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Discovering epitrochoid curves with STEAM-based learning methods*

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Abstract. In this paper, we present a new teaching-learning technique for drawing and identifying several members of an important family of parametric curves based on educational robotics supported by dynamic geometry software. Epitrochoid curves are essential in teaching first-year computer science and engineering students mathematics. From a methodological point of view, these are usually attractive and aesthetic curves suitable to capture students' interest and give first-hand experiences and activities. To implement a new STEAM-based learning method in this field, we created a drawing LEGO robot to visualise the epitrochoid curves and improved a virtual Epitrochoid Tracker with the Desmos graphing calculator to check the parametric equations for the drawn curve. The paper focuses on the two pillars of the developed STEAM-based learning method and their pedagogical aspects.

Keywords: STEAM-based education, dynamic geometry software, educational robotics, parametric curves, epitrochoid curves

AMS Subject Classification: 97I20, 14H50

1. Introduction

Nowadays, innovative teaching and learning methods can be applied in higher education thanks to technological advances and the spread of STEAM education. A trend in STEAM education is the integration of cutting-edge technologies [4, 17]. The necessity of integrating technology into education and using educational tech-

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nologies in the process of teaching-learning is a widely accepted idea in the field of education science [23]. Many studies demonstrate that integrating technology into mathematics education and its use in the teaching-learning process increases students' academic success and motivation, positively affects their attitudes towards learning, supports the development of students' problem-solving and cooperative learning skills, and provides teachers with more opportunities to guide their students [2, 7, 18, 21]. In today's highly digital age, portable info communication tools (ICT) have become essential to students' daily lives. Dynamic geometry software (DGS) that can also run on mobile devices helps to increase focus by capturing students' attention by providing visual elements [24]. However, digital content alone is insufficient for Generation Z students to sustain attention. Several studies have shown that first-hand experiences are the best way to achieve optimal learning outcomes for today's university students [1, 5, 10, 16]. This explains the re-emergence of hands-on and explorable models in universities and increased projects involving physical activity in mathematics courses. The 19^{th} century descriptive and differential geometry models, such as Schilling's kinematic models, are again used in university practice [19]. The limited accessibility makes it worth considering the possibilities of creating kinematic models that demonstrate a given problem well in the classroom environment. One of these possibilities is the innovative use of educational robot kits to provide didactic tools for specific chapters of mathematics.

Students at technical universities study curves of the Euclidean two-dimensional plane during their first semester, including trochoidal curves. These curves can be drawn with physical tools or virtually. We can draw by hand or with an easy-to-build device if we choose the first option. Drawing trochoids by hand requires tools; for example, we can use LEGO Technic gears to model the non-slip rolling, as demonstrated in [13].

As for drawing a virtual curve, maths teachers have plenty of possibilities to illustrate the current curriculum. In recent years, DGS has become increasingly popular because it effectively helps students understand basic and advanced geometrical concepts at all levels of education. However, because of its interactive nature, a complex DGS has many features beyond the simple representation of curves. Such a program can be used, among other things, to create spectacular animations that demonstrate the generation of curves in the process, providing a deeper understanding of this topic.

This paper focuses on one of the families of trochoidal curves, epitrochoids. We introduce a new STEAM-based method by integrating practical approaches to support the teaching-learning process.

2. Mathematical background

A roulette is a plane curve considered as the trajectory of a point rigidly connected with some curve rolling upon another fixed curve without slipping. Let us denote the fixed curve by F and the moving curve by M. The point P, which traces the roulette, is said to be the pole or the generator. If F and M are both circles, and

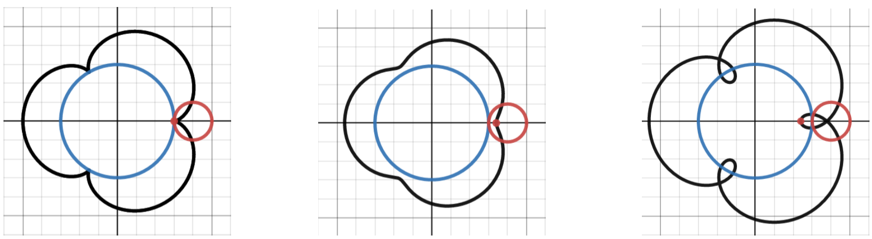
M is rolling outside of F, the obtained roulette is called an epitrochoid from the Greek words 'epi' (over) and 'trokhos' (wheel).

