

A Dynamic Balance: Leveraging HVDC and the Kuramoto Method in Power Grid Stability

Á. Weidinger¹, B. Hartmann¹ and I. Vokony¹

¹ Department of Electric Power Engineering
Budapest University of Technology and Economics
Műegyetem rkp. 3., H-1111 Budapest, Hungary

Abstract. Power grid stability has become an increasingly intricate challenge with the rising integration of renewable energy sources like wind and solar power. These energy sources, while essential for achieving sustainability goals, introduce significant variability and reduce the system's inertia due to the displacement of conventional synchronous generators. This paper explores the combined role of High-Voltage Direct Current (HVDC) systems and the Kuramoto model in addressing these challenges. The Kuramoto model, traditionally used to study synchronization phenomena, is adapted to represent power system dynamics, enabling simulation of grid stability under diverse conditions. HVDC systems, with their ability to regulate power flows dynamically and reduce instability, are evaluated for their potential to mitigate fluctuations caused by renewable energy. Key topics include the theoretical underpinnings of grid stability, detailed simulation methodologies, experimental scenarios, and results showcasing HVDC's stabilizing effects. Conclusions emphasize the synergy between advanced control systems and theoretical modelling, providing a pathway toward resilient and adaptable future energy networks.

Key words. HVDC technology, Kuramoto-model, stability analysis of power systems

1. Introduction

Modern electrical power systems are undergoing a fundamental transformation driven by the global push for decarbonization and the widespread adoption of renewable energy. These changes present a dual challenge: ensuring adequate system stability and maintaining reliable operation amidst increasing grid complexity. Traditional grids, dominated by synchronous generators, offered inherent stability through large rotating masses. However, the integration of power electronic converters and weather-dependent renewables disrupts these dynamics, necessitating novel solutions.

The Kuramoto model, a mathematical framework for studying synchronization in oscillator networks, offers a valuable lens to explore power grid stability. By modelling the dynamics of generator phases, this method can capture the intricate interdependencies within modern grids. Concurrently, HVDC systems, capable of precise power flow control, emerge as a practical tool to counteract

instability and enhance resilience. This paper bridges these theoretical and practical approaches, offering a comprehensive exploration of their combined application.

2. Methodology

A second-order Kuramoto model was employed to simulate grid synchronization under various operational scenarios. The model adapts classical oscillator dynamics to reflect power systems, incorporating variables like coupling strength, natural frequencies, and inertia terms. The Kuramoto model, first proposed to study phase synchronization in coupled oscillators as described in [1], is adapted here to analyse the dynamics of power grids.

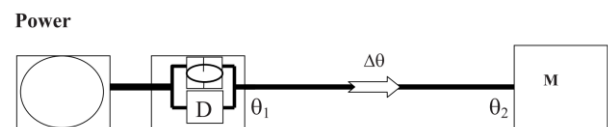


Fig. 1. Equivalent diagram of generator and machine connected by a transmission line. The turbine consists of a flywheel and dissipation D. [2]

The following detailed explanation of the adjusted second-order Kuramoto model is based on the literature review of the model presented in [2]. In Fig. 1, we consider the basic elements of a power grid: an active generator, denoted by a circle (G), and a passive machine, denoted by a square (M). The power generated, P_{source} is made by the turbine of the generator with a frequency, which is near to Ω (the standard frequency, typically 50 Hz). This energy at the output generator leads to a phase angle θ_1 . So, the equation that describes the system in Fig. 1 follows:

$$\theta_1 = \Omega t + \tilde{\theta}_1, \quad (1)$$

where Ω represents the system's standard frequency, and $\tilde{\theta}_1$ is a small perturbation, which causes the small frequency difference at the generator. During the operation, the turbine of the generator dissipates energy at a rate proportional to the square of the angular velocity, given by:

$$P_{diss} = K_D (\dot{\theta}_1)^2, \quad (2)$$

where K_D is a damping coefficient. Additionally, the turbine can store kinetic energy at a rate of:

$$P_{acc} = \frac{1}{2} I \frac{d}{dt} (\dot{\theta}_1)^2, \quad (3)$$

where I represents the moment of inertia. The power transmitted between the active generator and the passive machine depends on the phase difference $\Delta\theta = \theta_2 - \theta_1$ (as the oscillators have the same standard frequency, the mismatch between the perturbations $\Delta\tilde{\theta}$ is equals to $\Delta\theta$). As follows, the transmitted power is given by:

$$P_{transmitted} = -P^{MAX} \sin(\Delta\theta). \quad (4)$$

(The negative term is because of the producer point of view.)

