Estimation of Center of Inertia and Center of Power from WAMS and SCADA Measurements

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***Abstract*—The increasing share of distributed renewable generation is reshaping the dynamics of modern power systems, necessitating more accurate and adaptive monitoring solutions. Wide Area Monitoring Systems (WAMS), based on Phasor Measurement Units (PMUs), provide high-resolution system-wide observability, enabling advanced applications. A critical input to these functions is the system frequency, which is often taken to be equal to the frequency of Center of Inertia (COI), in which the exact estimation remains an open question due to varying definitions of inertia-based weighting. This paper investigates alternative formulations using inertia constants and kinetic energy terms to determine the most representative system frequency. To support the evaluation, the spatial behavior of the Center of Power (COP) is also analyzed using SCADA-based power flow data. Results derived from real PMU and SCADA measurements show that the H·S-based COI formulation better aligns with the actual measured COP behavior.**

***Keywords*— Center of Inertia, Center of Power, Wide Area Monitoring Systems, Phasor Measurement Unit**

I. Introduction

The increasing penetration of renewable energy sources is fundamentally transforming the operational dynamics of power systems, introducing variability, reducing inertia, and creating new challenges in maintaining system stability. Traditional grid architectures, characterized by centralized generation and unidirectional power flow, are no longer sufficient to ensure reliable operation in this new landscape. To address these challenges, enhanced system observability becomes crucial. Wide-area monitoring systems (WAMS), based on synchronized measurement technology, offer the capability to provide high-resolution, time-aligned data across geographically dispersed locations, enabling real-time assessment of system dynamics **[1][2]**. Phasor measurement units (PMUs), as the cornerstone of WAMS, have proven effective in increasing situational awareness and enabling advanced applications such as adaptive protection, stability monitoring, and state estimation, even under incomplete observability conditions **[3]**. As highlighted by industry developments and recent case studies, the deployment of WAMS is not merely a technological enhancement but a necessary evolution toward a smarter, more resilient grid, especially as systems face faster dynamic cycles and more complex interconnections **[4]**.

A. Center of Inertia and Center of Power

The Center of Inertia (COI) and it's frequency value offers a physically meaningful and system-wide representative indicator of grid frequency, particularly under dynamic conditions such as generator outages or abrupt load changes. Unlike individual frequency measurements, which can be highly location-dependent due to electromechanical wave propagation and oscillations, the COI frequency aggregates the behavior of all rotating masses in the system, weighted by their inertia **[5]**-**[12]**. However, the definition of inertia-based weighting is not always consistent. While **[5][6][7]** and **[10]** and introduce the COI calculation using inertia time constants as weights (see Eq. (1)), **[8][9][11]** and **[12]** define the weights based on the kinetic energy of the generators (Eq. (2)). In this paper, PMU measurements are utilized to compare the two approaches using real input data.

$$f_{COI} = \frac{\sum_{i=1}^{n} H_i f_i}{\sum_{i=1}^{n} H_i}\ [Hz] \tag{1}$$

$$f_{COI} = \frac{\sum_{i=1}^{n} H_i S_i f_i}{\sum_{i=1}^{n} H_i S_i}\ [Hz] \tag{2}$$

Where f_{COI} is the estimated frequency of the Center of Inertia [Hz], H_i is the inertia constant, S_i is the rated apparent power and f_i is the local frequency of the generation unit.

Since the PMU measurements used in this study are located at substations rather than directly at the generator buses, an alternative evaluation approach was required to assess and compare the resulting reference frequency representations. An extended analysis is carried out, incorporating power-based weighting in addition to the inertia-based approach. In this case, the so-called Center of Power (COP) frequency is derived, which reflects the influence of active power flow on the aggregated frequency dynamics. The COP frequency provides an alternative viewpoint for system-wide frequency representation, especially in cases where real power transfer plays a dominant role, such as in areas with high non-synchronous renewable penetration.

$$f_{COP} = \frac{\sum_{i=1}^{n} P_i f_i}{\sum_{i=1}^{n} P_i}\ [Hz] \tag{3}$$

Where f_{COP} is is the frequency of the Center of Power [Hz], P_i is is the active power associated with the i unit or measurement point [MW] and f_i is the local frequency measured at the i unit or substation [Hz].

