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A NUMERICAL MODEL FOR HOT ROLLING OF ALUMINUM STRIPS

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Abstract. In this paper, a p version finite element based numerical model is presented for the 3D steady-state analysis of the deformation and velocity field of aluminum strips during hot flat rolling. The material behavior is described by the Levi-Mises type constitutive equation considering no volume change. The assumed roll-strip interaction depends on the relative velocity between the strip and the roll (hydrodynamic lubrication). MATLAB mathematical software was applied for the implementation of the numerical algorithm.

Keywords: flat rolling, p version finite element, Levi-Mises constitutive equation

1. Introduction

The large-scale manufacturing of steel and aluminum rolled products has grown to enormous proportions and ranks as one of the largest industrial segments in modern industry. The major proportion of this market is flat rolled product and represents significant investment which needs to be efficiently operated and regularly upgraded to take advantage of the new technical inventions.

A wide spread form of upgrading in recent years has been the introduction of advanced automation systems with model-based setup and control functions. The design, application and maintenance of these systems have led to significant improvements in mill performance and product quality. The development of model-based systems requires a deep understanding of the physical phenomena involved.

Considering the market requirements for flat rolled aluminum products an increasing demand can be observed for tight dimension tolerance, especially for thickness distribution along the strip length and profile (thickness distribution along the strip width). The strip profile is determined essentially during the hot rolling operation.

Important pieces of information can be predicted by the developed models, such as separation force, separation force distribution in the roll gap, required torque, power, forward and backward slips. These can be utilized for developing and optimizing pass schedules and providing set up data to the mills renting powerful process tools to satisfy the requirement of product developments.

The first 1D numerical model valid both for hot and cold rolling was developed in the form of a non-linear first order differential equation, the so called Kármán equation. Several expressions for the calculation of the separation force were derived from the approximation of the solution of this differential equation. Due to the basic assumptions this model is incapable of an analysis through the thickness and along the width of the strip. It was applied mainly for modeling cold rolling. Significant efforts were made to improve the reliability of the model predictions, such as shear compensation, roll deformation and yield stress adaptation.

2D models were developed for the study of the plane deformation in the middle section plane of the strip perpendicular to the axis of the work rolls. This approach does not allow us to investigate the lateral spread, the separation force distribution and the velocity field variation along the work roll. Compared with the improved 1D model the yield stress compensation for the shear stress component is not required and the Mises type yielding criteria may be incorporated.

Another family of 2D models was derived in the middle plane of the rolled strip (defined by the traveling direction and the width of the strip). All the properties, parameters and variables are considered in average sense through the thickness requiring the yield stress compensation. These models are able to predict the lateral spread and provide certain information about the along the work roll force distribution and velocity field variation.

The problems and difficulties mentioned above initiated the development of a finite element based 3D numerical model for the calculation of velocity and deformation fields for aluminum strip during hot rolling.

2. Basic equations

The reference system (x, y, z) is defined by the traveling direction, the width and the thickness of the strip. The velocity at a given point P of the strip is given by

$$\mathbf{v}\,(x,y,z) = v_x\,(x,y,z)\,\mathbf{e}_x + v_y\,(x,y,z)\,\mathbf{e}_y + v_z\,(x,y,z)\,\mathbf{e}_z, \tag{2.1}$$

where $\mathbf{e}_x, \mathbf{e}_y$ and $\mathbf{e}_z$ are the unit vectors in the reference system.