Suppose that a circle of radius r is rolling around the outside of a fixed circle of radius R, and the pole P is attached to the moving circle a distance d from its centre. Then, the parametric equations of the epitrochoid traced by P are

$$\begin{aligned} x(t) &= (R+r)\cos t - d\cos\left(\frac{R+r}{r}t\right), \\ y(t) &= (R+r)\sin t - d\sin\left(\frac{R+r}{r}t\right), \end{aligned} \tag{2.1}$$

where the independent variable $t \in \mathbb{R}$ denotes the angle between a line through the centre of both circles and the x-axis [15]. In the sequel, the quantities R, r and d in (2.1) will be referred to as parametric constants and d will be called the pole distance.

One of the ways of classifying epitrochoids is based on the relation between the radius of the moving circle and the pole distance. If $d = r$, then the obtained special curve is called an epicycloid (Figure 1a). We speak about curtate or contracted epitrochoid if $d > r$ (Figure 1b), and in case of $d < r$, we obtain prolate or protracted epitrochoid (Figure 1c).

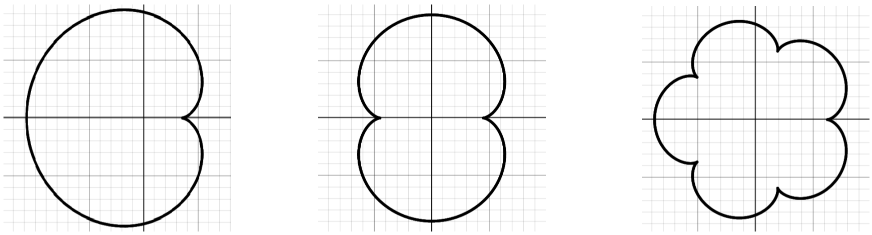


(a) Epicycloid. **(b)** Curtate epitrochoid. **(c)** Prolate epitrochoid.

Figure 1. Representative example of epitrochoid curves. An epicycloid touches the fixed circle; a curtate epitrochoid does not touch the fixed circle, while a prolate epitrochoid crosses it.

Remark 2.1. It should be noted that there is no complete agreement in the literature on the naming of cycloid-type curves. For example, in Hungary, the term epitrochoid is not used; instead, the dimpled and looped epicycloids' names are common for the curtate and prolate epitrochoids [8].

If the parametric constants of the epitrochoid are chosen specifically, we obtain special plane curves. Clearly, if $d = 0$, the trace of the pole is a circle of radius $R+r$. If the radius of the fixed and the moving circle are equal, we have a limaçon. In particular, a limaçon is called cardioid if $d = r$ also holds. For STEAM-based teaching of the cardioid curve combined with educational robotics, see [12, 14]. Some other epicycloids have been given their specific name depending on the ratio of the radii of the circles. For example, a nephroid is an epicycloid with $\frac{R}{r} = 2$ and we have a ranunculoid when $\frac{R}{r} = 5$ (Figure 2). In general, if R and r are relatively prime numbers, the curve closes on itself and has R cusps, where a cusp is defined as a point where the epicycloid meets the fixed circle. If $\frac{R}{r}$ is irrational, the curve will never return to the initial starting point.



(a) Cardioid ($R = r$). **(b)** Nephroid ($R = 2r$). **(c)** Ranunculoid ($R = 5r$).

Figure 2. Special epicycloids.

Remark 2.2. Hypotrochoids can be generated similarly to epitrochoids, but in this case, the rolling circle is inside the fixed one. For the parametric equations of hypotrochoids, see [11], where we outline a learning project to study these curves.