The concept of Center of Power (COP) was originally proposed in **[13]**, where it was developed as a novel frequency reference for defining dynamic stability metrics based on generator-level data. In that work, the COP was formulated as a power-weighted average of rotor speeds and internal angles, enabling the construction of indicators for transient instability detection. While their application focused on generator terminals and coherent machine groups, the COP framework is adapted here to substation-level measurements using PMU-based frequency data and SCADA-derived active power flows. The COP weighting used in source **[13]** is presented in

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equation (4), where P_e is the active power injection from generator, ω_j is the rotor angular frequency of the generator and ω_{COP} is is the COP angular frequency.

$$\omega_{COP} = \frac{\sum_{j=1}^{N_g} \omega_j \cdot P_{ej}}{\sum_{j=1}^{N_g} P_{ej}} \tag{4}$$

In this paper, the effect of different weighting strategies applied to WAMS frequency measurements on the estimation of system-wide reference frequencies is analyzed, and the temporal behavior of the COP location is examined under real operating conditions of the Hungarian transmission system. The analysis is based on actual PMU frequency measurements with 100 ms resolution and SCADA-based active power data with a 1-minute sampling interval. To evaluate the appropriateness of different system-wide frequency reference formulations, the Center of Inertia calculations based on inertia constants *(H)* and kinetic energy terms *(H·S)* are compared. The assessment was based on how well the resulting COI frequency aligned spatially and temporally with the independently computed COP, derived from actual power flow data. The results indicate that the H·S-based COI formulation exhibits a closer correspondence with the COP trajectory, supporting its use as a more accurate system-wide frequency representation under real operating conditions.

II. Methodology

This study utilizes high-resolution synchrophasor data collected from 16 PMUs deployed across the transmission network. The analyzed dataset spans a 25-hour and a 73-hour measurement window from January 31, 2025, 23:00 UTC to February 2, 2025, 00:00 UTC and from January 31, 2025, 23:00 UTC to February 4, 2025, 00:00 UTC, with a sampling resolution of 100 milliseconds. As several PMUs were located in the same substation, they were merged. Thus, 12 different measurement points were used for calculations.

A. *System frequency comparison published by TSO*

The System frequency published by the Hungarian TSO (MAVIR) is used as a system-wide reference signal. While the relationship between the PMU-based measurements and this reference is formally nonlinear, due to the weighted mean involving the weights both in the numerator and denominator, this nonlinearity can be bypassed using a linear least squares approach. Specifically, the weights can be determined so that the weighted average of the PMU frequencies best approximates the MAVIR-provided system frequency f_{sys} at each time step.

$$f_{sys_{TSO}}(t) = \frac{\sum_{i=1}^{12} f_i(t) \cdot w_i}{\sum_{i=1}^{12} w_i} \; every \; t = 1, \dots, 1499 \tag{5}$$

This is equivalent to the following (6) linear formulation.

$$f_{sys_{TSO}}(t) \cdot \sum_{i=1}^{12} w_i = \sum_{i=1}^{12} f_i(t) \cdot w_i \tag{6}$$

In this analysis, the weights assigned to each PMU location were determined using the least squares method, a standard linear regression technique. The objective was to find a weight vector w such that the weighted combination of 12 frequency measurements closely approximates the reference frequency published. Mathematically, this means solving the overdetermined system $X_w \approx y$, where X is a matrix of size 1499 by 12, containing 1-minute averaged frequencies from 12 PMUs, and y is the 1499-sample reference frequency vector. The least squares solution minimizes the squared error $\|Xw - y\|_2^2$. The resulting weight vector provides the optimal linear combination of the PMU measurements that best fits the target.

The calculated weight magnitudes are illustrated in Figure 1. Some measurement points have negative weights. This outcome is acceptable within the mathematical framework, as the weights were obtained using an unconstrained least squares method. Negative values may occur if a measurement point consistently deviates below the system frequency, thus minimizing the overall fitting error. The figure shows the percentage contribution of each PMU-equipped substation to the system frequency estimation. The values indicate how much each location contributes to the weighted combination that best approximates the TSO-published reference frequency. In this approach, the goal is to find the best linear combination of the PMU frequencies that approximates the reference frequency. Subsequently, the same mathematical procedure was applied to an extended 73-hour data (Figure 2).

