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CONSTRUCTION OF THE CONTACT ZONE OF A HELICAL CYLINDRICAL EXTERNAL GEAR PAIR WITH STRAIGHT TOP LAND MERIDIANS

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Abstract: The study deals with the construction of the contact zone of helical cylindrical external gears. The shape and size of the contact zone determine the phenomena that occur even in the case of error-free tooth design, installation error exemption and load-induced error exemption Such a phenomenon is the vibration excitation resulting from the connection, which also affects the identification of the drive elements. It is possible to deviate from the regular rectangular shape of the contact zone by modifying the top land surfaces. The change can mean a combination of the change in the total length of the contacting generating lines and the change in the contact ratio. This study deals with the construction of the generalized contact zone.

Keywords: *contact zone, contacting generating line, top land meridian*

1. THE RELATIONSHIP BETWEEN TOOTH DIRECTIONS AND THE COMPONENTS OF THE CONTACT ZONE

When designing a gear pair, the geometry of the gears, their position relative to each other, the direction of the teeth and their direction of rotation are fixed. With this knowledge, it is possible to decide where the point of meshing in and the point of leaving meshing of the contacting generating lines are located in the contact zone (Erney, 1983), (Litvin F. , 1972). Figure 1 shows the two basic cases that help determine the arrangement of the contact zone. The pinion (small wheel) is the drive wheel, on which the teeth can be left (L) or right (R) tooth direction. On the driven wheel (gear), in the case of an external connection, the tooth direction must be opposite to this. The red dashed line marked on the wheels indicates the position of the recommended top land meridian modification.

In the lower part of Figure 1, the contact zone belonging to the meshing gears can be seen in the yz coordinate system. The contact zone is located in the contacting plane, which is the common tangent plane rolling down from the base cylinders of the gears, the common tangent plane of the base cylinders.

The size of the contact zone is determined in the y direction by the common tooth width, and in the z direction by the distance that the addendum cylinders cut out of the contacting plane in the transverse plane(s) (point A in the case of the large wheel and point E in the case of the small wheel). These points mark the point of meshing in (A) and the point of leaving meshing (E) in the case of deceleration drive. (The figure does not show the apparently increased part of the contact zone.)

The origin of the yz coordinate system is located in half of the common tooth width and on the component containing the main point C of the meshing. The slanted lines passing through points A and E indicate the contacting generating lines (the contact line of the meshing tooth sides).

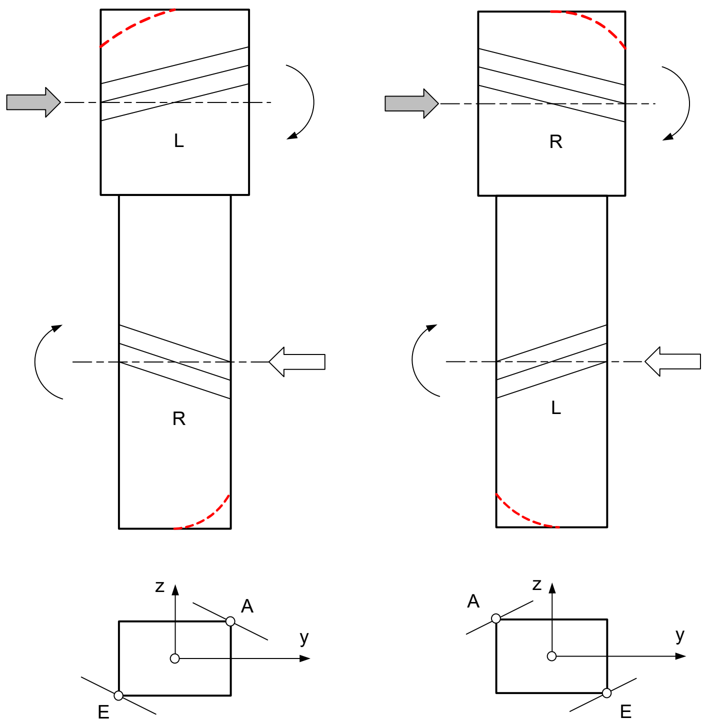

Figure 1. *Layout options for the contact zone*

2. KNOWLEDGE FROM GEAR PAIR DESIGN

There are many articles in the literature that talk about modifying the shape of the gear tooth. These usually deal with the modification of the profile (tip relief modification), crowning or end relief (Litvin, et al., 2003), (Litvin, Gonzalez-Perez, Fuentes, Hayasaka, & Yukishima, 2005), (Tran, Hsu, & Tsay, 2014), (Jamali, Sharif, Evans, & Snidle, 2015), (Yang, Wu, Li, & Liu, 2023). This article discusses the modification of the top land surface, as a result of which the shape of the contact zone changes.

