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On a method for measuring the effectiveness of mathematics teaching using delayed testing in technical contexts in engineering education

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Abstract. In this research, engineering topics with different levels of modeling are included in engineering mathematics courses and the effect of the method is tested through delayed tests in a technical context independent of the mathematics classroom. To address the difficulty of problem solving through modeling, three types (levels) of mathematical problems motivated by engineering tasks are defined. A library of problems was collected and the problems were systematically integrated into the classroom work. The long-term effect of the targeted application of professional problems was investigated by means of delayed tests in the context of statics (mechanics), which the students studied in the following semester. In our approach, the key efficiency factor is the extent to which students can apply the mathematical concepts and methods they have learned while studying professional subjects and later in their engineering work.

Keywords: teaching efficiency, delayed test, engineering mathematics, technical learning environment

1. Introduction

Since our teaching practice focuses on key engineering competencies in general, we consider engineering mathematics as a professional subject rather than a separate course from other modules in the curriculum, and our goal is to create synergy be-

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tween mathematical and professional subjects. The desired synergy can be achieved by linking topics when teaching new mathematical and technical materials, engaging in joint projects. Furthermore, the mathematical knowledge is required to be developed and evaluated in as many professional courses as possible during the training.

In this paper, we address the specific concept of teaching efficiency and the role of mathematics in engineering education. We also examine the relationship between the teaching method used in engineering mathematics courses and the ability to recall mathematical concepts and methods later in engineering courses.

Based on our several decades of experience, interviews, and daily communication with engineering students, two of the most important aspects to consider when discussing the efficiency of a methodology development in engineering mathematics are

- the expectations of engineering students (What motivates them?) and
- the desired role of Engineering Mathematics courses in modern engineering training.

The expectations of engineering students – After studying the attitudes of students in our engineering programs, we concluded that expectations (motivating factors) are changing rapidly. The current generations of engineering students increasingly prefer to learn things that are immediately applicable rather than focusing on studying for the future. Students need to feel that the material is useful to them, and this fact is more important than the difficulty of the material to reach our educational goals. We believe that any method of engineering education that fails to meet these expectations is inefficient. A few decades ago, it was quite natural for our engineering students to study pure mathematics for its beauty, regardless of its applications. In our experience, however, it now motivates only a few percent of them. In this research, the level of motivation was not studied directly, but indirectly by assessing the efficiency with the application of delayed test in a professional context.

The desired role of Engineering Mathematics courses in modern engineering training – We believe that engineering education is one of those areas where being a student is (or should be) an integral part of a professional career. In several ways students must already think as engineers and form professional opinions. Thus, the engineering approach must naturally be present in engineering education, where constant evaluation of efficiency should play a crucial role in the educational process. Our research presented in this paper is based on the idea that each step of education is as successful as it may serve the subsequent steps built upon it. Ultimately, the effectiveness of any engineering education depends on the application of the acquired knowledge in engineering work, at the desired time and under the desired circumstances. Accepting this principle, the efficiency of acquiring mathematics cannot be examined independently of the success in the application of the knowledge in the future.

The circumstances of engineering education have changed in the last decade to such an extent that it is necessary to respond to them by revising and developing didactic tools. The difficulties of fundamental courses also referred to as "barriers" to STEM degrees are discussed e.g. in [16]. The difficulties observed in the engineering mathematics courses replicate other work based on them according to [13]. Methodological issues of mathematics education for non-mathematics students, including engineering students, need to be brought to the forefront. The low level of achievements in mathematics subjects can be partly attributed to inappropriate teaching methods. It seems inevitable to broaden the range of didactic methods and to apply them regularly and in a varied way in engineering education. The learning process in basic courses (especially in mathematics and physics) should be brought closer to that used in secondary education in certain respects, such as interactivity, progress monitoring and regular assessment. In addition, the competencies to be acquired need to be identified more precisely and integrated into the curriculum through specific professional tasks. The way we teach is highly dependent on the level of mathematical knowledge of the incoming students, as discussed in a study by the Dublin Institute of Technology. It shows that the level of mathematics at entry is the strongest predictor of successful completion of the first year for an engineering student [7].