With the velocity field $\mathbf{v}$,

$$\dot{\varepsilon} = \frac{1}{2}\left(\mathbf{v}\circ\nabla + \nabla\circ\mathbf{v}\right) \tag{2.2}$$

is the strain rate tensor. According to the Levy-Mises assumption [2] the equation that relates the strain rate to the stresses for hot aluminum rolling is

$$\dot{\varepsilon}_d = \lambda\sigma_d \tag{2.3}$$

and

$$\dot{\varepsilon}_I = 0 \tag{2.4}$$

where $\dot{\varepsilon}_d$ and σ_d are the deviatoric parts of the strain rate and stress tensors, $\dot{\varepsilon}_I$ is the first scalar invariant of the strain rate tensor and λ is given by

$$\lambda = \sqrt{3}\frac{\dot{\varepsilon}_e}{\sigma_e} \tag{2.5}$$

in which $\dot{\varepsilon}_e$ is the effective strain rate

$$\dot{\varepsilon}_e = \sqrt{\dot{\varepsilon} \cdot \cdot \dot{\varepsilon}/2}\,. \tag{2.6}$$

Here double dots stand for the energy product of two tensors. Based on the hot twisting test the effective stress can be given in the following form

$$\sigma_e = \frac{1}{\alpha}\,\mathrm{ash}\left[\left(\frac{\dot{\varepsilon}_e e^{\frac{Q}{RT}}}{A}\right)^{\frac{1}{N}}\right] \tag{2.7}$$

where α, A, N and Q are constants determined from the tests, R is the universal gas constant and T is the absolute temperature.

Table of parameters for the yield stress model				
Alloy code	A	α	N	Q
1050	$3.8 \cdot 10^{11}$	0.0462	3.84	156700
3003	$8.0 \cdot 10^{11}$	0.0311	4.48	167600
545	$1.7 \cdot 10^{10}$	0.0551	2.74	165400

Making use of the notations

$$\overline{\sigma}^T = [\sigma_x,\ \sigma_y,\ \sigma_z,\ \tau_{xy},\ \tau_{xz},\ \tau_{yz}] \qquad \overline{\varepsilon}^T = [\dot{\varepsilon}_x,\ \dot{\varepsilon}_y,\ \dot{\varepsilon}_z,\ \dot{\gamma}_{xy},\ \dot{\gamma}_{xz},\ \dot{\gamma}_{yz}] \tag{2.8}$$

$$\mathbf{C}_0 = \langle\ 2,\ \ 2,\ \ 2,\ \ 1,\ \ 1,\ \ 1\ \rangle \qquad \mathbf{m}^T = [\ 1,\ \ 1,\ \ 1,\ \ 0,\ \ 0,\ \ 0,\ ] \tag{2.9}$$

equation (2.3) can be cast into the form

$$\overline{\sigma} = \frac{1}{\lambda}(\mathbf{C}_0 - \frac{2}{3}\mathbf{m} \circ \mathbf{m}^T)\overline{\varepsilon} + \mathbf{m}\overline{\sigma}_h = \frac{1}{\lambda}\mathbf{C}\overline{\varepsilon} + \mathbf{m}\overline{\sigma}_h \tag{2.10}$$

where $\mathbf{C} = \mathbf{C}_0 - 2\mathbf{m} \circ \mathbf{m}^T/3$ and $\overline{\sigma}_h$ is the hydrostatic pressure. The principle of virtual power with an additional penalty term representing the volume change is of the form

$$\int_V \delta\overline{\varepsilon}^T \frac{1}{\lambda}\mathbf{C}\,\overline{\varepsilon}\,\mathrm{d}V + \int_V \delta\overline{\varepsilon}^T \mathbf{m}\overline{\sigma}_h\,\mathrm{d}V + \beta\int_V \delta\overline{\varepsilon}^T\frac{1}{\lambda}\mathbf{m} \circ \mathbf{m}^T\overline{\varepsilon}\mathrm{d}V = \delta W \tag{2.11}$$

where β is a penalty parameter for the numerical calculations in the range of $10^4 - 10^6$ and δW is the power of the external forces. In addition, the weak form of the incompressibility condition can be formulated as

$$\int_V \delta\overline{\sigma}_h \mathbf{m}^T\overline{\varepsilon}\mathrm{d}V = 0\,. \tag{2.12}$$