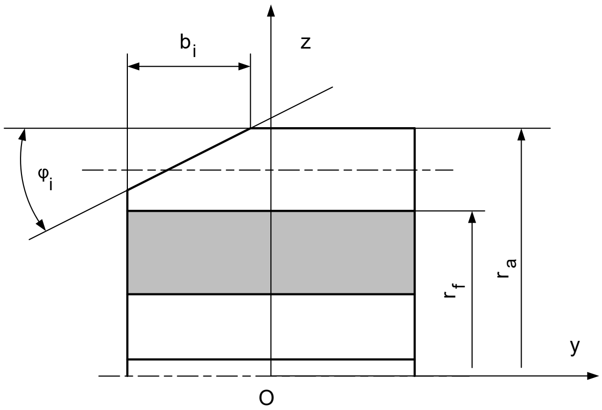

Figure 2. *Drawing representation of the top land meridian modification*

Based on the knowledge about the contact zone (Graf von Seherr-Thoss, 1965), (Debreczeni, 2021), the designer, deviating from the top land meridian that defines the regular rectangular contact zone, creates it with several well-defined lines compared to each other. Figure 2 shows a detail of a gear drawing, where a single line represents the modification, but there can be several of them. For further investigations, let's follow the drawing definition of a straight line. The gear should be defined in the „O" centred coordinate system with the parameters specified by the designer.

The top land of the tooth should be truncated by a straight line given by the parameters φ_i, b_i shown in Figure 2. To construct the contact zone, the design data must be transformed and the identification of the meridian consisting of several straight lines (Figure 3) must also be taken into account.

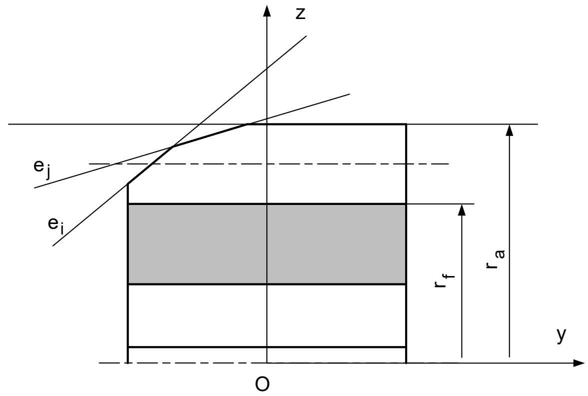

Figure 3. *Example of a meridian element consisting of several straight lines*

The i[th] line element of the meridian, which generates the top land surface and also maps the borders of the contact zone, is interpreted according to Figure 4. The meridian element also intersects the axis of the gear wheel, which should be marked by point $K_i(y_{Ki}, z_{Ki})$.

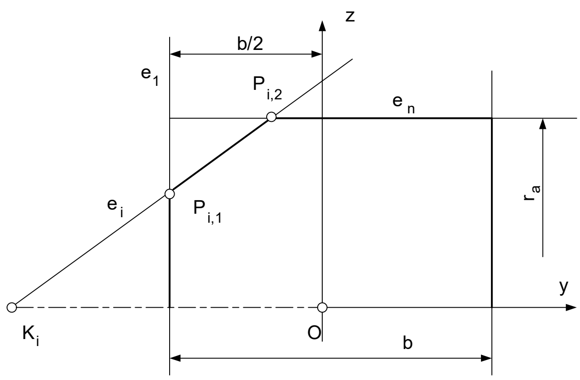

Figure 4. *Definition of a meridian straight line*

Let n be the number of straight lines that make up the axial section of the space enclosed by the transverse surface of the gear on the modification side and the addendum cylinder. Among the straight lines defining the transverse surface, the first $y = -\frac{b}{2}$ is parallel to the z axis. The last (n-th) straight line, usually parallel to the axis of rotation, is the generator of the addendum cylinder, whose equation is $z =$

r_a. The additional elements of the meridian have a number of $k = n - 2$, i.e. if the modification contains k elements, then the number of straight lines to be examined is $n = k + 2$. Points that define the i^{th} line have coordinates $P_{i1}(y_{i1}, z_{i1})$and $P_{i2}(y_{i2}, z_{i2})$. (Figure 5).