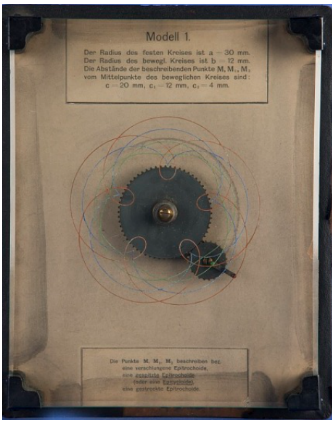
3. An innovative STEAM-based method for teaching epitrochoid curves

The approach proposed in this paper to teach and learn epitrochoid curves is based on the combination of educational robotics and dynamic geometry software.

3.1. Epitrochoid drawing devices

At the end of the 19$^{\text{th}}$ and the beginning of the last century, using physical models to illustrate certain mathematical phenomena was relatively common among mathematics teachers. These models were used mainly to represent curves and surfaces and other mathematical concepts useful to mathematicians, engineers and scientists. Most models were created in Europe, particularly in Germany, for use in school and college teaching. One of the most prominent manufacturers of the models was Martin Schilling's company. The kinematic models were designed by Frederick Schilling, a professor of mathematics at Göttingen. The 1911 Schilling Catalog lists 377 items divided into forty series. Series XXIV consists of kinematic models, of which the first group of 4 models is suitable for representing trochoids. The first model is a device for plotting epitrochoids, where a gear can be rolled along another fixed gear (Figure 3a).

While Schilling's kinematic models were designed primarily for educational purposes, the Spirograph toy was intended to entertain a wider audience. The Spiro-



(a) Kinematic model by Martin Schilling, Series XXIV, model 1, number 329.

(b) Original Spirograph set.

Figure 3. Devices for drawing trochoids.

graph kits contain serrated-edge plastic parts and can be used to create various patterns, including several trochoidal curves (Figure 3b). Spirograph parts contain small holes into which the tip of a pen can be inserted, and the pattern can be drawn by rolling the part over another fixed part. Obviously, the pole distance can only be less than the radius of the rolled circle, so only curtate trochoids can be drawn with the Spirograph toy [22].

3.2. Robot model implementation

Epitrochoid curves are derived by the non-slip rolling of a circle along another fixed circle. Non-slip rolling is achieved in most epitrochoidal drawing tools by the use of gears. Since LEGO sets contain a variety of different sizes of gears, they are suitable for building a drawing device. The prototype of our drawing robot (Figure 4) was built using elements from the LEGO SPIKE Prime set, but of course, elements from other robot kits can also be used. The factor that most determines which epitrochoids can be drawn with the robot is the size of the suitable gears in the set. The SPIKE Prime core set includes four double bevel gears with 36, 28, 20 and 12 teeth. These gears can be used to model the rolling circle in epitrochoid generation. The fixed circle was modelled with a different type of part. For this purpose, we applied LEGO Technic turntables, which are available in two sizes.

In the robot model, a motor was used to roll the moving gear along the fixed gear. The motor actually turns a lever to which the gear is attached. The connecting lever was designed to be adjustable in length to easily assemble the possible gear configurations. A drawing head is mounted on the moving gear, where the writing instrument can be placed, also at adjustable distances from the centre of the gear. This means that the pole distance can be equal to the radius of the

