EFFECTS OF CROWDING ON THE OPTIMAL SUBSIDY OF PUBLIC TRANSPORT

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ABSTRACT

Subsidizing public transport companies is a worldwide used policy due to the scale economies and low specific travel costs of these kind of transport services. Frequent use of subsidies, however, does not mean uniformity, as there are many methods of subsidizing transport companies. Recent studies has shown that the amount of subsidy applied varies greatly depending on the attributes of the city, service provider and transport mode. The purpose of the present study is to reconsider a model that optimizes fares, and modify the model's computation method by using new parameters that simplifies the management of crowding costs. The study presents the theoretical background of the topic, the important modifications made in the model, and also demonstrates usability of the model by examining the pricing of the Hungarian interurban transport system. The results recalculated with the modified model showed that crowding cost is a critical component of optimal pricing, as even at a moderate congestion level travel costs are increased by 30–50%. This means, that the optimal fares aimed to reduce the costs of crowding are higher than they were previously set. In case of Hungarian interurban transport, a subsidy rate of around 90% should be applied, with the exception of peak period bus transport, for which the calculated subsidy rate is 74%.

Keywords:
aggregate analysis, pricing policy, vehicle-size optimization, transit fares optimization, public transport regulations

1. INTRODUCTION

The extreme increase in the number, size and population of cities and megalopolises raises serious questions about their livelihood and sustainability. These questions can only be answered through prudent and watchful urban planning, by reaching the sufficient level of human flexibility (e.g. in housing, mobility), and improving the quality and efficiency of public and social services. The transport system can be able to compensate the shortcomings of urban design by providing high quality mobility services.

The smooth operation of the transport system is a cornerstone for the well-functioning cities and megalopolises. Since the recognition of this idea, greater attention has been focused on researching optimal planning and operational strategies for tasks of transport planning and management. Based on the commonly used microeconomical approach, travel pricing is a well-established and frequently used method to influence traveller decisions.

Our study examines the optimal subsidy of public transport operators. Although the daily operation of these companies are often supported by public funds, the accurate definition of subsidy—a more precise and easily usable formulae of the usual rate—is still missing. The appropriate rate of subsidies and the suitable financing method of public transport is important, because of these services' attractiveness and reliability—since both feature is strongly dependent on financial resources, and they are some of the key aspects of the transportation process. (The substantially unprofitable operation at a social level can also bring optimal welfare conditions, therefore subsidies are essential to the operation of transportation companies.) However, based on the results of recent studies, significant (proportional) differences can be observed between subsidies granted to companies providing services in similar circumstances (see for example the results of Boss and Rosenschon 2008; Doll and van Essen 2008; Parry and Small 2009; Tscharaktschiew and Hirte 2011). Although it is hard to draw far-reaching conclusions because of the variance between the compared systems' and countries' transport policy, economic situation, and the limits of subsidization, the topic of subsidies given to transportation operators functioning in similar circumstances is still worth examining.

The basis of our research was the computational model presented by Parry and Small (2009), which investigates the (ideal) pricing of travel. The model can be used to determine the subsidy rate and the expected operational-economic characteristics belonging to it under optimal conditions. The aim of our study is to extend this model with parameters describing crowding in public transport, and also the application of the model for a new context, the Hungarian interurban public transport system. The mode and purpose of adding new parameters to the model is to make the model able to handle both new and already-built-in parameters in the same environment, and thus be able to account for more detailed and accurate results of the effects of crowding. (Nowadays crowding is studied from many aspects from engineering and economics to psychology and ergonomics).

The paper is structured as follows: first the paper deals with the optimal pricing of public transport—the most influential factors of the calculation, and the basics of the model. The following sections introduce the needs that call for the modifications of the model, the use of crowding parameters and the further use of optimized parameters. Then the paper presents the basic data for the calculation of interurban traffic in Hungary and also the application of the model for the examined region. Finally we summarize all the results and conclusions that can be drawn from the results of the model and we suggest some further improvements.

2. OPTIMAL PRICING OF PUBLIC TRANSPORT

The benefits of public transport and the positive effects (e.g. lower local emissions, moderate fuel consumption and land use) are most dominant in networks with significant traffic and near bottlenecks. Even when utilizing the benefits of public transport, it is advisable to strive for an optimal operational situation and strategy, where the benefits of the daily use are not outweighed by the inherent negative phenomenon. All systems should avoid the state, when:

- the presence of public transport vehicles cause a major congestion in the road traffic of the city (or even cause significant delays of the public transport service itself),
- the level of crowding on public transport vehicles grow so high, that it causes a major decrease of utility, or when
- the contamination of vehicles and the decrease of passenger safety cannot be avoided (see the results of Perone 2002 on passenger preferences and mode choices).

2.1. Scale economies of scheduled transport services

In the case of public transport systems operating on a frequent basis and providing scheduled services, the phenomenon of scale economies prevails (Mohring 1972, 1976; Turvey and Mohring 1975). The additional operation costs of passenger services at higher passenger numbers, under idealized conditions, are balanced by the benefits of passenger time savings through the reduction of the average waiting time, i.e. the net marginal social cost of travel is lower than the marginal cost of operation. In other words, the marginal cost of using a service that operates under scale economies is always lower than the average cost, and therefore the optimal price, which is equal to the marginal cost, leads to a loss that must be compensated with subsidy.

Regarding the optimization of the social costs, the sum of operating and user costs, by the means of service frequency, we obtain simple relation that the optimal frequency—number of vehicles departing per unit of time—increases proportionally with the square root of the number of passengers, assuming a proportional relationship of vehicle amount and operating costs, and that passengers arrive to stops at random (Mohring 1972).

In addition, in public transport, due to fixed operating costs and waiting times, scale economies can also be observed in operating and user costs. Since both lead to subsidies, Mohring's model requires that the service should be subsidized even if the economies of scale in operation do not materialize, as the scale economy still applies to user costs.

2.2. Marginal cost pricing

Where user cost is a non-negligible part of social cost, as in the case of scheduled public transport services, the marginal cost based pricing should include two types of costs at the optimum, the service provider's and the passengers' costs (Jansson 1979; Vickrey 1980). This duality can be observed both in the amount of vehicles on the road, proportional with user costs and the size of the service area, and the size or capacity of the vehicles, proportional with the costs of the service provider.

