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A FINITE ELEMENT MODEL FOR STABILITY ANALYSIS OF SYMMETRICAL ROTOR SYSTEMS WITH INTERNAL DAMPING

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Dedicated to Professor István Páczelt on the occasion of his sixtieth birthday

Abstract. This paper deals with the stability analysis of self-excited bending vibrations of linear symmetrical rotor-bearing systems with internal damping using the finite element method. The rotor system consists of uniform circular Rayleigh shafts with internal viscous damping, symmetric rigid disks, and discrete isotropic damped bearings. The effect of rotatory inertia and gyroscopic moment are also included in the mathematical model. By combining the sensitivity analysis and the eigenvalue problem of the rotor dynamics equations presented in complex form, it is proved theoretically that the whirling motion of the rotor system becomes unstable at all speeds beyond the threshold speed of instability. In addition, the latter is always greater than the corresponding whirling speed. It is found that the rotor stability is improved by increasing the damping provided by the bearings, whereas increasing internal damping may reduce the stability threshold. It is also shown that the whirling speed of the rotor is higher than the first forward critical speed. Numerical examples are given to confirm the validity of the theoretical results.

Keywords: rotor dynamics, stability analysis, internal damping, threshold speed, finite elements

1. Introduction

It is well known that the stability of rotors is influenced by internal damping. The early works of Kimball [1] and Newkirk [2] showed that internal damping destabilizes the whirling motion of the rotor at speeds above the first critical speed. The stability problems of rotors with both internal and external damping have been discussed by several authors [3-7]. In most of the works by the investigators listed above, however, the gyroscopic effects are neglected.

Of the many researchers studying the stability problems of rotors using finite elements, Zorzi and Nelson [8] carried out first the numerical stability analysis of such rotor systems including the effects of rotatory inertia, gyroscopic moments, and both internal viscous and hysteretic damping. By using the numerical examples of a uniform circular shaft with viscous material damping, supported at its ends by two identical undamped isotropic bearings, they found that the first and second forward precessional modes become unstable at the first and second critical speeds, respectively. The author [9] of this paper generalized the above results for symmetric rotor systems with viscous internal damping, supported by isotropic undamped bearings. By applying the sensitivity analysis and the eigenvalue problem of the rotor dynamics equation in complex form, it has been proved that the *stability threshold speed,* at which the rotor loses its stability, coincides with *the first forward critical speed* regardless of the magnitude of the internal viscous damping coefficient.

The main purpose of this paper is to demonstrate that the finite element simulation and the sensitivity analysis are adequate methods to study the combined effect of internal damping and isotropic bearing damping on the stability of complex symmetrical rotor systems. By combining the sensitivity analysis and the matrix representation of the rotor dynamics equations in complex form to assess stability, it is proved theoretically that the whirling motion of the rotor system becomes unstable at all speeds above the threshold speed of instability. In addition, the latter is always greater than the corresponding whirling speed (frequency). It is found that the rotor stability is improved by increasing the bearing damping, whereas increasing internal viscous damping may reduce the stability threshold speed. Furthermore, it is shown that the whirling speed of the rotor is higher than the first forward critical speed. Numerical examples are given to show the validity of the theoretical results of the present work. The threshold speeds and the whirling speeds of the rotor model are calculated using a computer program written in real form, which utilizes a standard QR-algorithm and an iterative technique developed by the author [10].

2. Equations of motion in complex form

2.1. Preliminaries and notations. In this section, the equations of motion for a rigid disk, finite shaft element with internal viscous damping, isotropic damped bearing, and the complete rotor system are written solely in complex form by making use of a note by Nelson [11] and the paper by Zorzi and Nelson [8]. Note that the equation of motion for the shaft element in complex form [11] does not contain internal damping, whereas the effects of both viscous and hysteretic internal damping are included into the finite element model in the work by Zorzi and Nelson [8].