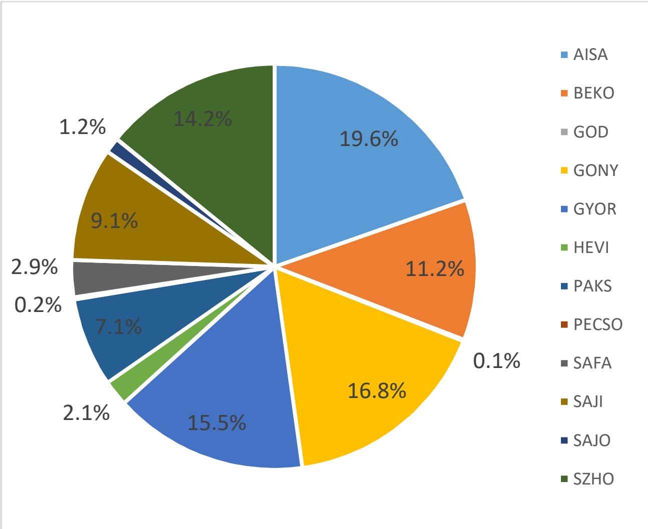

Fig. 1. Contribution to the value of the system frequency in abs value (25-hour measurement window)

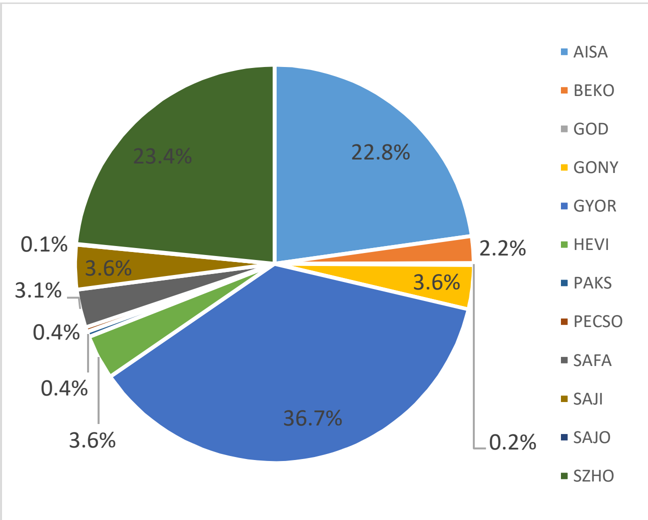

Fig. 2. Contribution to the value of the system frequency in abs value (73-hour measurement window)

B. *Mathematical center movement in time*

To track how the different system's reference frequency definitions changes over time, the weight estimation process was repeated using a 12-point moving window. For each window position, a new set of weights was calculated based on the PMU measurements and the MAVIR reference frequency. This resulted in a time series of weight vectors, capturing the temporal variation in the contribution of each measurement location. The evolution of these weights provides insight into how the system's frequency-dominant measurement regions shift over time. This is shown in Figure 3.

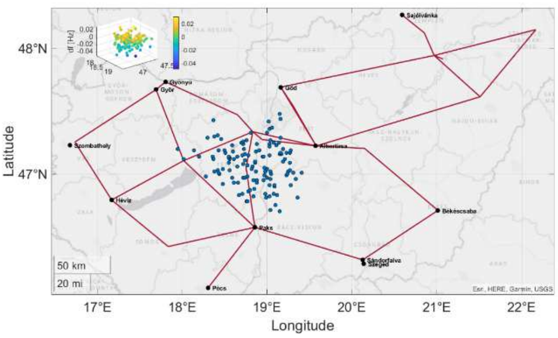

Fig. 3. Spatial Deviation of Reconstructed Frequency from Nominal 50 Hz (25-hour measurement window)

C. *Center of Power calculations*

Active power flow data provided by MAVIR served as the basis for calculating the Center of Power (COP) using two distinct methods. The first method involved computing half the sum of the absolute values of incoming and outgoing active power flows at each PMU-equipped substation. This value represents the total power volume passing through the node, independent of direction, and indicates the general magnitude of power flow. The second method considered only downward-directed active power, specifically the 400/132 kV transformer flows, which effectively reflect local energy consumption and enable a consumption-based interpretation of the COP. The analysis of both formulations allows for assessing how the definition of nodal power contribution affects the spatial behavior of the estimated reference frequency.