The uniqueness of the transport market is that the consumer (passenger) is also involved in the process of producing the product, since the locomotion only can be performed by the passenger's time spent. It is precisely because of passenger involvement that the right approach is to escape the notion that the only costs which are relevant to optimization are those of the transport operator. The time-costs of the passengers must be included too, and fares must be equated with marginal social costs. (Turvey and Mohring 1975) The significant change brought by the proliferation of information and communication technologies (ICTs) also has to be highlighted, as it has made multitasking a realistic option during travel (though learning, working, various communication forms etc., see Keserű et al. 2015; Keserű and Macharis 2018; Munkácsy, Keserű, and Siska n.d.). This change made travel time almost as valuable as other activities—in an appropriate travel environment—the value of passengers' travel time savings could drop significantly (International Transport Forum 2018).

2.3. Overview of the model of Parry and Small

One of the significant works of the recent years on the field of transport economics, subsidization and pricing policy was the paper of Parry and Small (2009). The paper highlights the importance of the question whether the subsidy of public transport companies needs to be reduced (or even abolished), or increasing subsidy level brings the system closer to the optimal operating conditions.

The authors stated that despite the differences between the subsidy rates of transport companies, there is no generally accepted, practicable calculation method that can be used to determine the ideal financial strategy for a given transport operator, i.e. the rate of subsidy. In most cases, only complex, and therefore location-specific simulation models are able to provide data on traffic flow. Since pricing policies are often based on these results of modelling it is difficult to determine whether public transport pricing fits well with a city or region's transport system.

The aim of the authors was to create an analytical model that is simple enough to use without highly specific data, i.e. it requires data that are typically available at all transport providers, statistical officest etc., and at the same time the model can manage the most important parameters describing the transport system. It was also targeted that the calculation method could be applicable to transport systems of significantly different size, structure, operation method, and vehicle occupancy, as well as to cities and countries with different levels of development.

2.3.1. Model structure

The model was built using aggregated data that incorporate peak and off-peak results of all transport modes. The advantage of this model structure is that it can replace network models significant in size and computational supplies, their need for detailed calibration, the necessarily integrated decision methods of traffic models etc.. It is important to note that aggregate data are sufficient for this level of analysis, and it is neither necessary nor advisable to subdivide the data, as in many cases aggregated results can be obtained from local transport companies, road transport organizations, statistical offices, etc., or censuses, surveys have already done these researches—which also simplifies the collection of data and thus the investigation and modelling, too.

One of the model's most important methodological feature is that all parameters are derived to vehicle-miles and the calculations are defined by these kind of parameters, too. The key components of the model can be calculated using the following parameters:

- user benefits (consumption of numeraire good and the utility of travelling);
- vehicle occupancy, service frequency, crowding and travel time;
- external pollution and accident costs;
- household budget constraint, the balance of income and expenditure;
- ways in which companies adapt the constraints of the service.

The model calculates values of costs and benefits and then aggregates these results using passenger-miles "travelled" in the system—passenger-miles per travel mode and time periods. Similarly, in-vehicle travel time (T), waiting time (W), access time (A), and crowding measure (C) for the extra cost of vehicle-crowding can be calculated as the product of specific values multiplied by the number of passenger-miles travelled during the time period under investigation, calculated for each transport modes. A combination of these factors can be used to produce the generalized (non-money) cost of travel, which is proportional to the time spent traveling: $\Gamma = \Gamma(T, W, A, C)$. The Γ function establishes the relationship between the costs received in time dimension.

The way to determine utility is to sum all the components in cost dimensions, which can help balancing the benefits and losses. According to the model, the user preferences and the benefits of their activities (U) can be modeled as the difference between the value of the utility function—based on the consumption of numeraire good (X), the sub utility from passenger miles travelled (M), and the generalized (non-money) cost of travel—and factor Z, which takes the magnitude of pollution externalities and accidents costs into account. In addition to the utility function, the balance of the representative household must be determined. This approach assumes that all households spend the tax-deductible portion of their income on traveling and consuming numeraire good.

The indirect utility function of households is calculated as the value of the maximized utility function within the budget constraints, see the first line of Equation (1), but an indirect utility function of a user (passenger) can also be used to measure social benefits by replacing the transit agency's budget constraint. Then, the amount of tax (TAX) expressed as a function of operating costs and ticket revenue can be inserted in the second expression of Equation (1). In which the sum of operating costs (OC) can also be divided into fixed and variable (service-dependent and independent) costs.

$$\tilde{U} = \tilde{u}(\{p^{ij}, t^{ij}, w^{ij}, a^{ij}, c^{ij}\}, TAX) - Z = \max_{X,M,\Gamma} \left\{ \begin{matrix} u[X, M, \Gamma(T, \mathrm{W}, \mathrm{A}, \mathrm{C})] + \\ +\lambda \cdot \left(I - TAX - X - \sum_{ij} p^{ij} M^{ij}\right) \end{matrix} \right\} = \max_{X,M,\Gamma} \left\{ \begin{matrix} u[X, M, \Gamma(T, \mathrm{W}, \mathrm{A}, \mathrm{C})] + \\ +\lambda \cdot \left(I + \sum_i \tau^{iH} M^{iH} - \sum_{ij \neq iH} OC^{ij} - X - \sum_i p^{iH} M^{iH}\right) \end{matrix} \right\} \quad (1)$$

As it can be seen from the presented model structure, one can analyse the journeys by transport mode and travel time period. This makes easier to examine the journeys' components in detail, but because of the aggregate level an optimal strategy can also be determined based upon the same data.