Consider a symmetric rotor system as shown in Figure 1. The rotor system consists of symmetrical rigid disks with negligible thicknesses, uniform circular Rayleigh shafts with viscous internal damping, and n isotropic damped bearings with stiffnesses k_i and damping coefficients $c_i\,(i = 1, 2, ..., n)$. The rotor is balanced, and rotates at a constant speed $\Omega(\Omega > 0)$. The reference system $Oxyz$ is fixed in space with the horizontal x-axis coinciding with the undeformed rotor centerline. The external damping, axial load and gravity are neglected.

Any node i of the rotor system has four degrees of freedom: two translations (v_i, w_i) in the (y, z) directions, and two rotations$(\varphi_{yi}, \varphi_{zi})$ about the (y, z) axes, respectively. The complex displacement vector of the ith node is defined by complex coordinates[11]

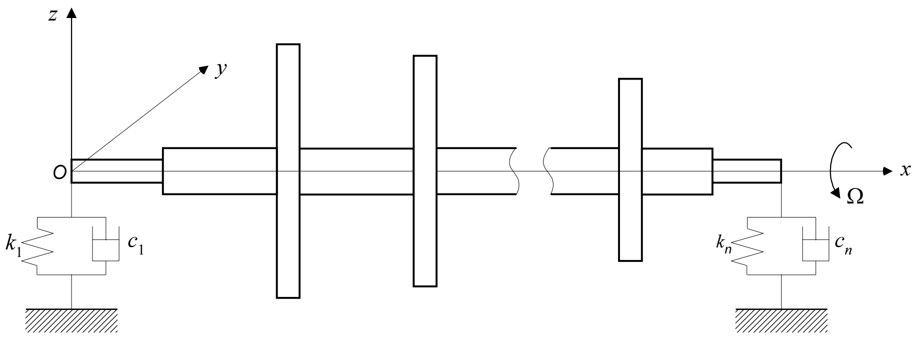

Figure 1: Symmetric rotor in isotropic damped bearings

as

$$\mathbf{p}_i = \begin{bmatrix} r_i \\ \varphi_i \end{bmatrix} = \begin{bmatrix} v_i & +iw_i \\ \varphi_{yi} & +i\varphi_{zi} \end{bmatrix}, \qquad i = \sqrt{-1} \tag{2.1}$$

The component equations and the system equation in complex form may be written as presented below.

2.2. Rigid disk. The equation of motion for a rigid disk in complex form is given by

$$\left(\mathbf{M}_t^d + \mathbf{M}_r^d\right)\ddot{\mathbf{p}}^d - \Omega \mathbf{G}^d \dot{\mathbf{p}}^d = \mathbf{F}^d \,, \tag{2.2}$$

where $\mathbf{p}^d$ is the complex displacement vector corresponding to the four degrees of freedom $\left(v^d, w^d, \varphi_y^d, \varphi_z^d\right)$ of the node at which the disk is attached. The translational and rotational mass matrices $\left(\mathbf{M}_t^d, \mathbf{M}_r^d\right)$, and the gyroscopic matrix $\mathbf{G}^d$ are defined as

$$\mathbf{M}_t^d = \begin{bmatrix} m^d & 0 \\ 0 & 0 \end{bmatrix}, \tag{2.3}$$

$$\mathbf{M}_r^d = \begin{bmatrix} 0 & 0 \\ 0 & J_D \end{bmatrix}, \tag{2.4}$$

$$\mathbf{G}^d = \begin{bmatrix} 0 & 0 \\ 0 & iJ_P \end{bmatrix}, \tag{2.5}$$

where m^d , J_D and J_P are the mass, the diametral and polar moments of inertia of the disk, respectively.

2.3. Finite shaft element. The equation of motion for the finite rotating shaft element with internal viscous damping takes the form [9]

$$\left(\mathbf{M}_t^e + \mathbf{M}_r^e\right)\ddot{\mathbf{p}}^e + \left(\eta \mathbf{K}_b^e - \Omega \mathbf{G}^e\right)\dot{\mathbf{p}}^e + \left(\mathbf{K}_b^e + \eta\Omega \mathbf{K}_c^e\right)\mathbf{p}^e = \mathbf{F}^e \,, \tag{2.6}$$

where

$$\mathbf{p}^e = \begin{bmatrix} \mathbf{p}_i \\ \mathbf{p}_j \end{bmatrix} \tag{2.7}$$

is the $(4x1)$ complex nodal displacement vector of the shaft element with nodes i and j, η is the internal viscous damping coefficient,

$$\mathbf{K}_c^e = -i\mathbf{K}_b^e \tag{2.8}$$

is the complex circulation matrix of the shaft element.