This transmission of power and the interaction between grid components form the basis for the following power balance equation, which describes the behaviour of each generator or machine:

$$P_{source} = P_{acc} + P_{diss} + P_{transmitted}. \quad (5)$$

Substituting the expressions for P_{acc} , P_{diss} and $P_{transmitted}$ into the balance equation above, we obtain:

$$P_{source} = I\ddot{\theta}_1 + K_D(\dot{\theta}_1)^2 - P^{MAX} \sin(\Delta\theta). \quad (6)$$

In the context of small deviations from the synchronous frequency, $\dot{\theta}_1 \ll \Omega$ (operating in close proximity to 50Hz), equation (6) can be further simplified to:

$$P_{source} = I\ddot{\theta}_1 + [I\ddot{\theta}_1 + 2K_D\Omega] \dot{\theta}_1 + K_D\Omega^2 - P^{MAX} \sin(\Delta\theta). \quad (7)$$

By assuming that the coefficient of the first derivative is constant, we can neglect the term related to acceleration ($\ddot{\theta}_1 \ll \frac{2K_D\Omega}{I}$). (Practically, the rate of energy storage in the kinetic term ($\approx I\ddot{\theta}_1$) is significantly lower than the rate of energy dissipation caused by friction ($\approx K_D(\dot{\theta}_1)^2 \approx K_D\Omega^2$)). Focusing on the dissipation and synchronisation dynamics, equation (7) simplifies to:

$$I\Omega\ddot{\theta}_1 = P_{source} - K_D\Omega^2 - 2K_D\Omega\dot{\theta}_1 + P^{MAX} \sin(\Delta\theta), \quad (8)$$

which leads to a normalized form:

$$\ddot{\theta}_1 = P - \alpha\dot{\theta}_1 + P^{MAX} \sin(\Delta\theta), \quad (9)$$

where $\alpha = \frac{2K_D\Omega}{I}$ represents the damping term, and the equation becomes formally identical to the fundamental component of the Kuramoto model.

Until this point, the analysis of stability focused on deviations from the predetermined frequency Ω of the grid, assessing the impacts of these deviations from uniform rotation. However, by mapping the system onto a Kuramoto model, a more extensive prediction of dynamic stability can be achieved, without relying on a fixed frequency assumption. The Kuramoto model aligns with the complex issue of mutual synchronisation among different oscillators, particularly in cases where frequency synchronisation cannot be assured. Assuming the common frequency Ω is subtracted, each oscillator tends to maintain a natural average velocity that is approximately equal to Ω , with the deviations represented by the frequency ω_i :

$$\langle \dot{\theta}_i \rangle = \Omega + \omega_i. \quad (10)$$

The ω_i values represent small spreads around zero, indicating that power generators can provide AC voltages with frequencies slightly deviating from the standard reference, as illustrated in Fig. 2.

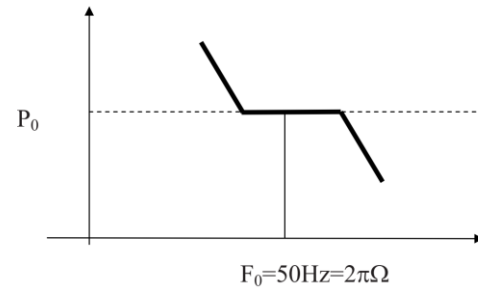


Fig. 2. Power-frequency characteristic of electrical plants. [2]

The frequency shift in model equation (8) arises from the diversity of the oscillators in the model, that represent generators and machines, which are mainly spread around two frequency peaks - positive for generators and negative for machines.