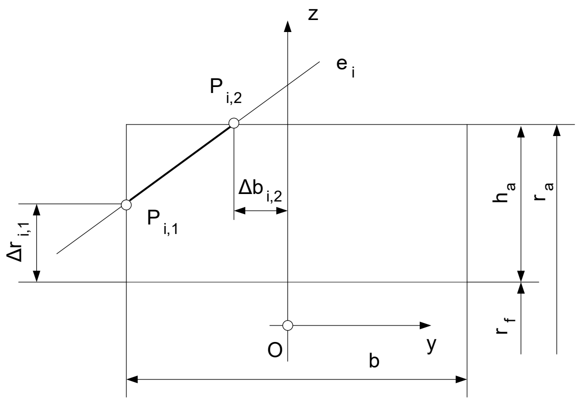

Figure 5. *Determining points of the meridian elements*

By definition, the possibility of changing the radial direction is possible in the range of height h_a designated by the dedendum and addendum cylinders:

$$h_a = r_a - r_f \tag{1}$$

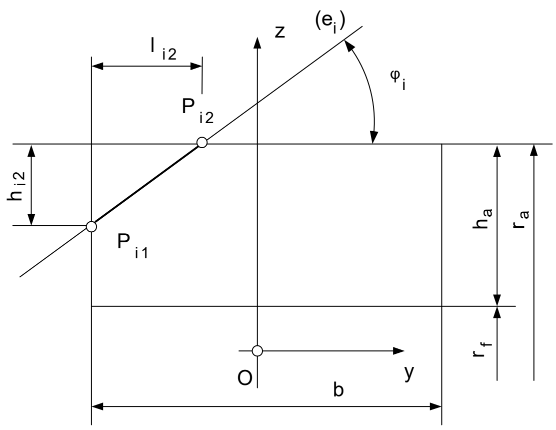

Figure 6. *Drawing geometry*

The coordinates of the two points of the i[th] line are $P_{i1}\left(-\frac{b}{2}, r_f + \Delta r_{i1}\right)$ and $P_{i2}(-\Delta b_{i2}, r_a)$. Values Δb_{i2} and Δr_{i1} can be obtained from the drawing documentation. In the drawing documentation, Figure 2 presented the design data for a meridian line of the wheel, which can be generalized to a straight meridian element of any position with the help of Figure 6.
With the dimensions given in the drawing

$$h_{i1} = l_{i2} \cdot \tan \varphi_i \qquad (2)$$

from which

$$\Delta r_{i1} = h_a - h_{i1} \qquad (3)$$

Forming a ratio factor a_i

$$a_i = \frac{\Delta r_{i1}}{h_a}, \quad (0 \leq a_i \leq 1) \qquad (4)$$

which can be used to select any point if a_i is chosen freely. The value of Δb_{i2} can be determined from half the tooth width:

$$\Delta b_{i2} = \frac{b}{2} - l_{i2} \qquad (5)$$

which can also be specified with a ratio factor:

$$c_i = -\frac{\Delta b_{i2}}{0{,}5b}, \quad (-1 \leq c_i \leq 1). \qquad (6)$$

3. Generalization of the formation of meridian straight lines

As can be seen in Figure 4, the supporting points of the straight lines forming the meridian of the top land are located on the transverse surface of the gear wheel $P_{i1}(y_{i1}, z_{i1})$ and on the addendum cylinder $P_{i2}(y_{i2}, z_{i2})$. It has already been seen in the previous sections that these points can be read from the drawing documentation of the gears or can be specified based on subsequent design modifications. However, it is advisable to calculate these from the zone modification inferred based on practice. The equation of the i-th generalizable line can be given according to the following procedure.
The coordinates of a pair of points can be systematized in the matrix:

$$\boldsymbol{P}_{(n\times 4)} = \begin{pmatrix} y_{11} & z_{11} & y_{12} & z_{12} \\ y_{21} & z_{21} & y_{22} & z_{22} \\ \vdots & \vdots & \vdots & \vdots \\ y_{n1} & z_{n1} & y_{n2} & z_{n2} \end{pmatrix},$$

the elements of which are denoted by $p_{ij}(i = 1,2,\dots,n; j = 1,2,3,4)$. The equation of the i^th^ line can be expressed as follows with the elements of the matrix:

$$\begin{gathered} \frac{p_{i4} - p_{i2}}{p_{i3} - p_{i1}} \cdot (y_i - p_{i1}) = z_i - p_{i2} \\ z_i = \frac{p_{i4} - p_{i2}}{p_{i3} - p_{i1}} \cdot y_i + p_{i2} - \frac{p_{i4} - p_{i2}}{p_{i3} - p_{i1}} \cdot p_{i1} \end{gathered} \tag{7}$$

from which:

$$z_i = m_i \cdot y_i + b_i, \tag{8}$$

where:

$$m_i = \frac{p_{i4}-p_{i2}}{p_{i3}-p_{i1}}, \tag{9}$$

$$b_i = p_{i2} - \frac{p_{i4}-p_{i2}}{p_{i3}-p_{i1}} \cdot p_{i1}. \tag{10}$$

Within the interpretation range of y_i, the z_i coordinate can be calculated for any y_i coordinate, which is always the current addendum circle radius interpreted on the gear meridian.

4. The interpretation range of the top land meridian, definition of intersection points

The top land meridian consists of straight lines whose intersection points determine the interpretation range of each line along the width. Figure 7 illustrates the interpretation range along the tooth width for the i^th^ line. By selecting the i^th^ line of the meridian, then adding the $(i-1)^{\text{th}}$ line and the $(i+1)^{\text{th}}$ line, they create the M_i, M_{i+1} intersection points with their intersection points. On the i^th^ straight line, the definition of the addendum circle radii is interpreted only in the range Δy_i where:

$$y_i \le y \le y_{i+1}. \tag{11}$$

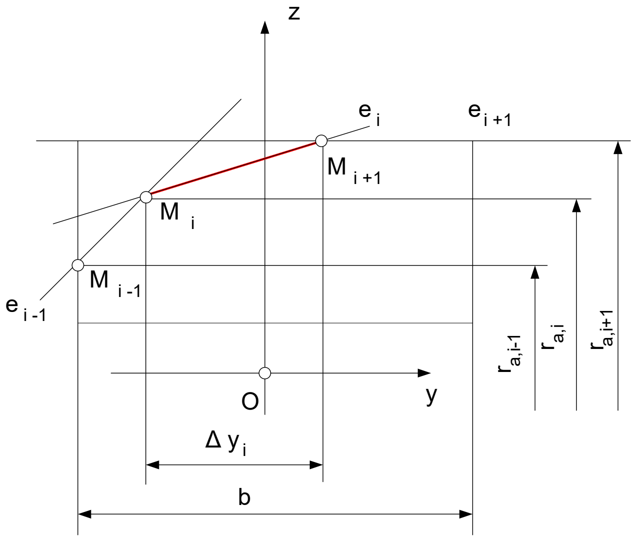

Figure 7. *Interpretation range of meridian elements*

The z coordinates determine the current addendum circle radii (r_a), i.e. $r_i \leq r_a \leq r_{i+1}$.
The intersection point of two arbitrary lines (i and $i+1$) can be determined from the equations of the lines (7-10) if $z_i = z_{i+1}$, i.e.

$$m_i \cdot y_i + b_i = m_{i+1} \cdot y_i + b_{i+1} \tag{12}$$

from which

$$y_i = \frac{b_{i+1} - b_i}{m_i - m_{i+1}}, \tag{13}$$

and

$$z_i = r_{a,i} = m_i \cdot \frac{b_{i+1} - b_i}{m_i - m_{i+1}} + b_i. \tag{14}$$

To facilitate further analysis, let's introduce $M_i^1(y_i^1, z_i^1)$ to mark the intersection point belonging to the drive wheel, and $M_i^2(y_i^2, z_i^2)$ to mark the intersection point belonging to the driven wheel.