The translational and rotational mass matrices $(\mathbf{M}_t^e, \mathbf{M}_r^e)$, the gyroscopic matrix $\mathbf{G}^e$, and the bending stiffness matrix $\mathbf{K}_b^e$ of the shaft element are defined as

$$\mathbf{M}_t^e = \frac{\mu l}{420}\begin{bmatrix} 156 & & Hermitian & \\ i22l & 4l^2 & & \\ 54 & -i13l & 156 & \\ -i13l & -3l^2 & -i22l & 4l^2 \end{bmatrix}, \tag{2.9}$$

$$\mathbf{M}_r^e = \frac{\mu r^2}{120l}\begin{bmatrix} 36 & & Hermitian & \\ i3l & 4l^2 & & \\ -36 & -i3l & 36 & \\ i3l & -l^2 & -i3l & 4l^2 \end{bmatrix}, \tag{2.10}$$

$$\mathbf{G}^e = i2\mathbf{M}_r^e\,, \tag{2.11}$$

$$\mathbf{K}_b^e = \frac{EI}{l^3}\begin{bmatrix} 12 & & Hermitian & \\ i6l & 4l^2 & & \\ -12 & i6l & 12 & \\ i6l & 2l^2 & -i6l & 4l^2 \end{bmatrix}, \tag{2.12}$$

where μ is the mass per unit length, l is the length of shaft element, r is the element radius, EI is the bending rigidity of the shaft element.

2.4. Linear isotropic damped bearings. The linear isotropic damped bearings can be modeled by the equation:

$$\mathbf{C}^b\dot{\mathbf{p}}^b+\mathbf{K}^b\mathbf{p}^b= \mathbf{F}^b\,, \tag{2.13}$$

where $\mathbf{p}^b$ is the complex displacement vector at the bearing location (node), $\mathbf{F}^b$ is the complex bearing force vector. The damping and stiffness matrices $\left(\mathbf{C}^b, \mathbf{K}^b\right)$ of the isotropic bearings are defined as

$$\mathbf{C}^b = \begin{bmatrix} c^b & 0 \\ 0 & 0 \end{bmatrix}\,, \qquad \mathbf{K}^b = \begin{bmatrix} k^b & 0 \\ 0 & 0 \end{bmatrix}, \tag{2.14}$$

where c^b and k^b are the direct damping and stiffness coefficients for the translational displacements, respectively.

2.5. System equations. The equations of motion of the complete rotor-bearing system can be obtained by assembling all component equations of the form equations (2.2), (2.6) and (2.13). The resulting equation is of the form

$$\mathbf{M}\ddot{\mathbf{p}} + (\eta\mathbf{K}_b + \mathbf{C} - \Omega\mathbf{G})\dot{\mathbf{p}} + [\mathbf{K}_B + (1 - i\eta\Omega)\mathbf{K}_b]\mathbf{p} = 0\,, \tag{2.15}$$

where

$$\mathbf{p}^T = \left[\mathbf{p}_1^T\mathbf{p}_2^T...\mathbf{p}_N^T\right] \tag{2.16}$$

is the $(2\,Nx1)$ complex nodal displacement vector of the rotor system (N equals the number of nodes), the letter "T" denotes the transpose.

2.6. Positive definite matrices. Since kinetic energy and strain energy cannot be negative, the system matrices $(\mathbf{M}, \mathbf{K}_b)$ are positive definite Hermitian matrices [9]. Thus the following relations hold:

$$\bar{\mathbf{p}}^T\mathbf{M}\mathbf{p} > 0\ , \qquad \bar{\mathbf{p}}^T\mathbf{K}_b\mathbf{p} > 0\ , \qquad (\mathbf{p} \neq 0)\ , \tag{2.17}$$

where the bar denotes the complex conjugate operator.