2.3.2. *Formulas derived from the model*

The net marginal social cost of each trip is obtained by totally differentiating the indirect utility function of Equation (1) with $-p$. The extreme is where marginal reduction or increase of fares no longer increases or decreases social welfare. For example, one can determine the extremes of the indirect utility function for peak-period rail travels by differentiating Equation (1) with $-p^{PR}$. The result is Equation (2), which shows how the value of utility changes for each component as a result of a single reduction in the travel fees of peak-period rail transport.

$$MW^{PR} \equiv -\left(MC_{supply}^{PR} - p^{PR}\right)\left(-M_{PR}^{PR}\right) + +\left(MB_{scale}^{PR} - MC_{occ}^{PR}\right)\left(-M_{PR}^{PR}\right) + \sum_{ij=PR,iH}\left(MC_{ext}^{ij} M_{PR}^{ij}\right) + + \sum_{ij=OR,PB,OB}\left(MC_{supply}^{ij} + MC_{ext}^{ij} + MC_{occ}^{ij} - MB_{scale}^{ij} - p^{ij}\right) M_{PR}^{ij} \quad (2)$$

In Equation (2) the first component (MC_{supply}^{PR}) reduces social welfare, since the greater the difference between the cost of transporting a passenger and the price of a ticket, greater losses the transport agency accumulates. The effect of the transport system's scale economies impacts through the difference between the marginal benefit of scale economies (MB_{scale}^{PR}) and the marginal cost of congestion (MC_{occ}^{ij}). When the Mohring effect applies, a marginal reduction of fares has a positive outcome, since the operator increases the service frequency, thereby reduces the waiting time of all the other passengers. In contrast, crowding has an opposite effect. With the marginal reduction of fares the number of passengers will increase, and in parallel the increase of crowding will cause losses of well-being and social benefits. The remaining components of Equation (2) expresses the benefits of changing externalities (MC_{ext}^{ij}) due to the expected reduction in the number of cars, and the impact of lower peak-period rail travel fees on bus and off-peak-period rail travel characteristics (via scale economies, appearance of new passenger, crowding on vehicles etc.).

2.3.3. *Results of previous work*

Parry and Small examined the transport systems of three cities having drastically different transportation systems (London, Washington DC, and Los Angeles). The subsidy rates for each city were arbitrarily modified by time periods and transport modes, because a one time intervention in the model would have caused unrealistic change in service levels, as well as users' expected decisions and mode choices.

The paper's results show that in most cases significant rates of subsidies are recommended for transport agencies. In 11 out of 12 cases, the optimal value of subsidy is more than two-thirds of the operating cost, and in more than half of the cases it reaches 90%, while in one case the model suggested a minor reduction. The model proves that new passengers appearing would have a negative impact on utility due to the difference between average and marginal cost. The results of the model also show that "revenues" are primarily come from scale economies and decreasing externalities.

The model predicts a major increase in passenger miles, more than 50% increase in off-peak periods. This result is perfectly rational, since it is particularly practical to facilitate the best use of capital (i.e. fleet of vehicles, stations and infrastructure) in less frequented periods. With significantly lower off-peak period transit fares and due to greater spatial and temporal coverage, on account of the Mohring effect, more passengers will choose public transport.

Based on the examples of the examined cities, it can be concluded that in such cases public transport should be given a decisive role, and the use of public transport should be significantly supported. However, it is worth emphasizing that this is a secondary optimum. Not only in terms of pricing, as this model ignores the expected impact of other actions taken (e.g. infrastructure or vehicle development). The primary optimum could be achieved through appropriate pricing of car use, but this kind of pricing is deliberately omitted by the study.

3. EXAMINATION OF THE CROWDING ELASTICITY

Determining the values of the parameters built in the model, Parry and Small (2009) made a suspicious finding that the proportion of congestion losses are "negligible" compared to other factors. However, studies over the last 10 years have shown that perceived travel costs can rise by around 50%, or even more according to some measures at a crowding level of 3 passengers/m^2. In addition, the original model used a nonlinear relationship between crowding and value of time, which is an inappropriate simplification based on the results of the relevant literature. (Björklund and Swärdh 2015; Hörcher, Graham, and Anderson 2017; Kroes et al. 2014; Tirachini et al. 2016; Whelan and Crockett 2009)

It is difficult to determine the actual characteristics of crowding, therefore our approach trace it back to the travel time. When supplemented by a multiplier term the effect of crowding can be calculated with the formula $c^{ij} = t^{ij}m(l^{ij})$. Since the relationship between vehicle occupancy and crowding factor can already be approximated by a linear function (see the results of the above-mentioned literature), a multiplier function of the form $m(l) = \alpha_c \cdot l$ has been used, where l expresses the degree of vehicle occupancy. We assume that the level of crowding does not affect travel time (i.e. neglecting the relationship between the crowding level and the aligning time). Thus the elasticity of α_c depends solely on the shape of the multiplier function, which in this case, due to the linear relationship $\eta_c = (\partial m(l)/\partial l) \cdot (l/m(l)) = 1$ (see the results of the relevant measures (Björklund and Swärdh 2015; Hörcher et al. 2017; Kroes et al. 2014; Tirachini et al. 2016; Whelan and Crockett 2009).

The model of Parry and Small (2009) takes only the most important parameters of the transport system into account, but the model is still simpler, and uses less data than the detailed traffic demand models. The analytical relationships of the model counts for different modes of traffic (rail, bus, and private car), and uses a simple method of welfare optimization while maintaining economic equilibrium. As well as the optimum depends on the costs of travel, it relies on the consumption of the numeraire good, the passenger miles travelled in the whole system, and the model also calculates the effects of pollution, accidents and other externalities.

The model traces back cost variables (waiting, access, crowding) to travel characteristics, such as waiting time to service frequency, accessibility to route density, and it gives the crowding costs as a function of load factor. The model takes travel time patterns into account specialized for every mode of transport.

By the time of Parry and Small's research there were no available measurement with valid results to quantify the access costs and the effects of crowding on vehicles, therefore the paper replaced this indicators with some simplifications. The applied assumption was that the service provider (travel agency) adjusts the vehicle route density (D) and service frequency (f^{ij}) according to the changes of travel demand, while responds to the changing intensity of crowding by optimizing vehicle size (l^{ij}) and congestion level (o^{ij}). These optimization conditions take the following form (see Parry and Small 2009, p. 708):

$$\varrho^W w^{ij}\eta_w^{ij} = \varrho^A a^{ij}\eta_a^{ij}, \quad (3)$$

$$\varrho^C c^{ij}\eta_c^{ij} o^{ij} = t^{ij}k_2^{ij}n^{ij}, \quad (4)$$

where the factors ϱ^k express the marginal (monetary) cost of each type of loss, and η^k their elasticity ($k = W, A, C$).

These equations can be turned into the relation that route density can be increased as long as the decreasing costs of access, through the fixed quantity of passenger miles can balance the effect of the decreasing service frequency, and thus the increasing waiting costs. Similarly, the increase in vehicle size pays off as long as the gains from the reduction of crowding can cover the rising operating costs. Based on these relationships a generalized user cost gas been expressed that can summarize all factors as a function of (travel) time:

$$q^{ij} = p^{ij} + \varrho^T t^{ij} + \varrho^W w^{ij}(1 + \eta_w^{ij}/\eta_a^{ij}) + t^{ij}k_2^{ij}n^{ij}/\eta_c^{ij}o^{ij}. \quad (5)$$

In the next chapter, we introduce the suggested modifications and changes in the aforementioned relationships and optimization parameters.