3. Finite element model

3.1. Finite element approximation. In a given finite element the approximation of the velocity field is calculated using the matrix of shape functions $\mathbf{N}(\xi, \eta, \varsigma)$ regarding the order of the shape functions (p) [3] (for the calculations presented in this paper $p=4$ and $p=5$ elements were applied) and the vector of coefficients $\overline{\mathbf{q}}$ such as

$$\mathbf{v}=\mathbf{N}\left(\xi,\eta,\varsigma\right)\overline{\mathbf{q}} \tag{3.1}$$

similarly, the hydrostatic pressure can be approximated by linear shape functions as

$$\overline{\sigma}_h=\mathbf{P}\left(\xi,\eta,\varsigma\right)\overline{\mathbf{p}}$$

where ξ,η and ς are the local coordinates of the element.

It follows from (2.2) by making use of the representation (3.1) that $\overline{\varepsilon}$can be expressed as

$$\overline{\varepsilon}=\mathbf{B}\left(\xi,\eta,\varsigma\right)\overline{\mathbf{q}} \tag{3.2}$$

where $\mathbf{B}\left(\xi,\eta,\varsigma\right)$ is the matrix of the modified shape functions. Substituting this expression into the variational equations (2.11) and (2.12) we have

$$\delta\overline{\mathbf{q}}^T\underbrace{\int_V\frac{1}{\lambda}\mathbf{B}^T\mathbf{C}_0\mathbf{B}J\,\mathrm{d}V}_{\mathbf{K}}\mathbf{q}+\delta\overline{\mathbf{q}}^T\underbrace{\beta\int_V\mathbf{B}^T\left(\mathbf{m}\circ\mathbf{m}^T\right)\mathbf{B}J\,\mathrm{d}V}_{\widetilde{\mathbf{K}}}\overline{\mathbf{q}}$$

$$+\,\delta\overline{\mathbf{q}}^T\underbrace{\int_V\mathbf{B}^T\mathbf{m}\mathbf{P}J\,\mathrm{d}V}_{\mathbf{G}}\,\overline{\mathbf{p}}=\delta W \tag{3.3}$$

and

$$\delta\overline{\mathbf{p}}^T\underbrace{\int_V\mathbf{P}^T\mathbf{m}^T\mathbf{B}J\,\mathrm{d}V}_{\mathbf{G}^T}\overline{\mathbf{q}}=0 \tag{3.4}$$

where J is the Jacobian of the element considered.

3.2. Boundary conditions. The work roll circumferential velocity $\mathbf{v}_r=v_r\mathbf{t}$ is given in the local coordinate system formed by the tangent of the roll $\mathbf{t}$, the width direction $\mathbf{e}_y$ and the normal vector of the roll surface $\mathbf{n}$. All the nodal point values of the roll-strip contact region are transformed into this local system. In the bite the velocity of the roll surface and the strip surface points slightly differ from each other. On the entry side the velocity of the roll point is higher (backward slip) and in the exit zone the strip point has a higher velocity (forward slip). The strip-roll interaction along the bite length depends on the relative velocity difference between the strip and the roll (hydrodynamic lubrication model). The traction at a point of the bite is given in the form

$$\mathbf{f}^*=\kappa\left(\mathbf{v}_r-\mathbf{v}\right)=\kappa\mathbf{v}_r-\langle\kappa,\kappa,0\rangle\,\mathbf{v}=\kappa\mathbf{v}_r-\mathbf{C}^*\mathbf{v} \tag{3.5}$$

where κ is the hydrodynamic coefficient. The power of the external forces is approximated as

$$\delta W=\delta\overline{\mathbf{q}}^T\int_A\kappa\mathbf{N}^T\mathbf{v}_rJ^*\mathrm{d}A-\delta\overline{\mathbf{q}}^T\int_A\mathbf{N}^T\mathbf{C}^*\mathbf{N}J^*\mathrm{d}A\overline{\mathbf{q}}=\delta\overline{\mathbf{q}}^T\left(\overline{\mathbf{f}}-\mathbf{K}^*\overline{\mathbf{q}}\right) \tag{3.6}$$

in which $\bar{\mathbf{f}}$ is the load vector of the element, J^* is the Jacobian of the surface element and $\mathbf{K}^*$ is an additional term which should be added to $\mathbf{K}$ in (3.3).