Having these power magnitudes allowed us to compute the spatial coordinates of the center of power at each time step by taking the power-weighted average of the EOV (Hungarian national grid coordinate system) coordinates of the substations. This resulted in a spatiotemporal trajectory representing the movement of the system's power center, as described in equations (7) and (8).

Furthermore, the same power values were used as temporal weighting factors to calculate the power-weighted average frequency of the system at each time step (3)..

$$x_{COP} = \frac{\sum_{i=1}^{n} P_i x_i}{\sum_{i=1}^{n} P_i} \tag{7}$$

$$y_{COP} = \frac{\sum_{i=1}^{n} P_i y_i}{\sum_{i=1}^{n} P_i} \tag{8}$$

III. Results and Discussion

Based on Eqs. (7) and (8), the spatial location of the COP can be determined for each time step. This calculation yields a dynamic trajectory that reflects the shifting center of real power flow within the transmission system. The resulting time-varying COP coordinates are illustrated in Figure 4. As power distribution changes across the substations, the COP moves accordingly, providing a spatial representation of system activity that complements frequency-based analysis.

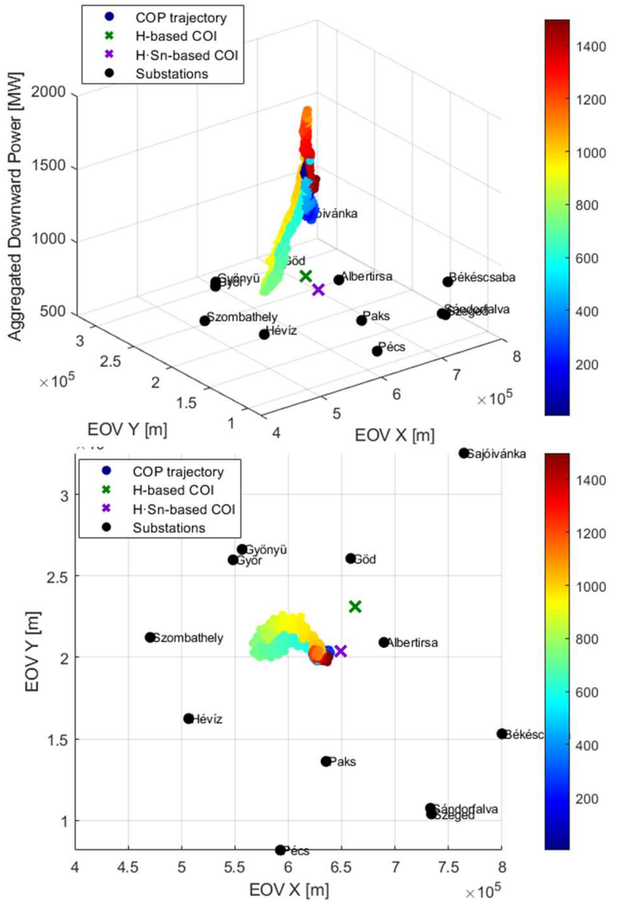

Fig. 4. Trajectory of Consumption-weighted Center of Power (400→120) (25-hour measurement window)

As illustrated in Figure 4, the spatial distance between the COP and the assumed COI increases when the aggregated 400/132 kV power flow decreases. This typically coincides with time intervals characterized by high solar generation, during which decentralized production leads to a lower net load on the transmission-distribution connection point.

