  $M_{jup}=6$  and an initial eccentricity higher 0.15 no continous stable region. The fact that we have a linear continous stable region for the Lagrangian points give us the possibility to depict  $\omega$  and M in a simple ratio. This could be used for future calculations to vary both orbital elements ( $\omega$  and M) during the integrations.

Table 3. Results of four systems listed in Table 2, which illustrates the extension of the stable region of the Trojan planets after 0.1 Myrs for the variation of the mean anomaly and the argument of perihelion. Ist column: name of the investigated system, 2nd column: inital eccentricity of the GG, 3rd column: new  $e_{\text{max}}$  of the stable region for the mean anomaly, 4th column: new  $e_{\text{max}}$  of the stable region for the argument of the perihel, 5th column: number of the calculated orbits, 6th column: Nr. of stable orbits for M, 7th column: Nr. of stable orbits for  $\omega$ , The number of the calculated orbits were changed, because of the different geometry of the stable regions.

System	$e_{ m ini}$	new	new	Nr. of	Nr. of	Nr. of
		$e_{ m max}$ of M	$e_{ m max}$ of $\omega$	calc. orbits	stable orbits for $M$	stable orbits for $\omega$
HD17051	0.24	0.4 - 0.5	0.06 - 0.32	1800	73	286
HD28185	0.07	0.1 - 0.2	0.02 - 0.19	1800	161	591
HD108874	0.20	0.3 - 0.4	0.11 - 0.29	2000	159	421
HD27442	0.07	0.1 - 0.15	0.00 - 0.19	2000	926	1259

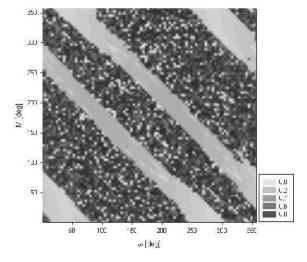


Figure 9. Stability region for the system HD17051 for a computation time of  $10^4$  yrs. The light region is the most stable whereas the dark region indicates chaotic motion.

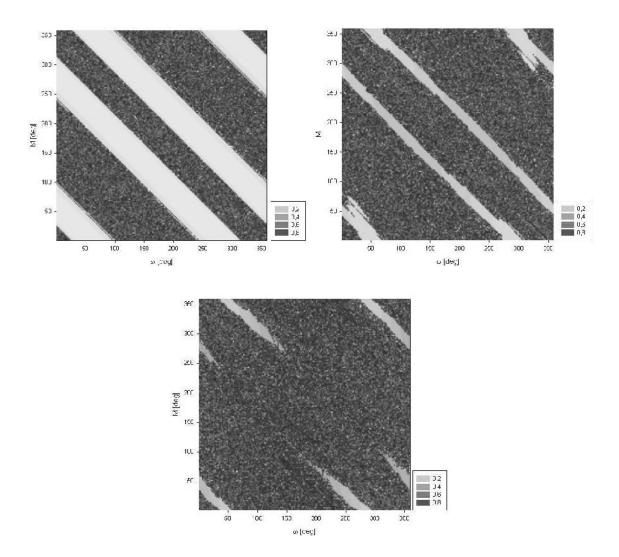


Figure 10. Stability region for the system HD28185 for a computation time of  $10^4$  yrs and different initial eccentricities:  $e_{\rm ini}=0$  (upper Figure),  $e_{\rm ini}=0.07$  (middle Figure) and  $e_{\rm ini}=0.15$  (lower Figure). The light region is the most stable whereas the dark region indicates chaotic motion.

#### 6. Discussion

We investigated Trojan like motion in 10 extrasolar planetary systems by using the restricted three body problem. The GG of the selected systems are partly or fully in the HZ and the initial eccentricity dont exceed the value of 0.3. We checked the extrasolar systems - by using the studies of Marchal - in the 1:1 MMR for the selected systems, where the gas giant moves near the HZ. We can conclude that only one of the investigated systems have no stable region (HD 141937) and that the stable region of the Trojan planets are getting smaller with larger values of  $\mu$  and e (see Fig. 2).

Numerical simulation were done to investigate the dynamical stability of six extrasolar planetary systems, which lie fully in the HZ. The MEM were

used to detemine the stability of the Trojan orbits. We find out that three systems dynamical lie completely in the HZ after a calculation time of 0.1 Myrs (HD28185, HD108815 and HD27442), but only two of them have Sun like stars (HD28185 and HD108815). The other three systems could also be candidates for habitable Trojan planets, because the stable orbits lie 87 percent (HD17051 and HD108874) and 96 percent (HD93083) in the HZ. Another part of this work was to investigate the interaction, if we change the initial mean anomaly (M) or the initial argument of perihel ( $\omega$ ) during the calculation. The comparison of both orbital elements was done for the extrasolar system HD17051 and HD28185 for an integration time of  $10^4$  years. We could find out that, if we vary the  $\omega$  there are much more stable orbits than for M. Because of that, future calculations should include both orbital elements, to become a more realistic simulation. Therefore further calculations should be done to analyse the stability region of  $\omega$  and M for different masses, eccentricties and inclinations.

### Acknowledgments

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#### **Notes**

- 1. i.e. the region where possible terrestrial plantes can have (a) liquid water on the surface and (b) a stable atmosphere shown in Fig. 1
  - 2. The Extrasolar Planets Encyclopedia at http://www.obspm.fr/encycl/encycl.html
- 3. A thrid body, which always remains in the orbital plane of the primaries, feels their gravitaional attraction, but does not influence their motion, because the mass is very small
  - 4. that means only one planet in these extrasolar systems is known.

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