In conclusion, in its second-order form, the Kuramoto model incorporates inertia, making it particularly suitable for modelling synchronous generators and consumer loads. The equations governing the model after final adjustments and simplifications, introduced in [3] is expressed as:

$$\dot{\theta}_i(t) = \omega_i(t), \quad (11)$$

$$\dot{\omega}_i(t) = \omega_{i,0} - \alpha\dot{\theta}_i(t) + \frac{K}{N_i} \sum_j A_{ij} \sin[\theta_i(t) - \theta_j(t)]. \quad (12)$$

where:

- θ_i : Phase angle of the i -th oscillator,
- $\omega_{i,0}$: Natural frequency of the i -th oscillator,
- K : Coupling strength,
- α : Damping coefficient,
- A_{ij} : Adjacency matrix weighted by admittance,
- N_i : Number of connected nodes at node i .

The natural frequencies $\omega_{i,0}$ (the frequency at which the oscillator would naturally oscillate if it were not influenced by other oscillators or external forces) are Gaussian-distributed, capturing the inherent heterogeneity in the grid. A fourth-order Runge-Kutta method is employed to solve these equations, ensuring numerical stability. Simulations begin from either fully synchronised or desynchronised initial states to explore how the system evolves towards synchronisation under varying coupling conditions.

This model captures both steady-state synchronization and transient behavior, making it invaluable for studying grid responses to disturbances. Consequently, the Kuramoto theory framework provides valuable insights into power grid dynamics. While the approximations employed may overlook certain active control mechanisms that maintain stability, they remain relevant for understanding the complexities of modern power grids, particularly those integrating multiple small power sources. The development of a proper Kuramoto-like model incorporating inertia suggests promising directions for establishing qualitative design rules that enhance stable

operation, warranting further investigation to ensure applicability in practical grid contexts.

3. HVDC Modelling Framework

HVDC transmission lines efficiently transfer large amounts of electricity over long distances with minimal losses. Their controllability enables dynamic power management, ensuring the potential to mitigate stability issues related to frequency deviations and voltage instabilities amid fluctuating renewable generation. By allowing precise power flow control, HVDC lines are able to reduce instability risks and blackout likelihood while supporting the integration of distributed energy resources. Unlike AC transmission, HVDC systems control power flow independently of phase synchronisation, mitigating disturbances without affecting network frequency. They dampen power oscillations, stabilise voltage levels, and reduce power losses, enhancing grid resilience. Their versatility supports long-distance power delivery and stabilises power flows in renewable-integrated systems. Controllable HVDC lines can also act as network isolators, segmenting grids into weakly connected zones to prevent cascading failures. By strategically placing HVDC connections, failure propagation can be blocked while maintaining interconnectivity. Simulations show that well-designed HVDC isolators effectively prevent cascading blackouts, making them essential for future large-scale renewable integration, as described in 0 and 0.

Integrating HVDC lines into power grid models involves various control strategies, each with distinct levels of complexity and system impact. The first and simplest option is a fixed power transmission control mode, where HVDC lines transfer a constant and predictable amount of power, maintaining load equilibrium between two (potentially asynchronous) networks without altering voltage, current, or other system parameters. This approach is particularly suitable for long-distance power transfers that require stable and predictable conditions.

A more sophisticated option is phase difference-based control, which dynamically adjusts power flow based on the phase angle difference between nodes. This method necessitates continuous monitoring and adjustments, enhancing the system's ability to manage grid imbalances. Implementing this strategy would require deep modifications to the second-order Kuramoto model, introducing a more complex control structure but potentially improving dynamic stability.

Another possible concept is frequency control, where HVDC systems provide a primary frequency response, adjusting power flow in response to frequency deviations. This approach is ideal for grids with frequent fluctuations, especially those with significant renewable energy penetration. Integrating frequency control into the Kuramoto model would significantly enhance its ability to stabilise the grid and improve synchronisation, though it would also demand considerable changes to the existing model structure.