The Hungarian government document entitled "Expected Learning Outcomes", which regulates engineering education in Hungary, places emphasis on defining the competencies to be acquired in engineering education, as well as the method of their assessment and control. Although this provides the framework for the competency-based methodology, a more specific and detailed system of competencies is needed to organize the educational process. The general mathematical competences are extensively discussed in the literature and an overview is provided in [1].

In our research, we focus on the concept of efficiency, in which a specific competence plays a central role: the ability of students to recall and use mathematical concepts and methods when solving professional problems or working as engineers. We emphasize the ability to recall the relevant mathematical topics rather than their advanced application. This competence can be studied primarily in engineering courses that follow the engineering mathematics course. In this paper we introduce a three-level database of mathematical tasks motivated by engineering problems that require different levels of model building. We present the result of a delayed test carried out in Statics to show the results of the systematic inclusion of selected engineering tasks. This test involved the application of various concepts and methods such as vector algebra, linear algebra, and differential and integral calculus.

2. Materials and methods

In order to assess the efficiency of the teaching process specific criteria for success must be established that may vary across different levels and fields. In our investigations in engineering education, we use a special concept of efficiency and a methodology that we employ to enhance efficiency in this context.

2.1. Literature review

Assessment and improvement of the effectiveness of higher education has become a widely discussed research topic from various perspectives. The peculiarities of vocational training of engineering personnel are discussed in [15]. The necessity of active attitude of engineering students and their motivated participation in the educational process is studied in [4]. A study on the introduction of research tasks into mathematics education in a bachelor program of Applied Mathematics and Computer Science is presented in [6]. In the experimental group the methodology of teaching higher mathematics was based on the introduction of research tasks to establish integrative connections, while in the control group, the mathematical disciplines were taught using traditional teaching methods. It was found that it was possible to develop the research potential of students effectively through the consistent organization of the educational process, including a holistic integrative construct in the mathematics curriculum.

Responding to the challenges caused by the rapidly changing expectations and circumstances, several papers have addressed the measurement and improvement of the learning process in engineering education, see e.g. [8, 10]. There is no question that engineering education needs to adapt to the radically changing needs of the engineering profession. There is a large body of research investigating the impact on effectiveness of different teaching methods, such as differentiated teaching, the inclusion of project work, increased student activity, and the integration of practical tasks into class work. These studies provide the theoretical background and motivation for the present research.

The level of professional competence of engineering students is studied in [15]. The research concludes that engineering education should be a system of educational activities that enables students to be professionally prepared for their future work. Therefore, the education should be oriented to the professional requirements, while the professional competence should be in the foreground in order to ensure efficient work performance.

In [18] the need for a more practical mathematical education in engineering is discussed. The MathePraxis project links the mathematical methods taught in the first semesters and practical problems from engineering applications. Within the project, first-year engineering students demonstrate clearly and convincingly where they will need mathematics in their later working life. In [2], the practice of spaced retrieval was investigated in nine introductory Science, Technology, Engineering, and Mathematics (STEM) courses. This practice involves repeatedly revising the same topics over time with intermittent delays.

Since success in mathematics is highly dependent on the initial level of knowledge and may be described in terms of the change in thinking and application skills, the question of efficiency may not be discussed without examining the mathematical knowledge of incoming students and how we can improve it through catch-up courses. Engineering relies heavily on mathematics, and a lack of basic math skills significantly hampers students' success. It has been observed that students who lack basic mathematical skills are more likely to perform poorly not only in mathematical modules but also in engineering modules such as thermodynamics, mechanics, and dynamics [12]. Their approach to improve educational efficiency involves assessing the need of individual support through online surveys and using the expertise of talented students to mentor their peers.

According to a UK study, a lack of adequate math skills not only affects students' performance in courses but also leads to higher dropout rates in the first two years of study. Many universities offer math support systems to address these issues, but the success of these programs varies. The research conducted by Gallimore and Stewart (see [9]) presents a novel approach to mathematics support developed and implemented at the School of Engineering, University of Lincoln. This approach provides students with a transition to bridge the gap between secondary school and university level mathematics, offers ongoing support through learning assessment and individual learning plans, and ultimately improves students' achievements, engagement, and retention.