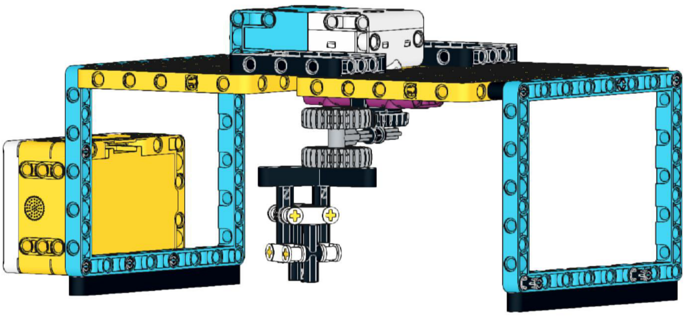
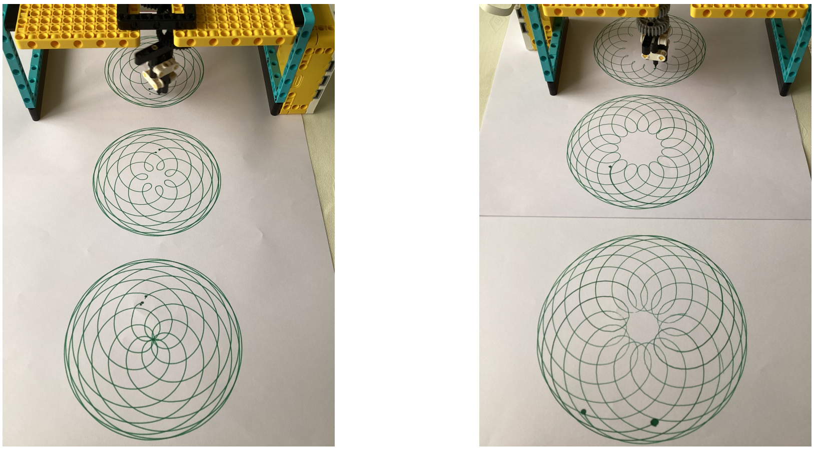


Figure 4. Epitrochoid drawing LEGO robot.

moving circle but can also be larger or smaller. This range of variability allows all three types of epitrochoids to be drawn. In Figure 5, the robot draws different curtate and prolate epitrochoids with pole distances of $d = 16\,\text{mm}$, $d = 24\,\text{mm}$ and $d = 32\,\text{mm}$.



(a) $R = 14\,\text{mm}$, $r = 18\,\text{mm}$. **(b)** $R = 30\,\text{mm}$, $r = 14\,\text{mm}$.

Figure 5. Drawing epitrochoids by different gear configurations.

The Spirograph kit contains parts of many different sizes, but of the three main types of epitrochoid, only the curtate ones can be represented. Schilling's teaching model shows a single representative of all three types because the gears built into the structure are not interchangeable, and the pole distances are fixed values. Conversely, the robot model can incorporate multiple gear combinations and variable pole distances, combining the advantages of the previous devices in a single device and far exceeding their capabilities.

3.3. Principles of STEAM-based learning method

In the Mathematical Analysis courses, first-year students have both theoretical and practical training. After a theoretical grounding, they can work on projects in small groups of students in practical sessions. Education experts say STEAM education is about more than developing practical skills alone. It also helps students develop the capacity to take thoughtful risks, engage in meaningful learning activities, become resilient problem solvers, embrace and appreciate collaboration and work through the creative process [3]. For teaching epitrochoid curves, a LEGO robot should not be used solely as a teacher demonstration tool but is best used as a drawing robot to be built, programmed and operated by the students. In this spirit, our STEAM-based learning method can be represented as a cyclic chain. Repeating segments of the chain consists of five main stages:

(1) Description of the problem, theoretical background, setting goals and tasks.

(2) Practical problem solving, i.e. build (or rebuild) and test a LEGO SPIKE Prime drawing robot.

(3) Determination of the parametric equations for the drawn epitrochoid curve.

(4) Checking the written parametric equations using dynamic geometry software.

(5) Summary of experiences, conclusions of the project, making a presentation.

The chain representing the method consists of repetitive parts because the robot needs to be modified in a targeted way to draw the different epitrochoid curves. The above method provides an opportunity to engage students in cooperative learning and teamwork, where they can share their ideas and apply new knowledge to gain a deeper understanding of the problem. A considerable advantage of epitrochoids is the wide range of curves that can be drawn, making it smooth and easy to perform cyclic steps. In terms of the teaching-learning process, this cyclicality has a positive effect as students become more proficient with both the drawing robot and the geometrical application. In this way, we can integrate educational robotics into the practices of Mathematical Analysis courses.

Building and rebuilding a drawing robot is one of the pillars of the method developed. It was important to collect the requirements that the robot has to meet. Our LEGO drawing robot was designed with the following goals in mind:

(1) to be as simple as possible for quick construction,

(2) to be precisely steerable and highly accurate,

(3) to be able to be equipped with many pairs of suitable gears to draw a variety of curves,

(4) the pole distance should be varied so that all three types of epitrochoid can be drawn,

(5) and the parametric equations of the curve drawn by the robot should be given immediately, knowing the parameters of the current configuration, i.e. the diameter of the mounted gears and the position of the pole.