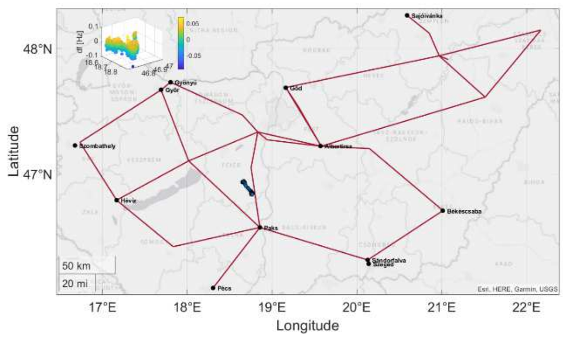

Fig. 5. Trajectory of Inertia-Weighted Center of Inertia Based on Real Measurement Data (25-hour measurement window)

The COI calculation based on Eq. (2) was applied using real measurement data. The results indicate that the COI position is strongly influenced by the largest synchronous generator within the examined network, which is the Paks Nuclear Power Plant. As shown in Figure 5, this dominant generation unit exerts a significant pull on the COI, shifting its spatial location closer to the site of major inertia and power contribution.

The observed shifts imply that with increasing levels of distributed generation, the actual center of system activity may differ from that inferred solely from synchronous generator inertia. This highlights the importance of integrating real-time power flow data into spatial system stability assessments. Figure 6 illustrates this effect by showing the COI–COP distances with hourly resolution.

It should be noted that the analysis is limited to the Hungarian part of the Continental Europe synchronous area, and the COI estimation is based on substation-level frequency measurements rather than direct generator data. As such, the computed COI represents an approximate reference point rather than a physically exact center of rotating inertia.

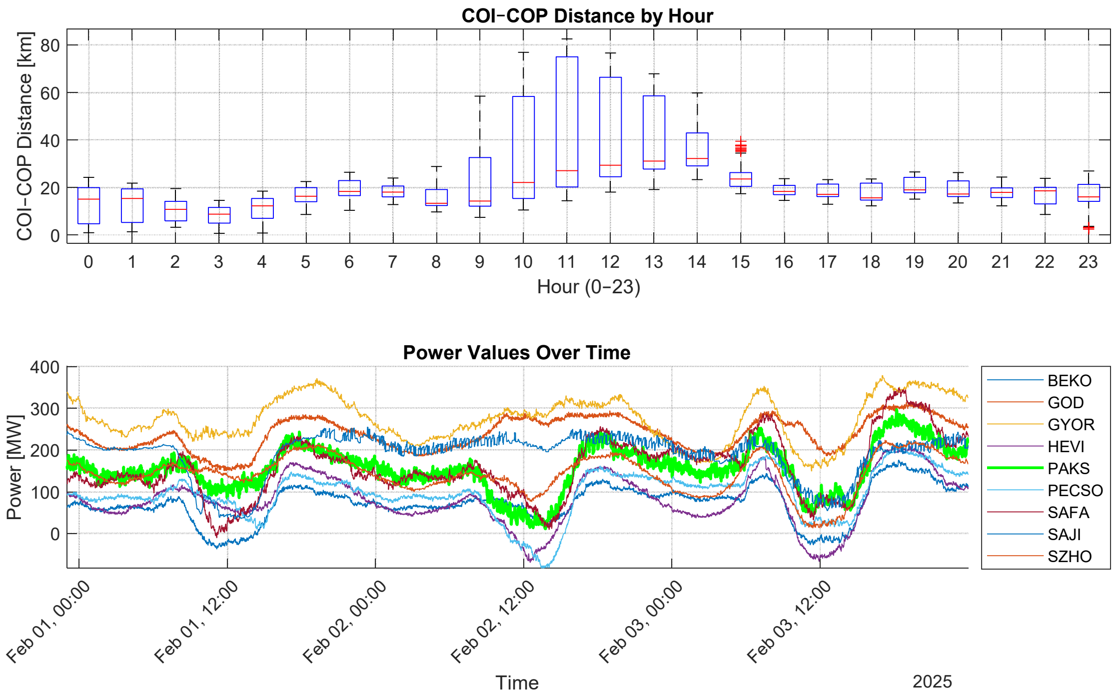

Fig. 6. Hourly COI–COP Spatial Deviation and Regional Consumption Patterns Based on 400/132 kV Transformer Flows (73-hour measurement window)