In a 2001 study (see [11]), a total of 95 UK universities were surveyed about the provision of mathematics support, and 46 reported that they provided support for their students. An update in 2004 found that 35 out of 106 UK universities still did not provide mathematics support (see [14]). However, a study [3] published in 2012 found that 88 out of 133 institutions had implemented mathematics learning support programs. A teaching model aimed to improve the quality of mathematics education is introduced and experimentally tested in [20].

2.2. The efficiency concept

In our practice, we measure the effectiveness of an educational activity by the extent to which knowledge is available when it needs to be used in professional subjects or in engineering work. In contrast, the usual assessments (tests, exams, class work) measure the success of learning mathematics only from a mathematical perspective; they say little about how successful engineering students are in applying their knowledge in a non-mathematical environment.

One of the most important tools for process control and improvement in engineering is feedback [5]. Although surveys and student evaluations are regularly conducted in higher education to obtain feedback without a precise formulation of the method and purpose of feedback. These are only formal activities and can provide rather general statements. In order to regulate the educational process, a deeper analysis of knowledge is required. A typical bad example of assessing conformity to expectations is when students are asked how useful they find the mathematical topic they are currently studying. This makes sense if the students has already been studying a subject based on acquired knowledge.

Mathematics is a subject that shapes one's perspective and increases one's professional intelligence, and it is also a preparation for learning engineering subjects. Assessments within subjects that focus on learnable and algorithmic knowledge do not show anything about the real usefulness and the ability of students to apply the knowledge in a long-term and creative way. A project at the Norwegian University of Science and Technology is presented in [17]. The aim of the project was for students to develop a deep understanding of mathematical concepts and processes, making them better equipped to use mathematics in applications. Digital technology was applied to free up teachers' resources and improve contact with students.

Our goal in teaching engineering mathematics is to help students recall the necessary knowledge in the context of engineering subjects. For this purpose, we use several tools and regularly check how successful our students are in professional applications of mathematics. In this study, we present a special approach to test the level of applicable mathematical knowledge in subsequent semesters in a professional context. In addition, we study the effect of integrating professional content into engineering mathematics courses on the results of delayed mathematics tests.

Our hypothesis was that the way we integrated engineering problems into the classroom as an element of our toolset would result students' better recognition of the necessary mathematical tools and better remembering the computational methods when they solve professional tasks, thus improving efficiency as we define it.

2.3. The framework of teaching Engineering Mathematics and the categories of engineering mathematics tasks

Due to the continuous improvement of the mathematics teaching methodology, a new didactic environment has been established at the Faculty of Engineering University of Debrecen in recent years. This environment includes the presentation of theoretical knowledge through methods such as the use of blackboards and data projectors accompanied by visual aids such as numerous figures and animations. In addition, examples of applications in science and engineering are shown, and related mathematical problems are solved interactively by the instructors or with the participation of students in practical classes. However, our experience in the classroom has shown that this commonly accepted and used method of covering the mathematics curriculum is no longer motivating for most engineering students. As a result, the absorption of new knowledge is not successful enough.

Furthermore, based on the entrance tests of first-year students and the experience of catch-up courses, it should be assumed when planning mathematics courses that students have incomplete knowledge of basic concepts, relationships, and computational methods. They have difficulties in recognizing the relationships between different mathematical topics and lack experience in the process of solving problems by creating and evaluating of models. Therefore, it is necessary to use a teaching method that can simultaneously convey new knowledge, applying it in an illustrative form, and can provide creative and motivating activities that prepare students for the application of mathematical knowledge in technical problem solving.

In our method we integrate engineering problems that require different levels of modeling, which is done in conjunction with simultaneous discussion of analytical and numerical methods, assigning team project tasks in mathematics and professional courses, and project-based learning. Based on our experience, these tools facilitate a deeper understanding and long-term retention of mathematical knowledge, as well as the development of the ability to apply it in learning technical subjects based on mathematics.

Our observations show that an intensive discussion of engineering applications related to each mathematical topic simultaneously with classical mathematical tasks is uncommon for most Hungarian engineering students, as several of them have not encountered mathematical modeling before.

Although there are tasks in secondary schools that would be suitable for introducing the steps of modeling, students are often unaware of them. The authors regularly offer "mini-courses" for high school students and have the opportunity to study the students' competencies and attitudes [19]. We found that high school students generally prefer solving application-oriented problems to purely mathematical ones. But the success rate is still higher when solving purely mathematical problems. University students also express a desire to solve application-oriented tasks, but their modeling skills are quite low and need to be developed.