The normal velocity of the contact points is zero.

3.3. Iterative algorithm. From the variational equations (3.3), (3.6) and (3.4) by assembling the corresponding matrices $\left(\mathbf{K} \to \mathbf{K}^{\Sigma}, \widetilde{\mathbf{K}} \to \widetilde{\mathbf{K}}^{\Sigma} \text{ and } \mathbf{G} \to \mathbf{G}^{\Sigma}\right)$ and introducing the global coefficient vector $\mathbf{q}$, the vector of global pressure coefficients $\mathbf{p}$ and the global generalized load vector $\mathbf{f}$, we obtain the equations

$$\left(\mathbf{K}^{\Sigma} + \widetilde{\mathbf{K}}^{\Sigma}\right) \mathbf{q} + \mathbf{G}\mathbf{p} = \mathbf{f} \tag{3.7}$$

and

$$\mathbf{G}^{\Sigma T} \mathbf{q} = 0\,. \tag{3.8}$$

which can be solved by applying an iterative approach [4] which fulfills a termination criteria $|\mathbf{q}_{n+1} - \mathbf{q}_n| \,/\, |\mathbf{q}_n| < \tau$ (for our numerical calculations $\tau = 10^{-3}$). The integrals are evaluated by using a Gauss type numerical quadrature of order $p+1$ where p is the order of the shape functions that approximate the velocity field.

In the $(n+1)$-th step the global velocity and pressure coefficient vectors $(\mathbf{q}_n, \mathbf{p}_n)$ are determined and

$$\mathbf{p}_{n+1} = \mathbf{p}_n + \rho \mathbf{r}_n \tag{3.9}$$

where $\rho = \lambda_{\min}$ is a factor to improve convergence. The residual of (3.8) is calculated by

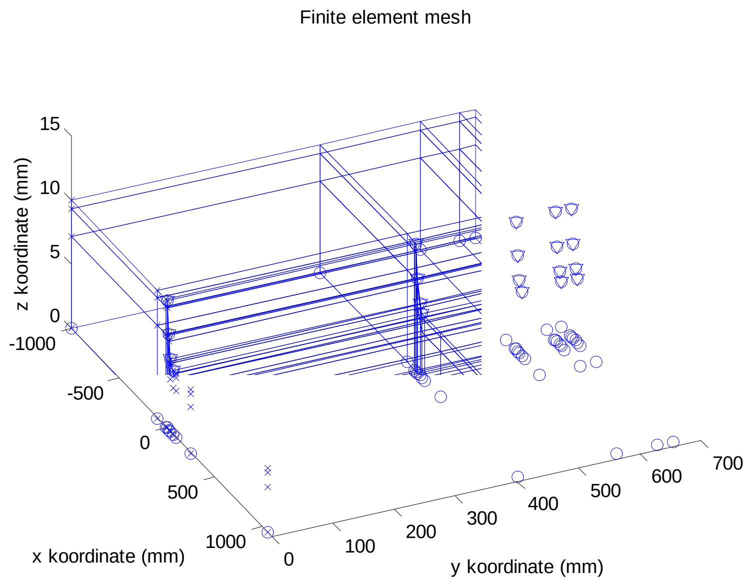
$$\mathbf{r}_n = \mathbf{G}^{\Sigma T} \mathbf{q}_n\ . \tag{3.10}$$