Note that the system gyroscopic matrix $\mathbf{G}$ is not Hermitian. However, by using the definitions of the component gyroscopic matrices presented by equations (2.5) and (2.11) it can be expressed as

$$\mathbf{G} = i\mathbf{M}_g\ , \tag{2.18}$$

where

$$\bar{\mathbf{p}}^T\mathbf{M}_g\mathbf{p} > 0, \quad (\mathbf{p} \neq 0)\ . \tag{2.19}$$

Evidently $\mathbf{C}$ and $\mathbf{K}_B$ are positive definite diagonal matrices, the nonzero elements of which are the damping coefficients and the stiffnesses of the isotropic bearings, respectively

3. Stability analysis

3.1. Stability threshold speed determination. On seeking a solution to equation (2.15) of the form

$$\mathbf{p} = \mathbf{P}e^{\lambda t}\ , \tag{3.1}$$

we obtain the eigenvalue problem

$$\left[\lambda^2\mathbf{M} + \lambda\,(\eta\mathbf{K}_b + \mathbf{C} - \Omega\mathbf{G}) + \mathbf{K}_B + (1 - i\eta\Omega)\mathbf{K}_b\right]\mathbf{P} = 0 \tag{3.2}$$

with $4N$ eigenvalues λ_j and the corresponding eigenvectors $\mathbf{P}_j$ $(j = 1, 2, ..., 4N)$. The eigenvalues λ are of the form

$$\lambda = \alpha + i\omega\ , \tag{3.3}$$

where α is the damping coefficient or decay rate, ω is the damped natural frequency or whirl speed.

For later use, the eigenvalue problem will be given in a modified form. To this end, we premultiply equation (3.2) by the complex conjugate eigenvector $\bar{\mathbf{P}}^T$. Then we obtain the following scalar equation:

$$\bar{\mathbf{P}}^T\left[\lambda^2\mathbf{M} + \lambda(\eta\mathbf{K}_b + \mathbf{C} - \Omega\mathbf{G}) + \mathbf{K}_B + (1 - i\eta\Omega)\mathbf{K}_b\right]\mathbf{P} = 0\ , \tag{3.4}$$

which can be rewritten as

$$m\lambda^2 + (\eta k_b + c - ig\Omega)\lambda + k_B + (1 - i\eta\Omega)k_b = 0 \ , \tag{3.5}$$

where the scalars m, k_b, c, g and k_B are in all positive real quantities defined by

$$\bar{\mathbf{P}}^T\mathbf{M}\mathbf{P} = m > 0 \ , \tag{3.6}$$
$$\bar{\mathbf{P}}^T\mathbf{K}_b\mathbf{P} = k_b > 0 \ , \tag{3.7}$$
$$\bar{\mathbf{P}}^T\mathbf{C}\mathbf{P} = c > 0 \ , \tag{3.8}$$
$$\bar{\mathbf{P}}^T\mathbf{G}\mathbf{P} = ig(g > 0) \tag{3.9}$$
$$\bar{\mathbf{P}}^T\mathbf{K}_B\mathbf{P} = k_B > 0 \ . \tag{3.10}$$

Note that the inequalities (3.6) - (3.10) hold on account of the positive definite matrices of the rotor system (see Section 2.5.).

Instability occurs if one of the eigenvalues has a positive real part. Thus, the problem of determining the limit of stability of the rotor system is reduced to finding the shaft speed Ω_s (stability threshold speed), at which the greatest real part of all eigenvalues λ_j equals zero. The corresponding imaginary part ω_s is the *whirling speed.*

For the possible limit ω , the substitution of the eigenvalue of the form

$$\lambda = i\omega \tag{3.11}$$

into equation (3.5) yields

$$-m\omega^2 + g\Omega\omega + k_b + k_B + i\left[(\eta k_b + c)\omega - \eta\Omega k_b\right] = 0. \tag{3.12}$$