4. OPTIMIZATION PROBLEMS

Crowding-related costs are difficult to measure and quantify in the way it was used in the model of Parry and Small. Based on assumptions confirmed by the literature we have modified and extended the original model using a linear multiplier function. These parameters and formulas measuring crowding can be used to optimize vehicle size and determine optimal fares. In this chapter we will present these questions, and also the related modifications in details.

4.1. Vehicle-size optimization

In the initial model, similarly to the other parameters, crowding is defined as a specific parameter of distance travelled, i.e. passenger miles. Despite the simplicity of this approach, in our opinion, the effects of crowding should rather be compared to travel time, since the extent of profit loss does not depend on the distance travelled, but rather on the duration of discomfort.

An important simplification of the original model is that the effects of crowding can be determined as a prerequisite for optimizing vehicle size with the balance of operating and crowding costs. According to this assumption, in order to increase passenger miles, the service provider must optimize the size of vehicles in such a way, that the resulting passenger-side benefits could compensate the increasing operating costs of larger vehicles. This relation would be a very practical solution, but its feasibility is highly questionable: it is not possible to react to the constant changes in passenger traffic by continuously changing the vehicle fleet, or intervening flexibly at a fixed infrastructure (e.g. by enlarging the stations of metro networks). Based on this consideration, it is advisable to abandon this equation and approach the question from the viewpoint of travel time.

The cost function was used by Parry and Small (2009) in the following form:

$$\Gamma\left(\sum_{ij} t^{ij} M^{ij}, \sum_{ij\neq iH} w^{ij} M^{ij}, \sum_{ij\neq iH} a^{ij} M^{ij}, \sum_{ij\neq iH} c^{ij} M^{ij}\right) \quad (6)$$

However, after substituting the term $c^{ij} = t^{ij} m(l^{ij})$, only the parameters in time dimension will remain:

$$\Gamma\left(\sum_{ij} t^{ij}\left(1 + m(l^{ij})\right), \sum_{ij\neq iH} w^{ij} M^{ij}, \sum_{ij\neq iH} a^{ij} M^{ij}\right) \quad (7)$$

In Equation (7) the measure of congestion is already time-related, hence the marginal monetary cost of travel time equals crowding's ($\varrho^C = \varrho^T$). If we derive the utility function supplemented with the crowding expression in a similar way to the term covering all travel costs expressed in Equation (5), and we derive the optimal vehicle size from this transformed utility function, then the maximum of the indirect utility function is as follows:

$$0 = \frac{\partial \widetilde{U}}{\partial n} = -\lambda \varrho^T t M \frac{\partial m}{\partial l}\frac{\partial l}{\partial n} - \lambda V t \frac{\mathrm{d}K}{\mathrm{d}n}. \quad (8)$$

By substituting $\partial l/\partial n = -l/n$ and $\mathrm{d}K/\mathrm{d}n = k_2$ in Equation (8), we can get a simplified form:

$$0 = -\lambda \varrho^T t M \frac{\partial m}{\partial l}\frac{l}{n} - \lambda V t k_2. \quad (9)$$

According to the formula (A7a) given in the appendix of Parry and Small (2009), the elasticity of crowding cost is:

$$\eta_c^{ij} = \frac{\partial c^{ij}}{\partial l^{ij}}\frac{l}{c} = \frac{t\,\partial m(l)}{\partial l}\frac{l}{t\,m(l)} = \frac{\partial m(l)}{\partial l}\frac{l}{m(l)} = \frac{\partial m}{\partial l}\frac{l}{m}. \quad (10)$$

If we use Equation (8) and (10) together, we get:

$$0 = -\lambda \varrho^T t M \eta_c^{ij} \frac{1}{n} m - \lambda V t k_2. \quad (11)$$

Rearranging Equation (11), dividing both sides by λM, then rearranging it again to get $\varrho^T t m$, which is practically speaking equals with $\varrho^C c$, one can get the formula that can be substituted in Equation (5) of the generalized costs.

$$\varrho^C c = \varrho^T t m = n t k_2 / \eta_c o. \quad (12)$$

Actually, we can express the result by using other terms, if we use the multiplier function $m(l) = \alpha_c \cdot l$ directly, since:

$$\lambda \varrho^T t M \frac{\partial m}{\partial l}\frac{l}{n} = \lambda V t k_2$$

$$\lambda \varrho^T t M \alpha_c l / n = \lambda V t k_2$$

$$\varrho^T t \alpha_c l o / n = t k_2$$

$$\varrho^T t \alpha_c m l = t k_2$$

$$\varrho^C c = \varrho^T t m = t k_2 / l. \quad (13)$$

Naturally the formula of generalized cost can be used with the result relationship from both types of derivation (only if a linear multiplier function is used):

$$q^{ij} = p^{ij} + \varrho^T t^{ij} + \varrho^W w^{ij} + \varrho^A a^{ij} + \varrho^C c^{ij} = p^{ij} + \varrho^T t^{ij}(1+m) + \varrho^W w^{ij}\left(1 + \eta_w^{ij}/\eta_a^{ij}\right) = p^{ij} + \varrho^T t^{ij} + \varrho^W w^{ij}\left(1 + \eta_w^{ij}/\eta_a^{ij}\right) + t k_2 / l. \quad (14)$$

It must be mentioned that a tractable formula can be derived for the costs of crowding, which directly gives the cost of crowding using only the travel time and the parameters m or l (latter is the widely used load factor).

4.2. Determining the optimal transit fares

The other formula where crowding plays a significant role is the one to determine optimal fares and subsidy rates. In this case it is necessary to examine where the extreme value of the indirect utility function is, where the equation $\partial \widetilde{U}/\partial p^{PR} = 0$ is met.