Ultimately, while the fixed power transmission control method offers straightforward implementation, phase and frequency-based control strategies present promising opportunities to enhance system stability and reduce the risk of blackouts and cascading failures. Considering the time

given for the research and the complex implementation of the adaptive control methods, the fixed power transmission control mode will be used in the followings. This means that the respective nodes which are connected through the HVDC links are considered in a power source and -sink approach. In practical terms this means, that there is no direct connection in the classical Kuramoto model sense between the HVDC end terminals, but the power flow is modelled by the incoming and outgoing energy, given for each node under consideration.

It is crucial to acknowledge that the model is dimensionless, which means that these values had to be properly normed. In the absence of data regarding the producers and consumers involved, a zero-centred Gaussian distribution is employed for the regular "AC nodes" for this purpose. For the standardisation intention of the HVDC [MW] node values, power-law fits were applied according to Figure 5. in 0 (most of the positive half of the zero-centred Gaussian distribution can be overlapped with these power-law fits). The A_{ij} weightings of the specific links were also used, as presented in 0, without modifications. In the following simulations, the thermalisation process is initiated from a phase-ordered state (meaning $R(t_k) = 1$ and $\theta_i(0) = 0$), where line-cuts are prohibited and $\alpha = 0,4$ is used as a relevant damping coefficient to represent power dissipation in the system. In every case, the simulation runs until a 300-iteration step limit, which seemed to be enough to reach steady state operation, where oscillations are no longer present to a significant extent. Out of these simulations for every scenario, 100 different samples were collected in order to rule out the possibility of events occurring by chance.

4. Simulation scenarios

The study employed a synthetic grid modelled on real-world data, incorporating nodes representing generation, load, and transmission lines. Scenarios tested included:

- Simulation of already existing HVDC lines
- Simulation of imaginary HVDC lines at critical connection points of the Hungarian power grid
- Fault-induced desynchronization events.

The main experiment was to investigate how the modelling of the existing HVDC lines in the European power system of 2016 (EU2016 dataset) would affect the characteristics of the system compared to the case, where HVDC lines are neglected, and simply modelled as regular AC lines, as in the study published in 0.

As an extension of the research, a couple of regular AC transmission lines, which are electrically linking neighbouring countries to Hungary were selected, shown in Fig. 3. The transmission lines were chosen primarily based on previous significant failure events in the Balkan region (to give one example, we may cite the 2021 electricity grid outage). In addition to that, the authors of the study published in 0 propose that the implementation of high-voltage direct current transmission lines to separate the Balkans could potentially mitigate blackout risk across the European electric power system. However, it is important to note, that the selection of the illustrated transmission lines was not supported by any numerical

calculations, and the chosen links were added to the list of HVDC lines that were existing in reality by 2016 for the purpose of the experiment. For the sake of simplicity, the input and output power values have been calculated from the average of the same parameters of all existing and operating HVDC lines. In order to enhance the visibility of the effects under consideration, all of the HVDC lines were modelled as entirely replacing the AC transmission lines, rather than merely providing supplementary support.

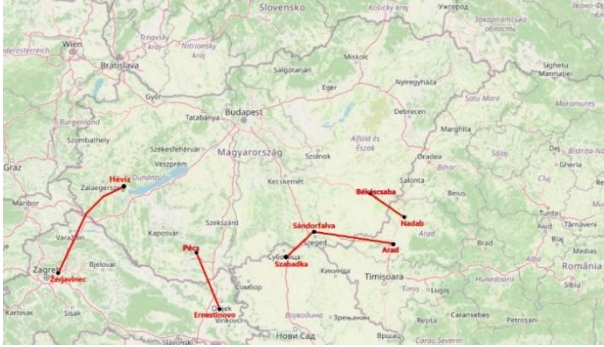


Fig. 3. The selected transmission lines between Hungary and neighbouring countries, which were converted to HVDC lines for a test scenario.