3.4. Using a dynamic geometry software

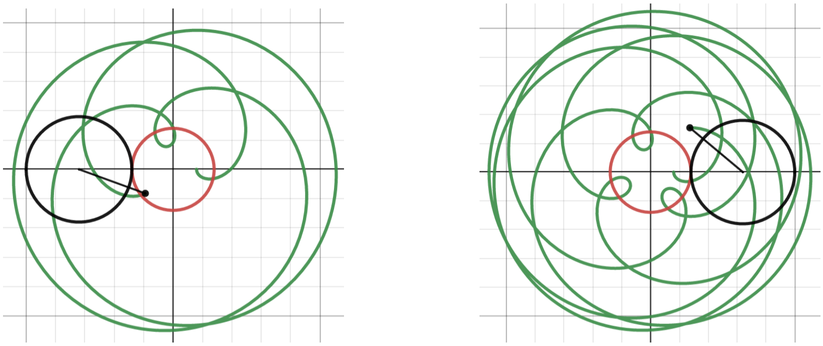
The other pillar of the method is a virtual app for the verification step. There are many applications for drawing and animating epitrochoids on the web, but integrating them into our method has, in general, posed some problems, so we decided to create a new application that can be used for robot-drawn curves during the identification step.

The teaching of parametric curves, particularly roulettes, can be made highly visual by incorporating moving animations that follow the formation of curves from point to point, showing how they are derived. We chose the Desmos graphing calculator among the numerous available options to support our work. Desmos is an easy-to-use free online application because it runs in a browser. Compatible with a wide range of devices, it is also available for smartphones and tablets, so students can use their preferred personal devices when completing assignments.

To plot a parametric curve using Desmos, we need to give the parametric equations $(x(t), y(t))$ and specify an interval for the variable t. By default, the domain of t is the interval $[0, 1]$, which can be modified according to the given curve. If the formula contains additional parametric constants, their values can be changed with a slider so that the effect of the change on the plotted curve is immediately visible. The appearance of the graph can also be modified by changing the colours, adding labels or customising the coordinate system.

In order to quickly identify the curves drawn by the robot, we created an application that takes advantage of the dynamic options offered by Desmos. This little program called Epitrochoid Tracker animates the process of how the curve is formed. It draws not only the trajectory of the pole but, according to the current parameter value, the position of the moving circle and the line segment connecting its centre with the pole are also demonstrated. In Figure 6, the Epitrochoid Tracker displays precisely the same curve that was previously drawn by the LEGO robot (see the middle image in Figure 5a).

We propose two different ways of using dynamic geometry software in the classroom. One is the post-check function. By this, we mean that, given the dimensions of the gears mounted on the robot as well as the pole distance, we can plot with Desmos the curve that we have drawn previously with the robot. This is very easy to do with the Epitrochoid Tracker; after entering the values of the parametric constants and an interval for the variable t, the actual curve is plotted on the screen point by point. The question arises: How do we specify the interval for t to plot the full curve? It depends on the $\frac{r}{R}$ ratio. If $\frac{r}{R} = \frac{p}{q}$, where p and q are relative prime numbers, then the moving circle needs p complete turns to return the pole to its initial position. Thus, a closed curve is obtained if the domain of t is set to the interval $[0, p \cdot 2\pi]$. In case of the epitrochoid shown in Figure 6, $\frac{r}{R} = \frac{9}{7}$, so its



(a) At $t = 5\pi$. **(b)** At $t = 10\pi$.

Figure 6. Two phases of the process when Epitrochoid Tracker plots a curve with parametric constants $R = 14$, $r = 18$, $d = 24$.

parametric equations are

$$x(t) = \frac{16}{5}\cos t - \frac{12}{5}\cos\left(\frac{16}{9}t\right),$$
$$y(t) = \frac{16}{5}\sin t - \frac{12}{5}\sin\left(\frac{16}{9}t\right),$$

where $t \in [0, 18\pi]$.