As part of our investigation, a task database was prepared, in which the tasks were divided into three categories:

- purely mathematical questions motivated by technical applications;
- technical questions with the model provided and only mathematical knowledge is needed for the solution;
- technical tasks formulated in a professional context requiring model creation and higher-level, complex mathematical knowledge.

For most engineering students, it is difficult to identify the appropriate mathematical concept or method related to the professional problem they need to solve. Similar to our observations in high schools, although more university students prefer solving real-world problems to purely mathematical problems, they are less successful in the former one. We believe that this phenomenon is due to the lack of experience with mathematical modeling in secondary education.

In order to prepare students to use mathematics as a tool, we must create a synergy between mathematical and professional subjects, emphasizing as many points of connection as possible. The gradual introduction of practice from the beginning of the study program is essential to develop the ability to use mathematical tools.

Our hypothesis was that regular discussion of professional problems from our three-level database in engineering mathematics classes would result students' better recognizing the mathematical tools needed and remembering the computational methods when they have to solve professional problems.

It is obvious that some students can recall the examples they studied even several semesters later when asked in the same context. Therefore, the delayed tests formally included questions on the professional topic that were different from both standard mathematical texts and engineering problems discussed in engineering mathematics classes. In the tests, they had to solve simple exercises related to the professional topic with emphasis on the mathematical content to assess the current mathematical skills.

In the experimental group, during the discussion of each mathematical topic, students were introduced to mathematical concepts in the usual way and solved typical mathematical examples in 75% of the time. After that, every week they were asked to solve technical problems from all categories using the mathematical tools they had just learned.

The goal is to simultaneously develop mathematical and professional intelligence by improving the ability to build models and demonstrate connections to professional topics. As a result, students can acquire deeper knowledge, and find answers to questions such as "What's the point of all this?" and "Why do I need to study mathematics?"

Below are examples of all three categories of tasks in the database.

Category 1: Purely mathematical questions motivated by technical applications

In the first part of the task collection, there are exercises that are purely mathematical problems. In some cases, they are formulated as technical questions, and in all cases, they touch on technical applications during the solution.

Example 1.1. A precise approximation of the curve of a corner of a Formula 1 racetrack is described by the graph $f(x) = x^2 + 2x$. If the car moving on the track drifts off at $x = 1$ along the tangent of the track, does it hit the column at the coordinate point $P = (2; 5)$?

Example 1.2. Engineers are planning a straight tunnel with an inverted parabolic cross-section under a mountain. The tunnel is 9 meters high and 6 meters wide at the bottom. What is the largest rectangular cross-section (width and height) of the truck that can still drive through the tunnel?

Example 1.3. The widths of two orthogonally intersecting corridors are 2.4 meters and 1.6 meters, respectively. How long is the ladder that can be taken from one corridor to another?

Category 2: technical questions for which the model is provided and only mathematical knowledge is needed for the solution

In the second category, we classified tasks that are technical or physical in nature but require mathematical knowledge to solve. We believe that it is important to provide students with practical tasks in the mathematics course that include technical examples beyond traditional mathematics education.

Example 2.1. An elevator whose motor is on the top floor is held up by a wire rope. We also know that a 1-meter piece of wire rope weighs 45 [N]. When the cabin is on the ground floor, 60 [m] of cable hangs down. By the time the elevator reaches the top floor, the cable is fully rolled up. How much work is required just to pull up the cable?

Example 2.2. The length of a spring in the unstretched state is 20 [cm]. To stretch it to a length of 30 [cm] requires a force of 40 [N]. How much work is required to stretch the spring from 35 [cm] to 38 [cm]?

Example 2.3. A lawnmower manual says to tighten the spark plug by a torque of 20.4 [Nm]. If the force is applied to the spark plug wrench from a distance of 25 [cm] from the spark plug, how much force is required to achieve the required torque?

Category 3: Technical tasks formulated in a professional context, requiring model creation and higher-level, complex mathematical knowledge

In the third category we have classified tasks that are no longer purely mathematical, but technical tasks that appear in other subjects. These tasks require the use of higher-level mathematical tools for their solution. Our goal was for students to develop a comprehensive understanding of the mathematical knowledge that appears in other subjects during their studies. We wanted them to be able to apply the methods they had learned in their mathematics courses, rather than just focusing on the process of solving problems.