Advantages of this modelling methodology include:

- **Enhanced Stability:** HVDC systems are able to consistently reduce instability caused by renewable energy fluctuations.
- **Dynamic Adaptability:** The Kuramoto model can effectively simulate diverse scenarios, providing insights into synchronization phenomena.
- **Practical Implications:** HVDC technology offers scalable solutions for large, interconnected grids, enabling reliable operation despite increasing renewable penetration.

5. Results

This section analyses the outcomes of the simulations conducted on HVDC-integrated power grids, focusing on key indicators such as system synchronisation, frequency stability, and resilience to cascading failures. The effectiveness of HVDC lines, both in their standard role and as network isolators, is assessed by comparing different scenarios. The results highlight potential enhancements in system stability and reliability, providing insights for future improvements and the practical implementation of HVDC technology. In the following three figures, the modelling results shown in black are the reference point, the full AC network case (provided by 0), which is used for comparison with the new scenarios using models of HVDC technology. As also indicated in [7], the most important characteristics of the modelling process consists of the Kuramoto phase-order parameter $R(t_k)$; the frequency-order parameter (frequency spread) $\Omega(t_k)$; and the average number of line faults $\langle N_f \rangle$ following a disturbance (line cut) depending on the different values of the coupling strength K , which is basically the maximum possible transmitted power between nodes (in this case practically the cross-section of the power lines).

Test 1: Frequency oscillations

As renewable energy sources displace traditional generators, system inertia decreases, leading to faster frequency deviations. [8] reveals that HVDC systems can significantly mitigate these effects by providing virtual inertia and enabling rapid power flow adjustments. Fig. 4. illustrates the frequency deviations of the system with HVDC integration compared to a purely AC grid. The frequency-order parameter $\Omega(t_k, N)$ is given by the equation:

$$\Omega(t_k, N) = \frac{1}{N} \sum_{j=1}^N \left(\bar{\omega}(t_k) - \omega_j(t_k) \right)^2 \quad (13)$$

where $\bar{\omega}(t_k)$ indicates the average frequency calculated for each individual sample at time step t_k according to [7] and [9]. This shows us the $1/N$ averaged variance of the frequencies of each N node of the system from the mean frequency within each respective sample.

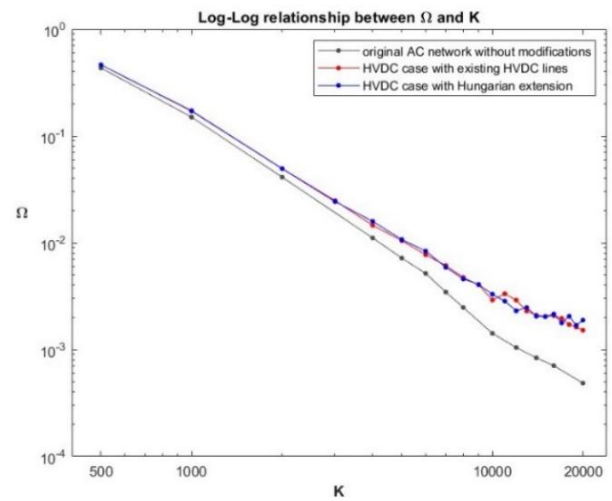


Fig. 4. Comparison of frequency deviation with and without HVDC integration.

Fig. 4 depicts the logarithmic relationship between the Ω frequency spread of each N node and the coupling strength K for three scenarios: the original full-AC network, the HVDC-enhanced network using existing lines, and the extended HVDC case with imaginary “Hungarian HVDC links”. Each data point represents average values derived from 100 samples, spanning K values from 500 to 20,000.

Test 2: System synchronisation

Oscillatory instability, characterized by growing amplitude swings, poses a significant risk in modern grids. HVDC lines are able to effectively dampen oscillations by absorbing and redistributing excess energy. Fig. 5 demonstrates the overall system synchronisation characteristics observed in the simulations.