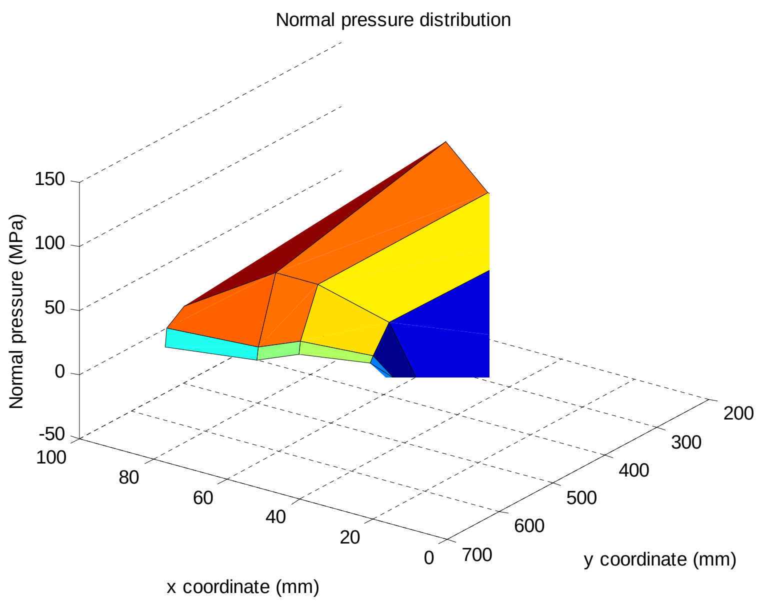
Utilizing equations (3.2) and (2.5)-(2.7) we can determine λ at the integration points. If we substitute these values into the expression of $\mathbf{K}$ then, according to the first term of (2.11), the following equation is obtained for $\mathbf{q}_{n+1}$:

$$\left(\mathbf{K}_n^{\Sigma} + \widetilde{\mathbf{K}}^{\Sigma}\right) \mathbf{q}_{n+1} = \mathbf{f} - \mathbf{G}\mathbf{p}_{n+1}\ . \tag{3.11}$$

4. Results

Computational results are compared with hot rolling data of aluminum strips on a four high single stand reversing hot mill. The applied finite element mesh can be seen in Figure 1. With regard to the symmetry conditions – the planes xy and xz are that of symmetry – only one quarter of the strip was considered in our computations. A summary of the measured parameters and the computed values for a 1320 mm wide 1050 alloy strip rolled at 1 m/s speed using 800 mm diameter rolls is given in the table below:


Finite element mesh
15
10
5
0
z koordinate (mm)
-1000
-500
0
500
1000
x koordinate (mm)
0
100
200
300
400
500
600
700
y koordinate (mm)


Normal pressure distribution
150
100
50
0
-50
Normal pressure (MPa)
100
80
60
40
20
0
x coordinate (mm)
700
600
500
400
300
200
y coordinate (mm)

Comparison between predicted and measured separation forces				
Temp. $[C^0]$	Entry thick. [mm]	Exit thick. [mm]	Predicted force [kN]	Measured force [kN]
280	16	8	11359	10050
325	16	8	9766	9621
370	16	8	8048	8447
325	16	10	6160	6366

The typical gap pressure distribution indicated in Figure 2 is comparable with the prediction of other types of models (for example: Kármán model) and measurements.

5. Conclusion

A new, effective 3D numerical model has been developed for the steady-state investigation of hot rolling of aluminum strips first time applying p version finite element successfully for the approximation of the velocity field and for the computation of the deformation rates. Hydrodynamic lubrication was assumed for the connection between the strip and the roll. By the development of this model all shortcomings of the 1D and 2D model discussed in the introduction have been eliminated.

The model allows us to predict the separation force, forward and backward slip and the required power/torque for the deformation. Those pieces of information can be utilized for developing, modifying and optimizing pass schedules.

Based on the comparisons of the calculated and measured separation force values it can be concluded that the model predictions are close to the real values and the differences are less than 12%. Based on the practical experiences the differences of the measured and predicted values are within the measurement error range and the predictions are suitable for practical applications.

The MATLAB software was applied for the implementation of the numerical calculations. The built-in spare matrix feature was utilized successfully to reduce the memory requirement of the computations. The graphical output provides a powerful tool for visualizing the computational results.

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