In addition to vehicle size optimization, crowding parameters also play important role in the marginal welfare formula's (A6) CROWD + VEHSIZE component. This sum contains the formula of the marginal welfare's crowding-dependent part, and the equation derived from the tax-relationship ($\mathrm{d}TAX/\mathrm{d}p^{PR}$), which is an indicator of the transport agency's operating cost, and therefore also depends on the vehicle size:

$$\sum_{ij\neq iH}\left[\varrho^C \frac{dc^{ij}}{dp^{PR}} M^{ij} + t^{ij} V^{ij} k_2 \frac{\mathrm{d}n^{ij}}{\mathrm{d}p^{PR}}\right]. \quad (15)$$

The further transformations of the equation require the relationships for elasticity that can be found in the appendix of Parry and Small (2009). Using these formulae, Equation (15) can be transformed:

$$= \sum_{ij\neq iH} \frac{n t k_2}{o}(1 - \varepsilon_V)\frac{\mathrm{d}M}{\mathrm{d}p^{PR}}. \quad (16)$$

From this equation, the form of the crowding factor (marginal cost of increased vehicle occupancy) is already apparent:

$$MC_{occ}^{ij} = \frac{n t k_2}{o}(1 - \varepsilon_V). \quad (17)$$

If we approach the question from the viewpoint of crowding multiplier, we get a similar result, but the parameters are divided into several other factors. At first, with only the CROWD component counted in, we will get the

$$\varrho^C \frac{\mathrm{d}c}{\mathrm{d}p^{PR}} M = \varrho^T m \frac{\partial t}{\partial p^{PR}} M + \varrho^T t \frac{\partial m}{\partial p^{PR}} M \quad (18)$$

equation, where the first term represents the crowding costs due to variable travel time, which, due to the similarity can be linked to the USERTIM designation in the marginal utility equation of the model. Thus, the crowding multiplier also appears in the relationship of marginal congestion costs

$$MC_{cong}^{iH} = \sum_{k=H,B} t_H^{ik} \varrho^T (1+m) M^{ik} + t_H^{iB} K^{iB} V^{iB}, \quad (19)$$

that can later be used to calculate the marginal congestion costs of bus transport: $MC_{cong}^{iB} = \alpha_B MC_{cong}^{iH}$.

The second component of the sum expresses that the change of fares also has an impact on the crowding multiplier; the equation can be further adjusted as follows:

$$\varrho^T t \frac{\partial m}{\partial p^{PR}} M = \varrho^T t \alpha_c \frac{\partial l}{\partial o} \frac{\partial o}{\partial M} \frac{dM}{dp^{PR}} M. \quad (20)$$

By substituting the modified Equation (20) in the CROWD + VEHSIZE component used in Equation (15), the derivation can be continued:

$$\sum_{ij \neq iH} \left[\varrho^T t \frac{\partial m}{\partial p^{PR}} M + tVk_2 \frac{\partial n}{\partial p^{PR}} \right] = \sum_{ij \neq iH} \left[\varrho^T t \alpha_c \frac{\partial l}{\partial o} \frac{\partial o}{\partial M} \frac{dM}{dp^{PR}} M + tVk_2 \frac{\partial n}{\partial o} \frac{\partial o}{\partial M} \frac{\partial M}{\partial p^{PR}} \right]. \quad (21)$$

All the partial derivatives, $\partial l/\partial o = (1-\varepsilon_n) \cdot l/o$, $\partial n/\partial o = \varepsilon_n \cdot n/o$ and $\partial o/\partial M = (1-\varepsilon_V) \cdot o/M$ can be substituted in Equation (21), we get a quite compact relationship for the marginal welfare effects:

$$= \sum_{ij \neq iH} \left[\left(\varrho^T \alpha_c \frac{\partial l}{\partial o} M + Vk_2 \frac{\partial n}{\partial o} \right) t \frac{\partial o}{\partial M} \frac{dM}{dp^{PR}} \right] = \sum_{ij \neq iH} \left[(\varrho^T \alpha_c (1-\varepsilon_n) l + k_2 \varepsilon_n n/o) t (1-\varepsilon_V) \cdot \frac{dM}{dp^{PR}} \right]. \quad (22)$$

It is important to emphasize that in order to carry out the optimization, it is necessary to determine the occupancy of the vehicles, i.e. the load factor (l). Since the crowding level is often expressed by fraction of the number of passengers and the vehicle's (floor) size, it is necessary to determine this quotient in order to the further use of these parameters. Accordingly, the quotient of the average number of passengers and the average surface area of the vehicles (calculated for the vehicles of each city), or even the formula ($ml = k_2/\varrho^T = \alpha_c l^2$) retrieved from Equation (13) can give us the followings:

$$l = \sqrt{k_2/(\varrho^T \alpha_c)}. \quad (23)$$

Applying one of the two ways of approach (calculation of vehicle capacity for the whole vehicle fleet, or using Equation (23) with the parameter values defined in the model) can determine the required values of vehicle occupancy and the load factor, so the value of the crowding factor can also be calculated.

5. APPLICATION OF THE MODEL IN A HUNGARIAN INTERURBAN ENVIRONMENT

The uniqueness of the Hungarian settlement system is its strong capitalization, the effect of which can be obserbed both in the structure of the transport network and also in the main connections of transport services. Due to the country's radially structured transport network, the lack of transverse connections, and the poor permeability of the Danube River—which divides the country in halves—the vast majority of road and rail traffic flows directly through, or in the immediate vicinity of the capital. This effect puts a huge weight on the road and rail network of Central Hungary. One quarter of the country's population lives in Budapest and its metropolitan area (Hungary had a population of 9.77 million in 2019). Beside the complex and overcongested transportation system of Budapest and its agglomeration, the biggest traffic appears on the roads and rails between the central region of Hungary and the bigger cities of the country. Although of lesser importance in terms of traffic volume, railway lines and bus routes serving rural settlements also play an important role in the country's transport system.

An important feature of public transport in Hungary is that state-owned service providers still dominate the market. The Hungarian state-owned company group, the MÁV-Volán Group's subsidiaries are responsible for the passenger transport on rail and for the interurban bus transit—and in some cities for the local transport service also, on the basis of a contract between the municipality and the company. The MÁV-Volán Group has an important role to play in providing public transport in cities and metropolitan areas, between the regions of the country, and as well as in connecting villages, towns and cities to the transportation system.

The examination of the subsidization issue is complicated by the fact that in the European Union, hence in Hungary also, transport service providers often operate on a regulated market within the framework of public service. In a regulated market like this, there is competition when one comes to entering the market, but companies already in the market are providing the service on a yet exclusive basis. In addition, the state, the procurer of the transport services may impose a public service obligation (according to Regulation (EC) No 1370/2007), but in this case the transport company's burdens resulting from this service has to be balanced. (Jászberényi and Pálfalvi 2009) In other words, it is a practical solution for a state to sustain public transport services through subsidization, but this method of operation also fixes the dependence of the service providers on state resources.