After seperating equation (3.12) into real and imaginary parts, we obtain

$$-m\omega^2 + g\omega\Omega + k_b + k_B = 0, \tag{3.13}$$
$$\omega(\eta k_b + c) = \eta\Omega k_b \tag{3.14}$$

It is clear from equation (3.14) and inequalities (3.7) and (3.8) that

$$\Omega = \omega\left(1 + \frac{c}{\eta k_b}\right) > \omega(\omega > 0). \tag{3.15}$$

Consequently, the threshold speed is greater than the corresponding whirling speed. Furthermore, from inequality (3.15) it is seen that the particular undamped whirl mode induced at the stability threshold speed is forward and asynchronous. It is noteworthy that all backward precessional modes of the rotor are stable for any rotational speed.

Now we shall prove that the rotor loses its stability at all speeds above the possible stability limit. Here, we apply the eigenvalue sensitivity analysis. Let us suppose that the shaft speed Ω is an independent parameter, and let us differentiate equation (3.5) with respect to Ω :

$$\lambda'(2m\lambda + \eta k_b + c - ig\Omega) - ig\lambda - i\eta k_b + m'\lambda^2 + \\ + (\eta k_b' + c' - ig'\Omega)\lambda + k_B' + (1 - i\eta\Omega)k_b' = 0 \ , \tag{3.16}$$

where primes denote differentiation with respect to Ω . The quantity $\lambda' = \partial\lambda/\partial\Omega$ is referred to as an eigenvalue sensitivity coefficient [12], which can be written, with the aid of equation (3.3), in the form:

$$\frac{\partial\lambda}{\partial\Omega} = \frac{\partial\alpha}{\partial\Omega} + i\frac{\partial\omega}{\partial\Omega} \; . \tag{3.17}$$

To calculate $\partial\lambda/\partial\Omega$ from equation (3.16) at the possible limit Ω , we substitute again equation (3.11) into equation (3.16):

$$\begin{aligned} \tfrac{\partial\lambda}{\partial\Omega}[\eta k_b + c + i(2m\omega - g\Omega)] + g\omega - i\eta k_b + (\underline{-m'\omega^2 + g'\omega\Omega + k'_b + k'_B}) \\ \underline{+i[\omega(\eta k'_b + c') - \eta\Omega k'_b]} = 0 \; . \end{aligned} \tag{3.18}$$

Since the eigenvalue derivative $\partial\lambda/\partial\Omega$ represents the unique solution of equation (3.18) at the possible limit of stability and hence its value is not influenced by any normalization criterion for the eigenvector $\mathbf{P}$, therefore the underlined terms will vanish:

$$-m\acute{\omega}^2 + \omega\Omega g' + k'_b + k'_B \;\; = \;\; 0 \; , \tag{3.19}$$

$$\omega(\eta k'_b + c') \;\; = \;\; \eta\Omega k'_b \; . \tag{3.20}$$

We then obtain the following expression for the damping sensitivity coefficient $\partial\alpha/\partial\Omega$:

$$\frac{\partial\alpha}{\partial\Omega} = \frac{2\eta k_b(m\omega - g\Omega)}{(\eta k_b + c)^2 + (2m\omega - g\Omega)^2} \; . \tag{3.21}$$

It is easy to show that the nominator of the above ratio is positive. By using equation (3.13), the bracketed term in the nominator can be written as

$$m\omega - g\Omega = \frac{k_b + k_B}{\omega} \; . \tag{3.22}$$

By making use of inequalities (3.6) - (3.10), it is clear that the right-hand side of equation (3.22) is positive. Therefore, the damping sensitivity coefficient $\partial\alpha/\partial\Omega$ is positive at each possible limit of stability. Thus, the lowest value of the above stability limits for the particular forward whirl modes is considered to be the *stability threshold speed* of the rotor-bearing system. Consequently, the whirling motion of the rotor becomes unstable at all speeds above the stability threshold speed.

3.2. Effect of bearing damping on rotor stability. Now we shall prove that an increase in the bearing damping coefficients results in an increase in the whirl threshold speed, thus the rotor stability will be improved.