As introduced in 0, the Kuramoto phase-order parameter $R(t_k)$ is intended to display the phase synchronisation properties of the system within each respective sample at time step t_k . The method of calculation follows:

$$z(t_k) = r(t_k) e^{i\theta(t_k)} = \frac{1}{N} \sum_j e^{i\theta_j(t_k)}, \quad (14)$$

where a $1/N$ averaging is used over all nodes in the system, and $R(t_k) = \langle r(t_k) \rangle$, is the real part of the order-parameter

averaged over 100 samples. In this case, a slight increase in system synchronisation can be observed, for low K values.

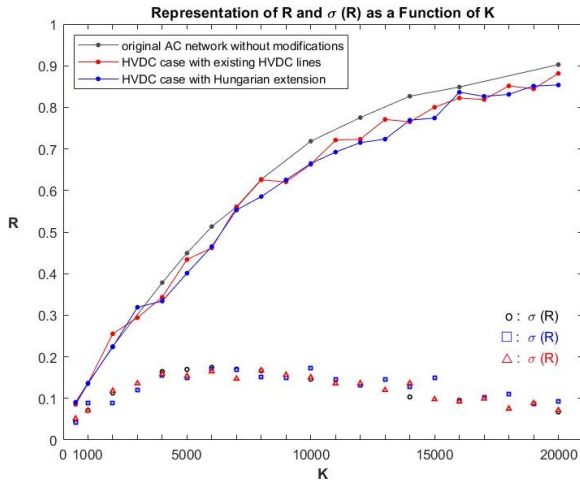


Fig. 5. Comparison of synchronisation strength with and without HVDC integration.

Test 3: Fault Isolation

HVDC systems can act as network isolators at some cases during fault events, segmenting the grid to mitigate disturbances as indicated by [4] and [5]. Fig. 6 shows a segmented grid topology with HVDC links, illustrating how fault propagation could be limited to a localized area.

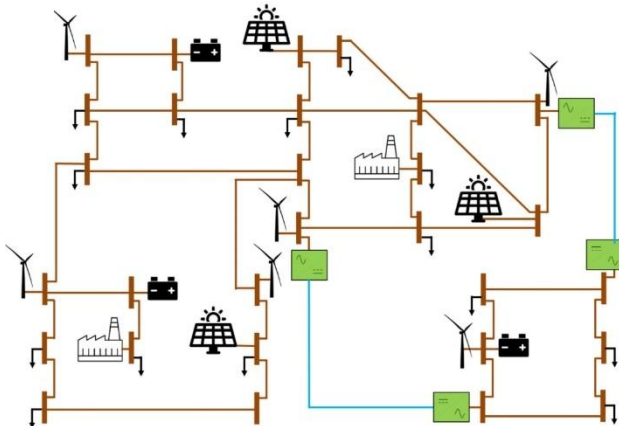


Fig. 6. Grid segmentation with HVDC isolators. [8]

Therefore, the potential of HVDC systems in mitigating failure spreading risks has been investigated through the simulation of cascade failures in the power system by perturbing the system. In this case, after the thermalisation, a power failure event is artificially produced by removing a randomly selected power line. Subsequently, if the resulting power flow on a line between adjacent nodes exceeded a defined threshold $F_{ij} = |\sin(\theta_i - \theta_j)| > T$, causing the line to be considered overloaded, the link was permanently removed from the graph, and the total number of line failures N_f was recorded for each simulated blackout cascade. $\langle N_f \rangle$ represents the number of these line faults averaged over 100 samples of the simulation.

Fig. 7 illustrates the correlation between the average cascade size $\langle N_f \rangle$ (number of faults following the perturbation of the system) and the coupling strength K on a logarithmic scale for the same three scenarios previously described. Each data point represents the mean of one

hundred samples, with K values still ranging from 500 to 20,000.