Example 3.1. Regarding the DC circuit given in Figure 1 solve the problem listed below. Data: $U_{b_1} = 20q$ [V], $U_{b_2} = 10$ [V], $U_{b_3} = 5$ [V], $R_1 = 2$ [Ω], $R_2 = 4$ [Ω], $R_{b_1} = 7$ [Ω], $R_{b_2} = 6$ [Ω], $R_{b_3} = 4$ [Ω]. Apply Kirchhoff's first rule for node B. Apply Kirchhoff's second rule for loops $A-B-E-F-A$ and $B-C-D-E-B$. Give the matrix of the obtained system of linear equations. Calculate the unknown current intensities.

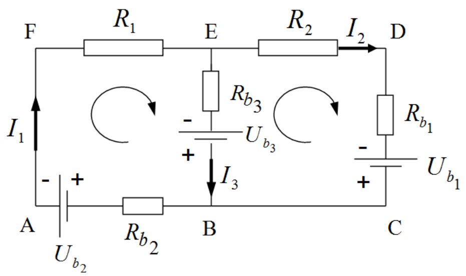

Figure 1. Electric circuit referred to in Example 3.1.

Example 3.2. The stress tensor elements at point P of a structure are demonstrated in the elementary cube in Figure 2. Data: $\sigma_x = 50$ [MPa], $\sigma_y = -30$ [MPa], $\sigma_z = 25$ [MPa], $\tau_{xy} = \tau_{yx} = 30$ [MPa], $\alpha = 30°$. Give the coordinates of unit normal vector $\bar{n}$ if it is in the $x-z$ plane and its angle with the x axis is α. Give the matrix of the stress tensor at point P. Calculate the stress vector $\bar{\rho}_n$, the normal stress σ_n and the shear stress. Determine the magnitude and the direction of the principal stresses.

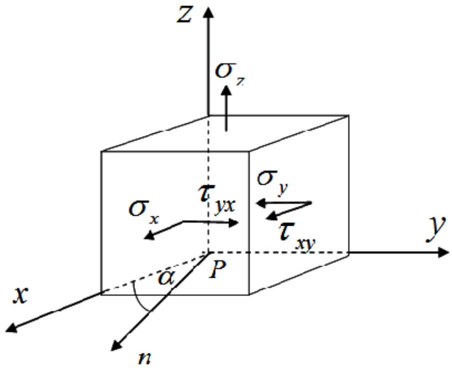

Figure 2. Stress state of a body referred to in Example 3.2.

Example 3.3. Suppose that there are three rotating parts in a machine generating harmonic vibration of the machine structure. The rotational speed values of the three parts are 600 rpm, 720 rpm and 1100 rpm, respectively. The effective velocity values of the three harmonic vibrations are 5.4 mm/s, 3.9 mm/s, 6.0 mm/s. Give the vibration state of the machine in the time domain and in the frequency domain with the velocity-time and the velocity-frequency diagrams.

2.4. Delayed tests

In this research, the delayed test consisted of mathematical questions based on the material covered in Engineering Mathematics I, but formulated as technical problems using the terminology of Statics. The students were not informed about the nature of the questions either before or during the test; therefore, they had to interpret the situations themselves. Although minimal knowledge of the subject Statics was required to provide answers, the presence of this knowledge was a prerequisite for passing the course. It was therefore safe to assume that the students had this knowledge. Once the questions were interpreted, solving them required only the use of purely mathematical tools. The test questions were as follows.

Question 1 (Q1)**.** Force F acts on a material point which is on the surface of the incline in Figure 3. The angle between the horizontal plane and the incline is 25°. The coordinates of force F in the blue coordinate system are $x = 2$ and $y = 5$.

Give the coordinates of F in the red coordinate system. Consider that the red coordinate system can be obtained by the rotation of the blue one.

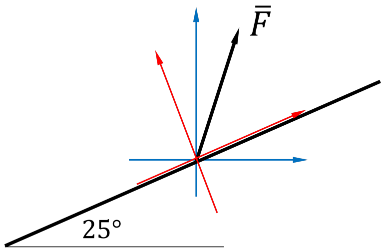

Figure 3. Force system referred to in Question 1.