Let us consider the bearing damping coefficient $c_i (i = 1, 2, ..., n)$ of the i th isotropic damped bearing as an independent parameter, and differentiate equations (3.13) and (3.14) with respect to c_i :

$$\omega'(g\Omega - 2m\omega) + \omega g\Omega' + (\underline{-m'\omega^2 + \omega\Omega g' + k'_b + k'_B}) \;\; = \;\; 0 \; , \tag{3.23}$$

$$\omega'(c + \eta k_b) - \eta k_b\Omega' + \underline{\omega(\tilde{c} + \eta k'_b) - \eta\Omega k'_b} \;\; = \;\; -\omega c^* \; , \tag{3.24}$$

where prime denotes differentiation with respect to c_i ,

$$\tilde{c} = \frac{\partial \bar{\mathbf{P}}^T}{\partial c_i}\mathbf{C}\mathbf{P} + \bar{\mathbf{P}}^T\mathbf{C}\frac{\partial \mathbf{P}}{\partial c_i} \tag{3.25}$$

and

$$c^* = \bar{\mathbf{P}}^T\frac{\partial \mathbf{C}}{\partial c_i}\mathbf{P} > 0\ . \tag{3.26}$$

By using the same reasoning that we have applied in connection with equation (3.18), it is clear that the underlined terms in equations (3.23) and (3.24) will vanish at the threshold speed Ω . The whirling speed sensitivity coefficient ω' and the threshold speed sensitivity coefficient Ω' can now be obtained from the above two equations as

$$\frac{d\omega}{dc_i} = \frac{gc^*\omega^2}{2(m\omega - g\Omega)\eta k_b}\ , \tag{3.27}$$

$$\frac{d\Omega}{dc_i} = \frac{\omega(2m\omega - g\Omega)c^*}{2(m\omega - g\Omega)\eta k_b}\ . \tag{3.28}$$

By using equation (3.22) and inequalities (3.7) and (3.9), it is easy to see that the above sensitivity coefficients are positive. Thus, the addition of bearing damping improves the rotor stability. It is also clear that the whirling speed is always greater than the first forward bending critical speed of the rotor system. The latter statement follows from the fact that the threshold speed of symmetrical rotors with viscous internal damping, supported by undamped isotropic bearing, coincides with the first forward critical speed [9]. It can further be concluded from equation (3.27) that when the gyroscopic moments of the rotor are neglected ($g = 0$), then the whirling speed remains constant (the first critical speed of the rotor) regardless of the magnitude of the bearing damping coefficients.

3.3. Influence of internal damping on threshold speed. We shall now prove that increasing the internal viscous damping coefficient η causes reduction in the stability threshold speed of the rotor. We assume that η is an independent system parameter. By differentiating equations (3.13) and (3.14) with respect to η , we get

$$\omega'(g\Omega - 2m\omega) + \omega g\Omega' + (\underline{-m'\omega^2 + \omega\Omega g' + k_b' + k_B'}) = 0\ , \tag{3.29}$$

$$\omega'(c + \eta k_b) - \eta k_b\Omega' + \underline{\omega(c' + \eta k_b') - \eta\Omega k_b'} = (\Omega - \omega)k_b\ , \tag{3.30}$$

where prime denotes differentiation with respect to η . Since the underlined expressions vanish at the stability threshold, the whirling speed and threshold speed sensitivity coefficients are determined by

$$\frac{d\omega}{d\eta} = -\frac{\omega g(\Omega - \omega)}{2(m\omega - g\Omega)\eta}\ , \tag{3.31}$$

$$\frac{d\Omega}{d\eta} = -\frac{(2m\omega - g\Omega)(\Omega - \omega)}{2(m\omega - g\Omega)\eta}\ . \tag{3.32}$$

By using inequalities (3.9), (3.15), and equation (3.22), it is clear that both the whirling speed sensitivity coefficient and the threshold speed sensitivity coefficient are negative at the stability threshold speed $\Omega = \Omega_s$. Consequently, internal viscous damping has a destabilizing effect on the rotor stability.