Table 1: Parameter values used by the calculations for the Hungarian interurban transport system

Parameters	Hungary		Unit	Source of data
	Rail	Bus		
Median wage rate	7340	7340	HUF/hour	KSH, 2017.
Number of unlinked trips	146.9	490.6	millions/year	KSH, 2017.
Annual passenger miles	7366	7397	millions	ITM, 2017.
Annual rail car / bus miles	82	367	millions	ITM, 2017.
Fleet size	1677	5239	–	ITM, 2017.
Transit speed	51.9	35.5	km/h	MÁV-Start, 2016., KTI
Purchase cost of rail car or bus	350	62	million HUF	webpages of vehicle manufacturers
Total operating cost	243	161	billion HUF	ITM, 2017.
Total fare revenues	039	57	billion HUF	ITM, 2017.
Parameters	**Road transport**		**Unit**	**Source of data**
	Peak	Off-peak		
Annual vehicle miles	82.2		million/year	(Magyar Közút 2018)
Average trip length	41.1		km	KTI, 2016.
Fuel tax	228.0		HUF/liter	estimation
Occupancy	1.32	1.32	pass/vehicle	previous results of Hungarian measurements (from 2016)
Auto average speed	50	60	km/h	estimation
Fuel efficiency	6		l/100 km	estimation

Among the European Union member states Hungary has one of the highest rates of public bus transport (Statistical Office of the European Communities 2019), a kind of service that is available at all settlement. This level of coverage is probably caused by the fact, that the motorization level is well below the European average, and the fares are cheaper compared to other countries'. Moreover, the Hungarian State provides travel discounts to several social groups. For example, citizens under the age of 6 and over 65 can use almost all kind of public transport for free, and people with a student card also receive a 50% discount on their travel.

Although this paper primarily deals with the development of methodology of calculating optimal public transport subsidies, the Hungarian adaptation of the modified calculation method was also an important element of the research. The application of the methodology for Hungary has been carried out on a national level, primarily in order to ensure that the local characteristics of the large-scale transport system do not significantly influence the results of the calculations.

By the time of our research, the operating and budget data of the national service providers were available for the years 2016/17, so we were able to perform domestic calculations with quite recent data. To describe the operating conditions of the Hungarian transport system we used the parameters and the values shown in Table 1, the other parameter values were the same as in the original model. We used primarily the data provided by the Ministry of Innovation and Technology, transport service providers, the Central Statistical Office (KSH) and the Institute of Transport Science (KTI).

It is important to emphasize that the Hungarian adaptation mainly remains on theoretical level, and does not include a detailed analysis of the domestic circumstances and characteristics of the local transport system. For the purpose of calculation and the correct interpretation of results, it should be also noted that the data used for the calculations mainly describe the characteristics of the interurban transport system. These data also describe all settlements where the service was ran by state-owned bus and train operators.

6. RESULTS OF THE CALCULATION

The results of the calculations are discussed in two sections, differentiating the effects of modifying the crowding formulas and the results of the model's application for the transport system of Hungary.

6.1. Adaptation of crowding relationships

Reconsidering the relationships and formulas used in the model was a practical decision. The original study stated that the costs of congestion were "relatively small", and since then a couple of measurements found the exact opposite, i.e. that the economic losses caused by crowding cannot be neglected, so a revision of the crowding relationships and the model was highly appropriate. After the modifications, the costs of crowding can be included as a stand-alone element of travel decisions, rather than a proportion of other travel-time components. As a result, the previously presented crowding parameters also appears in the MW formula (see Eq. (2)), creating a direct relationship between user costs and objective congestion metrics.

The results obtained by the model using the modified cost and crowding formulas are summarized in Table 2. Compared to the results of Parry and Small (2009), it can be seen that the discomfort caused by crowding already accounts for a significant proportion of travel

Table 2: Results of modelling with modified cost and congestion relationships

	Washington, DC				Los Angeles				London			
	Rail		Bus		Rail		Bus		Rail		Bus	
	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak	Peak	Off-peak
Current subsidy, percent of operating costs	47	55	80	76	83	82	79	69	67	72	59	40
Marginal welfare effects												
MW/W at current subsidy	–0,21	–0.24	–0.51	–0.50	0.34	–0.10	–0.18	4.21	0.51	0.01	0.09	1.38
Marginal cost/price gap	–0.03	–0.16	–0.94	–1.42	–0.57	–1.22	–0.87	–2.36	–0.20	–0.55	–0.25	–0.09
Net scale economy	0.09	0.41	0.51	1.99	0.18	0.86	0.45	5.90	0.04	0.28	0.30	1.74
Crowding costs	–0.08	–0.06	–0.20	–0.14	–0.17	–0.14	–0.22	–0.55	–0.07	–0.06	–0.21	–0.17
Externality	0.20	0.07	0.13	–0.05	0.79	0.32	0.46	0.44	0.57	0.35	0.14	–0.52
Other transit	0.04	–0.02	–0.02	0.13	0.10	0.07	–0.01	0.77	0.18	–0.01	0.12	0.41
MW/W at 50% subsidy	–0.21	–0.26	–0.05	–0.56	0.37	0.18	0.09	3.25	0.49	0.17	0.14	1.45
Optimum subsidy, percent of operating costs	>90	80	42	>90	>90	78	64	>90	>90	73	71	>90
Proportion of subsidy due to												
Marginal cost/price gap	0.45	0.57	0.40	0.40	0.38	0.48	0.42	0.49	0.31	0.55	0.50	0.39
Net scale economy	0.18	0.44	0.77	0.61	0.12	0.41	0.40	0.45	0.04	0.22	0.42	0.74
Crowding costs	–0.16	–0.06	–0.29	–0.04	–0.11	–0.07	–0.19	–0.04	–0.07	–0.04	–0.29	–0.07
Externality	0.45	0.08	0.15	–0.02	0.54	0.15	0.38	0.04	0.55	0.28	0.20	–0.30
Other transit	0.08	–0.03	–0.02	0.04	0.07	0.03	0.00	0.07	0.17	–0.01	0.17	0.24
Percent change is passenger miles	23.9	34.1	–48.3	28.6	11.6	–12.8	–23.6	51.0	20.3	1.7	12.6	142.5

costs (+20–60% increase), even at moderate congestion levels. It can also be observed that a significant increase in crowding-related (marginal) costs is to be expected, regardless of city, transport mode and travel period. The growing number of travels in the off-peak period may be more likely to be caused by the "replenishment" of crowding costs that were completely neglected before. The results show that marginal welfare gains, optimal subsidies and the expected passenger number are reduced in most of the studied cases. The direction (and magnitude) of the subsidy change is in line with previous expectations, as the transport system is able to optimally operate with fewer passengers due to the increase of travel costs, where the losses are mostly caused by crowding disutility.