Question 2 (Q2)**.** A distributed force system given by the intensity $f(x) = x\sin\left(\frac{\pi}{2}x - 2\pi\right)\left[\frac{\mathrm{N}}{\mathrm{m}}\right]$, $4 \leq x \leq 6$ acts on a 2 meters long segment of the supported beam in Figure 4. Calculate the resultant of a distributed force system.

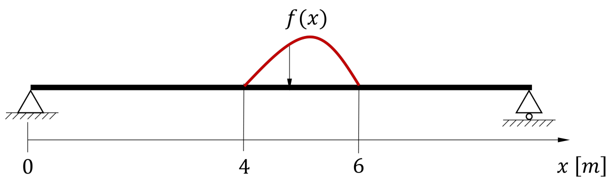

Figure 4. Beam loaded by a distributed force system referred to in Question 2.

Question 3 (Q3)**.** The bending moment acting on a prismatic beam is given as a function of coordinate x as $M_b(x) = -x\sqrt{100 - 4x^2}$ [Nm], $0 \leq x \leq 5$. Calculate the value of the shear force at $x = 3$.

Question 4 (Q4)**.** Calculate the moment vectors of forces $\bar{F}_1$ and $\bar{F}_2$ (Figure 5) relative to point O, and calculate the angle between the two-moment vectors. Data:

$$\bar{F}_1 = \begin{pmatrix} -2 \\ 5 \\ 1 \end{pmatrix}, \quad \bar{F}_2 = \begin{pmatrix} 2 \\ 8 \\ -4 \end{pmatrix}, \quad \bar{r}_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad \bar{r}_2 = \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}.$$

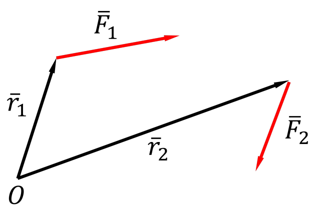

Figure 5. Forces referred in Question 4.

The mathematical knowledge needed to answer the questions:

Q1: linear transforms of the plane; the required coordinates can be obtained by rotating the force vector $-25°$, to get it we need the matrix of the rotation.

Q2: calculation of integrals; the resultant of a distributed force system with force density f can be calculated as $F_{\text{res}} = \int_a^b f$.

Q3: differentiation; the shear force function can be given as the negative derivative of the bending moment function.

Q4: vector operations; moment vectors can be given as vector product of force vectors and position vectors; the angle can be calculated with scalar production.

2.5. The experiment

80 students majoring in vehicle and mechanical engineering participated in the study: 40 students in the experimental group and 40 students in the control group. In these majors, the course Engineering Mathematics I consists of 4 hours of lecture and 4 hours of practical. The inclusion of technical examples of different levels serves to increase the efficiency according to our concept.

At the Faculty, the level of knowledge of the incoming students is checked every year with an entrance test consisting of high school exercises. The students in the two groups achieved almost identical results in this entrance test. The two-sample t-test indicated that there was no significant difference between the scores of the Experimental group ($M = 44.95$, $SD = 24.96$) and the Control group ($M = 49.18$, $SD = 25.70$), $t(78) = 0.75$, $p = 0.458$ (two-tail, $d = 0.17$).

Both the experimental group and the control group studied Engineering Mathematics I according to the same curriculum and for the same number of hours. However, the students in the experimental group spent 1 hour of the 4-hour practical class each week studying models and solving engineering problems, while the students in the control group only solved classical mathematical problems.

The subject Engineering Mathematics I covers the following topics of linear algebra and mathematical analysis: matrix algebra, linear spaces, linear functions; real functions, properties, elementary functions, composition, and inverse of functions; continuity, limit, derivative, linear approximation; Taylor polynomials, analysis of differentiable functions; Riemann integral; anti-derivative; Newton-Leibniz formula; numerical integration; applications of integral calculus.

Our two hypotheses in this research were as follows:

H1: Incorporating engineering problems of different categories into classroom work helps students understand the course material resulting better performance in Engineering Mathematics I for students in the experimental group.

H2: The students in the experimental group, for whom engineering problems are systematically integrated into the Engineering Mathematics I course according to our methodology, perform better on the delayed mathematics tests in the Statics course than the students in the control group.

