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On the irregular light variation of RU Camelopardalis

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Abstract. RU Camelopardalis is a W Virginis variable which has light curve with irregularly variable amplitude, mean value and phase. The Fourier-spectrum of the light variation shows a complex structure with the characteristic period of 22 days. The visual observations before 1960 also show this complex variation, but a dominant regular component was found in these data, too. There are some possible mechanisms underlying the erratic variations of stars: deterministic chaos, stochastic excitation and multiperiodicity with unresolved frequencies. In order to choose from these mechanisms we applied different kinds of analyses and numerical tests. The time-frequency analysis of the light curve indicates that the variation cannot be modelled by a multiperiodic variation. Phase-space reconstruction of the data was made in order to prove the existence of low dimensional chaos, but the phase portraits do not show any regular structure, *i.e.* the data cannot be modelled with a low dimensional ($\sim 2-3$) chaotic system.

Key words: stars: oscillations – stars: Cepheids – methods: data analysis – stars: individual: RU Cam

1. Introduction

RU Camelopardalis was discovered to be variable by Ceraski (1907) on Blazhko's plates, but it was concluded from searches of the older literature that RU Cam was several times exceptionally faint during the last decades in the 19th century. Blazhko (1907) determined the period of the light variation (22.27 d).

The weak nonstationarity (variation of the amplitude and period) of the light curve was already suspected by Leiner (1923) and confirmed by further investigations (Haas 1923; Leiner 1929; Edelberg 1932; Nielsen & Sjögren 1943).

Although the star was included in the general list of Cepheid variables, Robinson (1933) called the attention to the peculiarities of RU Camelopardalis in its spectrum and the period and shape of the light and radial velocity curves. Payne-Gaposchkin (1941) noted that W Virginis resembled RU Cam in the peculiarities.

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Demers & Fernie (1966) reported that they were able to detect only irregular light fluctuation of the order of 0.1–0.2 mag in 1965. This observation initiated a series of speculations about ceasing pulsation or the temporary decrease of the amplitude in RU Cam. Tracing back on Sonneberg archival plates Huth (1966) ascertained that the light amplitude of the star might undergo considerable changes within several months.

The first complex analysis of RU Camelopardalis was made by Broglia & Guerrero (1972). They found period changes from the $O-C$ diagram and interpreted the cessation of the pulsation by a transitory stage in the evolution of the star. The quiescence of the pulsation was also interpreted as the result of strong evolutionary effects by several authors (e.g. Wallerstein 1968).

Perdang (1985) drew the attention to the irregular light variation of RU Cam in the context of chaotic pulsation. The nature of the irregular light variation of several types of variable stars has been a matter of dispute for a long time. The W Virginis stars are between the almost regular (δ Cepheids) and rather irregular (RV Tauri, SRd, Mira) variables in the HR-diagram. The numerical survey of Buchler & Kovács (1987), Kovács & Buchler (1988) suggests that the irregular behaviour of the pulsation models in the W Virginis and RV Tauri regime is low dimensional chaotic. The analysis of the RV Tauri star R Scuti (Kolláth 1990a) also suggests that the underlying mechanism of the irregular light variation of this star is a chaotic process. The observed pulsation, however, is more complicated than the predicted low dimensional chaotic variation. The analysis of some Mira stars also failed to indicate low dimensional chaos (Canizzo et al. 1990).

Now we present a detailed investigation of the light variation of RU Camelopardalis including time–frequency and phase–space analysis.