The exception is the off-peak bus service of Los Angeles—we can also observe a minimal increase in London's peak rail marginal welfare gains—the model predicts significant subsidy and passenger growth compared to previous results. This may be caused by the fact that decreasing peak-time subsidies and significantly increasing off-peak period subsidies may lead to a higher proportion of trips taking place in the latter time period.

Comparing the results with the unmodified model's, it can be clearly seen that even in cases of moderate congestion and crowding level, the passengers' travel costs significantly increased. As a result, in most cases a substantial part of the benefits arising from scale economies are neutralized by crowding losses. In all the modelled cities crowding losses approach the level of peak rail transport's benefits coming from scale economies, and in the case of London it also outweighs it. Therefore, as a results of the model modification, we conclude that at all (public) transport systems with heavy traffic the negative effects of crowding, and all the related losses should always be taken into account.

6.2. Results of the model's application in Hungary

With the use of the presented parameter values (Table 1), the results of the optimization show that a subsidy rate of around 90% should be applied at almost all of the cases. There is an exception of the peak period bus transit, where a subsidy rate of 74% is recommended. This is 2 percentage points lower than the applied value of subsidy.) The forecast for change of passenger miles are in line with the change of subsidy rate. It predicts lower growth in the rail sector, while the rise by off-peak period bus travel, with the highest increase of subsidy that exceeds 90%. (A smaller decline in passenger miles is expected at the peak period bus transport, because of the optimal subsidy rate is smaller than the current one.)

The disaggregated results of Table 3 also show that similarly to the studied cities abroad, the benefits of the reduction of externalities also play a decisive role in the Hungarian transport system. Compared to the other transport systems however, in Hungary relatively high subsidy rates are applied, so the situation does not provide the opportunity for a significant increase of subsidy—with the exception of off-peak bus transport. Therefore the congestion losses can virtually neutralize the benefits of scale economies. It can be undoubtedly concluded, that similarly to other cities, a high subsidy rate is a necessity of the Hungarian interurban public transport system.

The results of the modelling with Hungarian data show many similarities with the characteristics of foreign cities, although the model basically aims to examine the transport system of the urban and suburban environment. The similarity between the results of the model variants (the original and the modified version) shows that the model can be

Table 3: Results of the application of the model in Hungary

	Hungary			
	Rail		Bus	
	Peak	Off-peak	Peak	Off-peak
Current subsidy, percent of operating costs	87	87	76	64
Marginal welfare effects				
MW/W at current subsidy	0.27	0.01	–0.02	0.38
Marginal cost/price gap	–0.85	–1.74	–0.71	–0.69
Net scale economy	0.02	0.13	0.12	0.69
Crowding costs	–0.11	–0.09	–0.15	–0.12
Externality	1.01	0.64	0.58	0.05
Other transit	0.20	1.07	0.15	0.46
Optimum subsidy, percent of operating costs	**>90**	**88**	**74**	**>90**
Percent change is passenger miles	5.7	3.8	–3.2	92.4

adapted and used in a geographical, social, economic, technical environment that is significantly different from the original cities. Naturally, the accuracy of the results could be improved by specifying the data, or by a detailed review of the factors adjusted to the features of other (Hungarian) cities. Nonetheless, the calculation method and results are perfectly consistent with the aims of the study.

7. SUMMARY, CONCLUSIONS, OPPORTUNITIES OF DEVELOPMENT

The results of the paper has been presented in two clusters. Firstly, we described the results of the transformations of the crowding formulae and all the modifications carried out in the model. Second, we presented the conclusions of the Hungarian adaptation. Finally, the suggestions and possible directions of further development of the model and the research area are presented.

7.1. Effects of crowding

The revision of the original study have shown that by taking the costs of crowding into account, it would be advisable to apply a smaller subsidy at peak periods. This would obviously lead to an increase in fares, resulting in fewer passengers and thus less crowding on public transport vehicles. At the same time, higher off-peak subsidy levels and cheaper fares would allow some travels to be shifted to this period, which would provide better overall transport conditions—naturally these provisions would require time-differentiated pricing which is currently unavailable in Hungary. It is also important to note that in Hungary this theory may be hampered by the fact that those social groups, whose travels could be relatively easily redistributed, such as people over the age of 65, currently do not have to pay fares, and hence the changes of pricing would have no effect on their travel habits.

From the results of the model modification and supplementation, we concluded that in these cases of heavy traffic and crowded transport systems the negative effects of the crowding phenomenon and the resulting losses should always be taken into account.

7.2. Interpretation of results in Hungary

The results of adapting the model for Hungary showed that although the model is primarily designed for urban and suburban traffic, it can be applied to national level service providers as well. As the collected technical and economic data primarily described the operation of the national railway and bus transport system, we could analyze the data of interurban traffic at a national level. In our opinion, the most important requirement for the model is that data shold come from cities, counties, regions etc. with similar economic, social, and transport conditions—this limitation prevents important features from being “averaged”, otherwise the data would already bring distortions into the calculation.

The results for the Hungarian transport system are clustered around very high subsidy rates. For example, providing a larger subsidy rate in the off-peak period could cause the higher occupancy of vehicles outside peak hours—for example, by shifting a part of peak period bus travels to the off-peak period. According to the model this transmission could utilize idle capacities, and thus it would make a substantial improvement of the transport system and would also achieve the reduction of social losses.

We suggest that the state should also promote the more even use of capacities in other ways to prevent harmful levels of crowding. Gradual shift start supported by the state, postponing the beginning of school, or by streamlining the flow of information by organizing and standardizing a real-time information management systems—e.g. presenting the usual occupancy of vehicles, or the actual values using ticket purchase or data of vehicle sensors. Additionally, transformations caused by the coronavirus epidemic coluld be also good examples. Changing working, commuting and travel habits, accelerating digitalization and the possibility od home office provide an opportunity to a major review of transport strategies.