From equation (3.31) we also see that a change of $\Delta\eta$ leads to an opposite change in the whirling speed ω_s . Further, when the gyroscopic moments are neglected ($g = 0$), the whirling speed remains constant regardless of the magnitude of the internal viscous damping coefficient.

4. Numerical examples

4.1. To demonstrate the validity of the above theoretical results, two numerical examples are provided. In both examples, the simply supported uniform shaft studied by Zorzi [8] is considered. The rotor model consists of a 10.16 cm diameter and 127 cm long steel shaft supported by two identical isotropic damped bearings at both ends. The stiffnesses of the bearings are: $k_1 = 1.75 \times 10^{11}$N/m. The material properties of the shaft are: Young's modulus $E = 2.06 \times 10^{11}$N/m^2 , and density $\rho = 7800$ kg/m^3. The rotor is modeled as an assembly of four finite elements of equal length. In the calculations, the damping coefficients c_1 of the bearings and the internal viscous damping coefficient η for the shaft are considered to be parameters.

4.2. As a first example, we shall examine the influence of the damping coefficient c_1 on the rotor stability for the viscous internal damping coefficient $\eta = 0.0002$ s. Table 1 shows the numerical values of the whirling speed ω_s and the stability threshold speed Ω_s of the rotor for different values of c_1 . The first forward bending critical speed of the rotor was found to be $\Omega_{F1} = 521.392$ rad/s.

bearing damping (Ns/m)	threshold speed (rad/s)	whirling speed (rad/s)
0	521.392	521.392
100	544.085	521.397
200	566.798	521.410
300	589.518	521.430
400	612.254	521.457
500	635.006	521.492

Table 1. Effect of bearing damping (c_1) on rotor stability

As can be seen from Table 1, the introduction of bearing damping will increase the stability threshold speed, thus the stability of the rotor system will be improved. The numerical results also illustrate that the threshold speeds are greater than the corresponding whirling speeds, which are only slightly greater than the first forward critical speed of the rotor. Clearly the numerical results of Table 1 are in quite good agreement with the theoretical results obtained in Section 3.2.

4.3. As a second example, we consider the influence of the internal viscous damping η on the rotor stability for the bearing damping coefficient $c_1 = 1750\,\text{Ns/m}$. Table 2 presents the calculated values of the threshold speeds and whirling speeds for different values of η .

viscous internal damping (s)	threshold speed (rad/s)	whirling speed (rad/s)
0.0002	922.335	522.565
0.0003	789.012	522.556
0.0004	722.375	522.551
0.0005	682.396	522.548

Table 2. Effect of internal damping on rotor stability

The numerical results show clearly that the stability of the rotor is greatly reduced by increasing internal damping. For example, for the viscous internal damping coefficient of $\eta = 0.0002\,\text{s}$, the rotor becomes unstable at the threshold speed $\Omega_s = 922.335\,\text{rad/s}$. By increasing the viscous internal damping coefficient to $\eta = 0.0003$ s, the rotor stability threshold speed will be reduced to $\Omega_s = 789.012$ rad/s. It should be noted that increasing internal damping produces only a small reduction in the whirling speed ω_s . Table 2 also confirms the validity of inequality (3.15). ($\Omega_s > \omega_s$). Evidently the numerical results summarized in Table 2 are in good agreement with the theoretical results derived in Section 3.3.

5. Summary and conclusions

In this paper a finite element stability analysis of self-excited bending vibrations of symmetric rotors with viscous internal damping, supported by isotropic damped bearings has been presented. By combining the sensitivity method and the eigenvalue problem of the rotor dynamics equations in complex form, it is proved theoretically that the whirling motion of the rotor becomes unstable at all speeds above the stability threshold speed.

In addition, the latter is always greater than the corresponding whirling speed. Further, the rotor stability is improved by increasing the damping provided by the bearings, whereas internal viscous damping destabilizes the whirling motion of the rotor.

It is also shown that the whirling speed of the rotor system is higher than the first forward bending critical speed. Numerical examples are provided to confirm the validity of the above theoretical results.

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