5. THE RELATIONSHIP BETWEEN THE INTERSECTION POINTS OF TOP LAND MERIDIANS AND THE CONTACT ZONE

The top land meridian is rotated around the axis of the gear wheel to create the top land surface (Roth, 1989a), (Roth, 1989b). The parts of the top land surface marked by intersection points are cone and cylinder surfaces. One boundary of the contact zone is determined by the intersection of these surface elements and the contacting plane (zone lower border), the other boundary is formed by the top land surface of the meshing gear with the contacting plane (zone upper border) (Figure 8). The driving wheel creates the zone lower border, the driven wheel creates the zone upper border. Contacting generating lines defining a meshing are interpreted only in the zone.

If the teeth of a gear wheel are not symmetrical, i.e. the base profile angles of the tooth sides are different, then depending on the direction of rotation, two different contact zones are formed (Drágár & Kamondi, 2018).

The intersection points determined on the top land meridian also form intersection points in the contact zone. Between the intersection points, which are formed as a section of a conic surface, a hyperbola section is the zone wrapper. In the future, it must be verified whether these sections can be replaced by straight lines, because then the meshing phenomena taking place in the contact zone can be examined more simply.

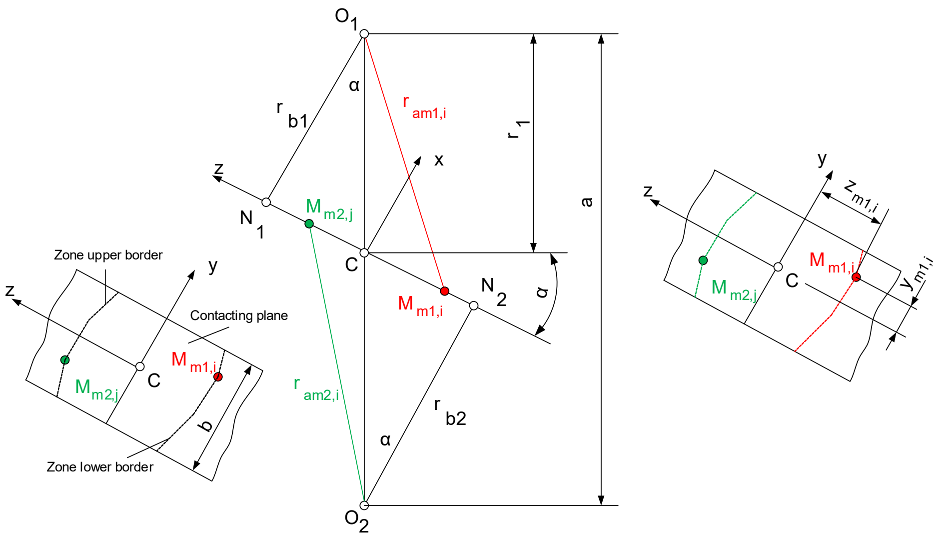

Figure 8. *Intersection point determined by the z coordinate of the contact zone*

The intersection point (M_i) determined on the meridian curve appears in the contact zone as the zone border point (M_{mi}) and its position can be calculated using the geometric data of the gear wheel (Drágár & Kamondi, 2021). Among the coordinates of the zone point, the y direction is identical to the similar coordinate of the point of intersection, z is to be determined, the generalized shape of which is

$$z_{mj,i} = \sqrt{\left(r_{amj,i}\right)^2 - \left(r_{bj}\right)^2} - \sqrt{\left(r_j\right)^2 - \left(r_{bj}\right)^2}, \tag{15}$$

when j = 1 drive wheel, j = 2 driven wheel, and i is the generalized intersection point.

6. CONCLUSIONS, RESULTS

The study presented the process in which the contact zone can be determined by knowing the geometry of the gears and the necessary tests can be performed in it. The meridian of the top land surface can be described in an easy-to-use and generalizable form with the presented description. The shape of the meridian elements (straight or higher order) between the intersection points (intersection planes) can also be chosen freely. The designer of the gear wheel has the freedom to shape the top land meridian according to his own vision, but at the same time, due to the role of the gear wheel in the drive, he may also require a special modification of the top land meridian. The presented procedure can help the designer to make quick and targeted modifications, as the method is also suitable for an automatic (expert) solution procedure, therefore, it gives the opportunity to be part of the planning assistance system in a computer program.

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