7.3. Directions for further improvement

By the further development of the model, it could more effectively predict the impact of strategic pricing decisions at a given city or at a regional (national) level. Currently the model cannot really account for the capacity constraints of the transport system. Thus, for example with increasing peak frequencies the user can't be certain that the system can manage the increasing demand without costly infrastructure (or traffic organizational and controlling) investments. Take the rail network of Budapest for example, which is currently suffering from the lack of capacity, so the model would also require a parallel examination of road and rail capacities. Despite the model can control the occupancy of a particular traffic mode and in a given period by optimizing crowding, but there may be situations where this method is no longer sufficient and enough for an optimal solution.

The main question of the modelling carried out during the research was to examine the kind of pricing that could be used to achieve the (second best) social optimum. Though, changing prices necessarily imply some kind of traffic reorganization, e.g. the increasing occupancy of public transport due to the reduction of cars in areas prone to congestion, but there are indirect effects that do not appear in the model. The calculation does not include the gains from detached and unused parking and transport spaces in downtown areas due to reduced road traffic, or the potential benefits of using these yet free spaces for other purposes—the benefits of increased local trade due to bigger pedestrian and bicycle traffic, or the rise in property prices.

Further questions are raised, since due to the use of aggregated data, it is not possible to select or further divide users, i.e. the impact of pricing cannot be examined separately on several social groups. It would be important to analyse in detail how social groups respond to a transport policy pricing decision, since in most cases there is a correlation between the quality of transport system and the settlement environment, thus housing prices, and so indirectly with the financial situation of residents. For these reasons, it is not certain that groups which are still distinguishable in reality will behave according to the model as a result of pricing. Therefore, it is questionable whether the social and the political-economic optimum do not slip, or intentionally can be slipped away. For example, the price-sensitive, but typically more mobile people can be "supported" by the overcharging of transport services for people with less income and reduced mobility. But this method can be used for good reasons as well, if we influence the travel patterns of the more price-sensitive social groups by pricing to disencumber the overcongested and crowded peak periods.

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REFERENCES

[1] Björklund, Gunilla, and Jan-Erik Swärdh. 2015. 'Valuing In-Vehicle Comfort and Crowding Reduction in Public Transport'. *Working Papers in Transport Economics* 2012(12).

[2] Boss, Alfred, and Astrid Rosenschon. 2008. *Der Kieler Subventionsbericht: eine Aktualisierung*. Kiel: Inst. für Weltwirtschaft.

[3] Doll, Claus, and Huib van Essen. 2008. *Road Infrastructure Cost and Revenue in Europe*. Delft, The Netherlands: CE Delft.

[4] Hörcher, Daniel, Daniel J. Graham, and Richard J. Anderson. 2017. 'Crowding Cost Estimation with Large Scale Smart Card and Vehicle Location Data'. *Transportation Research Part B: Methodological* 95:105–25. doi: 10.1016/j.trb.2016.10.015.

[5] International Transport Forum. 2018. 'Zero Value of Time Roundtable'. *ITF*. Retrieved 6 November 2019 (https://www.itf-oecd.org/zero-value-time-roundtable).

[6] Jansson, Jan Owen. 1979. 'Marginal Cost Pricing of Scheduled Transport Services'. *Journal of Transport Economics and Policy* 13(3):268–94.

[7] Jászberényi, Melinda, and József Pálfalvi, eds. 2009. *Nemzetközi közlekedés és turizmus*. Budapest: Aula Kiadó.

[8] Keserű, Imre, Jeroen Bulckaen, Cathy Macharis, Joeri Minnen, Ignace Glorieux, and Pieter Theun van Tienoven. 2015. 'Is Travel Time Wasted? Evidence from a Time Use Survey in Flanders, Belgium'. Windsor, United Kingdom.

[9] Keserű, Imre, and Cathy Macharis. 2018. 'Travel-Based Multitasking: Review of the Empirical Evidence'. *Transport Reviews* 38(2):162–83. doi: 10.1080/01441647.2017.1317048.

[10] Kroes, Eric, Marco Kouwenhoven, Laurence Debrincat, and Nicolas Pauget. 2014. 'Value of Crowding on Public Transport in Île-de-France, France'. *Transportation Research Record: Journal of the Transportation Research Board* 2417(1):37–45. doi: 10.3141/2417-05.

[11] Mohring, Herbert. 1972. 'Optimization and Scale Economies in Urban Bus Transportation'. *The American Economic Review* 62(4):591–604.

[12] Mohring, Herbert. 1976. *Transportation Economics*. Ballinger Publishing Company.

[13] Munkácsy, András, Imre Keserű, and Miklós Siska. n.d. 'Travel-Based Multitasking on Public Transport: An Empirical Research in Hungary'. *Periodica Polytechnica Transportation Engineering* (megjelenés alatt).

[14] Parry, Ian W. H., and Kenneth A. Small. 2009. 'Should Urban Transit Subsidies Be Reduced?' *The American Economic Review* 99(3):700–724. doi: https://doi.org/10.1257/aer.99.3.70.

[15] Perone, Jennifer S. 2002. *Advantages and Disadvantages of Fare-Free Transit Policy*. Tampa, FL: University of South Florida.

[16] Statistical Office of the European Communities. 2019. *Energy, Transport and Environment Statistics*.

[17] Tirachini, Alejandro, Lijun Sun, Alexander Erath, and Artem Chakirov. 2016. 'Valuation of Sitting and Standing in Metro Trains Using Revealed Preferences'. *Transport Policy* 47:94–104. doi: 10.1016/j.tranpol.2015.12.004.

[18] Tscharaktschiew, Stefan, and Georg Hirte. 2011. 'Should Subsidies to Urban Passenger Transport Be Increased? A Spatial CGE Analysis for a German Metropolitan Area'. *Technische Universität Dresden, Faculty of Business and Economics*.

[19] Turvey, Ralph, and Herbert Mohring. 1975. 'Optimal Bus Fares'. *Journal of Transport Economics and Policy* 9(3):280–86.

[20] Vickrey, William. 1980. 'Optimal Transit Subsidy Policy'. *Transportation* 9(4):389–409. doi: 10.1007/BF00177700.

[21] Whelan, Gerard, and Jon Crockett. 2009. 'An Investigation of the Willingness to Pay to Reduce Rail Overcrowding'. Harrogate, England.