Another application of Desmos is for pre-planning. This means that the Epitrochoid Tracker can visualise in advance the curves that the robot can draw, given the available gears and pole distances. With this simulation, the robot can be set to draw the curves considered most exciting or important. The Tracker also has a feature that allows one or more parametric constants to be run with a given step size. With this function, students can explore additional interesting and spectacular epitrochoids.

4. Advantages and disadvantages

Robotic mathematical models are an effective tool to promote learning in mathematics education. The success and impact of STEAM learning, skills and competencies may not easily be captured with existing assessment and evaluation tools [4]. Educational theorists believe that robotic activities have tremendous potential to improve classroom teaching [6, 9]. However, studies in which robots are integrated into the curriculum in university practice as a pedagogical tool and examine its effect on twenty-first-century skills are limited. In the spirit of [6], we have gathered our experiences on the advantages and disadvantages of STEAM-based education for drawing robots related to developing 21$^{\text{st}}$-century skills. Our observations are summarised in Table 1.

Table 1. Advantages and disadvantages of the work with drawing robot models related to developing 21^{st}-century skills.

Skills	Advantages	Disadvantages
Knowledge construction	Subject-specific skills development can be achieved by integrating educational robotics.	Integration of educational robotics is tool-dependent.
Interdisciplinarity	Mathematics is combined with engineering and computer science.	A level of knowledge should be reached where the student is able to make connections between the knowledge constructs of different disciplines.
Critical thinking and problem-solving	Visual thinking and modelling enrich the mathematical and technical imagination.	Preparing and executing robotics projects can be time-consuming, impacting the scheduling of teachers.
Using new technologies	The LEGO SPIKE Prime kit is one of the latest educational robot sets. It can be programmed in Scratch or Python.	Due to rapid technological advancements, robotics tools can quickly become outdated.
Innovation and creativity	Educational drawing robots support students' creativity by offering the possibility to develop new and innovative solutions through their versatile re-building.	No structured guidance is available for innovative projects, as they are based on students' initiative.
Cooperation and flexibility	Participation in robotics projects allow students to work in teams, share ideas, and collaborate.	Students can only work with robots in a classroom environment.

5. Conclusions and future work

Methods of teaching mathematics in higher education must align with the possibilities offered by new didactic tools. Examples of such new tools are educational robots. In our paper, we presented a complex teaching-learning technique that can be used with first-year engineering and IT students when discussing one of the notable families of parametric curves, the epitrochoids. Using available technologies and tools, we have effectively combined educational robotics and dynamic geometry software to make STEAM-based learning of this highly visual and exciting subject. Applying the developed method allows students to improve their cognitive and creative activities, enhance their performance, and confirm their mathematical competencies. The main challenge was to create a STEAM-based method for epitrochoids that could be embedded in the classical university education framework while using non-traditional teaching aids to achieve the subject objectives. The relatively large number of curves that can be drawn with the LEGO robot allows groups or pairs of students to be assigned different problems, which is a significant advantage when defining project-based tasks.

When using a STEAM-based method, the proportions of the components are not constant. Also, when using our method, it is evident that for engineering students, for example, the construction of a robot is discussed in more detail, i.e. its technological and engineering aspects are more thoroughly covered than for computer science students. One of the crucial features of STEAM education is that the digital world is added to the analogue object creation. In our method, the same epitrochoid curves are created in two ways: the LEGO robot and a DGS. So, students will experience the joy of creating with LEGO elements, learn various useful building tricks, gain theoretical knowledge of epitrochoid curves, and gain up-to-date digital knowledge through a DGS. We have shown that the Epitrochoid Tracker can be used easily to check the parametric equations of the drawn curve and is also helpful in designing new curves.

In addition to providing an aesthetic experience, epitrochoid curves are also helpful in practical life: they can be used, for example, in surveillance or spatial coverage applications, as periodic motion primitives for human dancers or for performing complex choreographic patterns in small autonomous vehicles [20]. Real-life applications allow us to formulate exciting and meaningful projects related to epitrochoids for first-year engineering and IT students. In our further work, we will collect and systematise concrete practical applications for student projects where the developed method can be used.

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