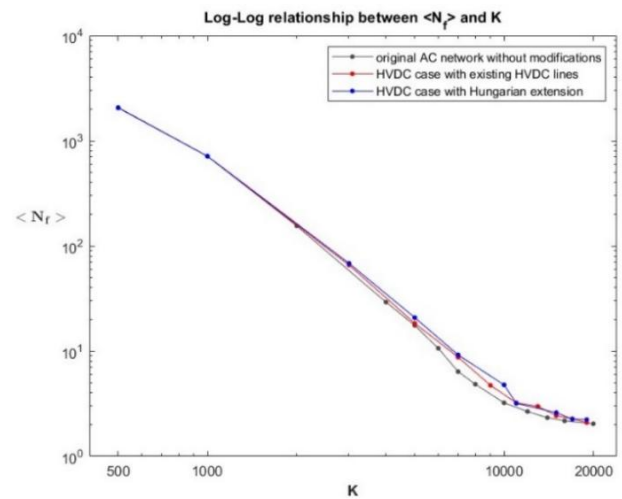


Fig. 7. A comparison of the dynamic simulation results for the average cascade sizes $\langle N_f \rangle$ after the perturbation of the system.

The results of the dynamic simulations indicate that the utilization of this specific type of HVDC modelling methodology results in a slight increase in the average cascade size at higher K values, while remaining unaffected at smaller K values. The results point to a significant controversy between the simulation outcomes and the theoretical expectations, given that HVDC lines are designed and used to mitigate the risk and size of cascading failures in many cases.

A review of all three graphs reveals that the extension of the list of HVDC lines by the Hungarian case study highlights the effects described above even more, but the relatively small number of "Hungarian HVDC lines" compared to the realistic number of DC links in the first HVDC case mitigates the potential for significant divergences.

6. Conclusion

However, in modelling complex power grid dynamics, several assumptions and simplifications were necessary to maintain feasibility and computational efficiency. These assumptions primarily concern data availability and the limitations of the applied Kuramoto-based HVDC modelling. As already mentioned previously, only real-world data were used as an initial point, in order to specify the most accurate power in- and output data possible but identifying the direction of the power flow was challenging due to the lack of available information regarding this topic. As a result, the direction of the energy flow was defined randomly, which is certainly an imperfect approach. Additionally, HVDC control mechanisms were not modelled adaptively. In practice, HVDC systems often regulate power flow based on real-time phase-angle and frequency differences between nodes. However, this study adopted fixed parameters, limiting its ability to reflect dynamic grid conditions. Overall, these assumptions, while necessary for computational feasibility, introduce limitations that could impact the accuracy of the simulation outcomes.

Addressing these constraints would require more detailed data and the incorporation of adaptive control strategies into future models. The validation process highlights several important considerations regarding the limitations and outcomes of the HVDC modelling approach. Based on the study conducted in [7], the Nordic region emerges as the community with the weakest synchronisation properties, coinciding with the highest concentration of HVDC lines in the EU2016 dataset. This means that although the results of the experiment are not favourable, the placement of the HVDC lines is likely to play a crucial role in the final outcome. The severance of the graph in this modelling scenario (using power source and -sink nodes, without any link connecting them) also introduces difficulties in achieving synchronisation, which reflects the practical constraints of this method. Though it is important to note that HVDC lines are not always implemented to enhance synchronisation; they often fulfil other engineering or technical requirements. However, given the critical need to mitigate cascading failures, their ability to control power flows is essential, knowing that this modelling methodology does not make it possible. As it can be seen through the graphs of the results, the current modelling approach is suboptimal in practical terms, as it does not effectively reduce the propagation of cascades. Future efforts should focus on adaptive power control strategies that regulate the power flow based on actual phase-angle and frequency differences between nodes. In conclusion, while the model provides valuable insights in cases where the fully AC network approach is changed to a theoretically more realistic network model, to improve its practical relevance, more sophisticated control mechanisms tailored to real-world operational needs will be required.