3. Results

Although our main goal was to examine our teaching methodology in terms of our efficiency concept, we were also interested in its effect on the test scores of the math course.

The results of the two tests written in the "standard" mathematical context indicate that there is no significant difference between the two groups based on the total scores obtained. The two-sample t-test indicated that there was no significant difference between the scores of the Experimental group ($M = 41.85$, $SD = 23.00$) and the Control group ($M = 47.03$, $SD = 23.70$), $t(78) = 0.995$, $p = 0.325$ (two-tail, $d = 0.22$). Thus hypothesis H1 was rejected.

It should be noted that this is not particularly surprising, as we have observed that the mathematics test scores are mostly correlated with the amount of time spent practising computational steps, rather than with the specific knowledge required to study engineering subjects.

The post-measurement was conducted in the frame of the Statics subject, which is based on Engineering Mathematics I and takes place one semester later. The experimental and control groups studied Statics under identical conditions. The post-measurement was conducted with the first test of Statics subject and it was called "extra test" (hiding the research purpose) for extra points. Students were allowed to earn 10% of the total points with this part.

Regarding the second hypothesis (H2), it should be emphasized that both groups received the same mathematical knowledge in the first semester and the same professional knowledge in Statics in the second semester. Both groups had to answer the same questions in the delayed test. The questions were not covered in Engineering Mathematics I for either the experimental group or the control group, thus preventing the students from recalling the answers.

The scores were compared in the two groups; the result of the two sample t-test confirmed our second hypothesis. The students who studied mathematics in a way that regularly involved solving technical problems of different modeling levels during a part of the lessons (Experimental group) ($M = 54.15$, $SD = 24.17$) achieved significantly better results in the subsequent assessment of their mathematical knowledge in the Statics subject than students in he Control group ($M = 37.03$, $SD = 21.24$), $t(78) = 3.36$, $p = 0.001$ (two-tail, $d = 0.75$).

4. Conclusions

In our study, we formulated our definition of the efficiency of teaching engineering mathematics and presented our teaching method aimed to improve efficiency in this sense. Among the three main elements of our methodology, we analyzed the effect of the targeted application of different categories of engineering tasks using a delayed test conducted in the context of a professional subject to be studied in the following semester.

In cooperation with the lecturer of the engineering course we prepared a special delayed test with new types of questions for this study. The test focused on mathematical knowledge but the questions were presented as technical texts. The test questions differed from the exercises discussed in the mathematics classes of both the experimental and control groups, as well as from the questions in the regular mathematics and statics tests. Some of the delayed test questions asked in the Statics course are presented in Subsection 2.4.

To implement the method of "Integration of engineering problems into classwork" we created a collection of tasks consisting of three groups: purely mathematical questions motivated by technical applications, professional questions with given models requiring only mathematical knowledge for their solution, and engineering tasks presented as professional texts requiring model building and higher-level, complex mathematical knowledge. While various collections of engineering problems for discussion in mathematics classes are mentioned in the literature, we also categorized the problems according to the level of modeling required and prepared a unique collection of tasks organized by topic and modeling difficulties.

In the experimental group, we specifically involved professional tasks, dedicating 1 hour of each 4-hour practical class to them. We compared the test results of the two groups within the Engineering mathematics course (normal test) and the Statics course (special post-test). Our results showed that, although there was no significant difference between the two groups in terms of the regular tests of the Engineering Mathematics I course, the experimental group performed significantly better in the mathematics survey of the Statics course that took place one semester later.

Based on our results, to improve the effectiveness of engineering mathematics education, we recommend to conduct mathematics post-tests in the context of professional. If the effectiveness of educational activity is measured by the extent to which knowledge is available for practical use, our study suggests that dedicating a portion of class time to posing and solving professional problems with a focus on modeling significantly increases the ability to recognize the necessary mathematical tools and the effectiveness of knowledge retrieval in the professional environment.

For a more in-depth analysis of the impact of our methodology on the effectiveness of teaching mathematics we are preparing post-tests for further engineering courses and we plan to request more detailed derivations and explanations to allow for a qualitative analysis of mathematical knowledge one or more semesters after learning the subject. Although the result of the t-test and our subjective analysis confirmed our second hypothesis, larger groups would be involved in further research to increase the reliability of our findings.

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