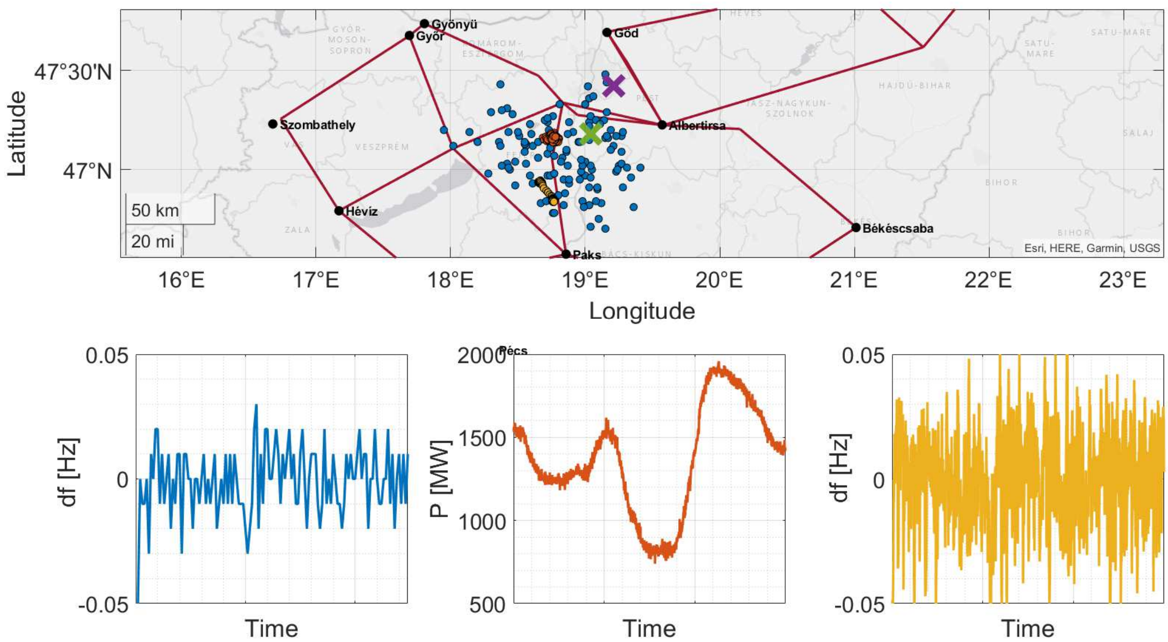

Fig. 7. Summary figure of previous investigations 25-hour measurement window

Figure 7 summarizes the main results of the 25-hour measurement window. The top panel shows the Hungarian transmission network with the locations of the PMU-equipped substations. The blue, red, and yellow markers indicate the centers calculated using mathematical weighting, 400/120 kV power-based weighting, and inertia-based weighting, respectively. The bottom panels present the difference between the reconstructed and the actual system frequency. The left figure corresponds to the mathematically weighted reconstruction (blue), while the right figure shows the same difference for the inertia-weighted reconstruction (yellow).

IV. Conclusion

As power systems continue to evolve due to the increasing share of distributed renewable generation, system-wide monitoring becomes essential to understand the changing frequency behavior. Wide area monitoring systems offer high-resolution, geographically distributed data that enable the evaluation of aggregated frequency indicators such as the Center of Inertia and the Center of Power. A key input to these evaluations is the COI frequency, yet its appropriate formulation remains an open question due to inconsistencies in the literature regarding inertia-based weighting methods.

This study compares two commonly used approaches: one based solely on inertia constants (H), and another incorporating kinetic energy terms (H·S). In addition, a COP-based formulation is introduced as a novel, power-flow-driven reference, offering an alternative perspective by accounting for the distribution of active power. The spatial alignment between the COI and the COP under real operating conditions is analyzed, revealing that the H·S-based formulation provides a more consistent spatial correspondence with the power distribution. While the comparison provides useful insights, the methodological ambiguity in COI estimation remains unresolved. The closer alignment of the H·S-based COI with the COP suggests that kinetic energy–based weighting may offer a more physically meaningful alternative, but further research is needed to validate its general applicability.

These findings suggest that power flow–based indicators, such as the COP, can provide valuable complementary insights when assessing system-wide frequency behavior. Although the movement of the COP does not directly imply inaccuracies in the COI, it reflects the shifting centers of system activity, especially under high levels of non-synchronous generation. Therefore, COP monitoring may serve as a useful complementary tool in future WAMS-enabled grid applications.

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