7. Summary and outlook

This study demonstrates the complementary roles of HVDC systems and the Kuramoto model in addressing modern power grid challenges. In conclusion, the paper emphasizes the growing need for rethinking the stability analysis of modern power grids, particularly with the integration of renewable energy sources such as wind and solar power. As traditional stability concepts are expanded to include the challenges posed by converter-coupled generation units and reduced system inertia, the significance of HVDC systems becomes increasingly evident. However, the modelling results in this study did not fully demonstrate the expected stabilising effects of HVDC systems, as theory would suggest, due to technical limitations inherent to the modelling process. Despite this, the potential benefits of HVDC transmission systems in reducing instability risks and improving the flexibility of power networks cannot be dismissed. To make the results align with the theoretical expectations, future research should focus on improving the accuracy of HVDC system models and simulations, particularly in dynamic grid conditions, to fully realize their stabilising potential, focusing on balancing frequency and voltage in the context of high renewable energy penetration. Additionally, exploring the role of energy storage and smart grid technologies, as well as developing predictive tools for adaptive control and enhancing the resilience of low-inertia systems will be essential in addressing the unique challenges posed by the rapid growth of renewable energy

sources. Potential future simulation scenarios to consider include high renewable penetration levels and sudden load variations in the observed system.

HVDC technology, with its flexibility and precision, provides practical tools for mitigating instability, while the Kuramoto framework offers a robust theoretical basis for understanding grid dynamics. Further research should focus on:

- Integrating advanced forecasting techniques with HVDC controls.
- Expanding the Kuramoto model to include additional grid elements like energy storage systems.
- Real-world implementation studies to validate simulation findings.

Acknowledgement

Gratitude is expressed to the HUN-REN Centre for Energy Research for their generous scholarship support, which played a crucial role in facilitating this research. The collective expertise of their colleagues and the resources provided have been instrumental in supporting the research efforts presented in this paper. The contributions of Dr. Géza Ódor in coding of the model utilized in this research is also gratefully acknowledged.

István Vokony acknowledges the support of the Bolyai János Research Scholarship of the Hungarian Academy of Sciences (BO/50/24).

This publication was supported by the HYPNET - this project has received funding from the European Union's Horizon Europe research and innovation programme under grant agreement No. 101172757.

References

- [1] Acebron, Juan & Bonilla, Luis & Vicente, Conrad & Ritort, Felix & Spigler, Renato. (2005). *The Kuramoto model: A simple paradigm for synchronisation phenomena*. Reviews of Modern Physics. 77. 137.
- [2] G. Filatrella, A.H. Nielsen, and N.F. Pedersen: *Analysis of a power grid using a Kuramoto-like model*, Eur. Phys. J. B 61, 485–491 (2008)
- [3] Ódor Géza, Hartmann Bálint: *A heterogenitások hatásai villamos hálózati modelleken*, Fizikai Szemle 2019 / 2, pp. 50-54
- [4] D. Gomilaa, B.A. Carrerasa, J.M. Reynolds-Barredo, P. Colet, O. Gomis-Bellmunt: *Analysis of the blackout risk reduction when segmenting large power systems using lines with controllable power flow*, International Journal of Electrical Power and Energy Systems, Volume 148, June 2023, 108947
- [5] Franz Kaiser, Vito Latora & Dirk Witthaut: *Network isolators inhibit failure spreading in complex networks*, Nature Communications, Volume 12, article id. 3143 (2021)
- [6] Bálint Hartmann, Géza Ódor, István Papp, Kristóf Benedek, Shengfeng Deng, Jeffrey Kelling: *Dynamical heterogeneity and universality of power-grids*, Sustainable Energy, Grids and Networks (SEGAN) 39 (2024) 101491
- [7] Géza Ódor, István Papp, Kristóf Benedek, and Bálint Hartmann: *Improving power-grid systems via topological changes or how self-organized criticality can help power grids*, Physical Review Research 6, 013194 (2024)
- [8] D. Gomilaa, B.A. Carrerasa, J.M. Reynolds-Barredo, M. Martínez-Barbeito, P. Colet, O. Gomis-Bellmunt: *Reducing blackout risk by segmenting the European power grid with HVDC lines*, 2024; Available at SSRN: <http://dx.doi.org/10.2139/ssrn.4739526> (November 2024)
- [9] Hartmann, Bálint & Ódor, Géza & Benedek, Kristóf & Papp, István: *Power-grid modelling via gradual improvement of parameters*, (2024). 10.48550/arXiv.2409.02758.