2. The data

Soon after Demers & Fernie (1966) reported the decrease of the amplitude of the light variation of RU Camelopardalis, photoelectric photometry of the star was started at several observatories. The investigation of RU Cam was also commenced by Detre & Szeidl (1967) at Konkoly Observatory. The observations were carried out by the 24" telescope at Budapest and by the 20" telescope at Piszkéstető Mountain Station in the pe-

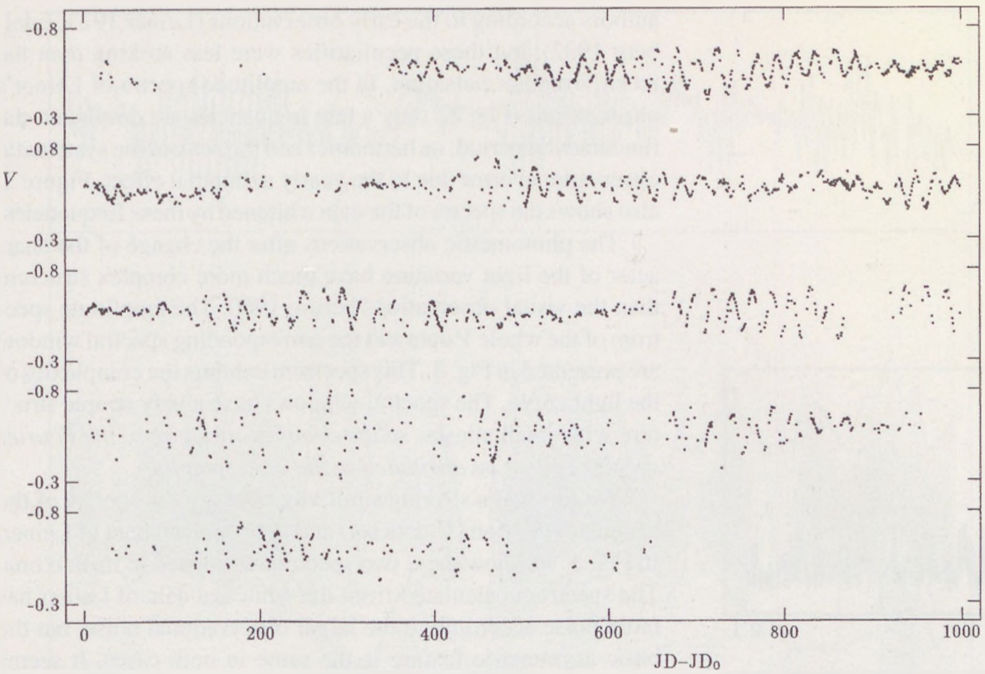

Fig. 1. Light curve of RU Camelopardalis (V data set) (JD_0 = 2 439 000, 2 440 000, 2 441 000, 2 442 000 and 2 443 000 for the segments of the figure from the top to bottom)

riod 1966-1982. The observations have recently been published (Szeidl et al. 1992).

The light curve of RU Cam in V colour is shown in Fig. 1. The time series consists of the data observed at the Konkoly Observatory complemented by the data of Wamsteker (1968), Broglia (1969), Cester (1968,1969), Broglia & Guerrero (1969,1972,1973) and Broglia et al. (1978). The data set consists of 2177 individual observations. For the simplicity we refer to this light curve by V data set hereinafter. The picture clearly shows the variation of the amplitude and the existence of some "quiet" regimes which were also found by other observers (e.g. Broglia & Guerrero 1972). It seems that the nearly zero amplitude at certain times is a consequence of some amplitude modulation and not of the standstill of pulsation. A part of the light curve plotted with a compressed time scale (see Fig. 4.) shows the complicated temporal variation of the amplitude and the average brightness.

We also analyzed some of the visual observations of RU Camelopardalis published by Shapley (1913), Silva (1922), Leiner (1923,1929), Edelberg (1932), Florja & Kukarkina (1953). The longest homogeneous visual observational sequence (Leiner's data) has almost the same length (3350 days) as our V data set but sampled more sparsely (649 observations).

3. Time-frequency analysis

In order to check the existence of multiperiodic variation, as the origin of the irregularity, the Fourier-spectra of various segments of the observations were calculated by the MUFRAN programme-package (Kolláth 1990b).

We analyzed the visual observations of RU Camelopardalis to get information about the stability of the pulsation before 1965. The irregularity of the variability was reported by several

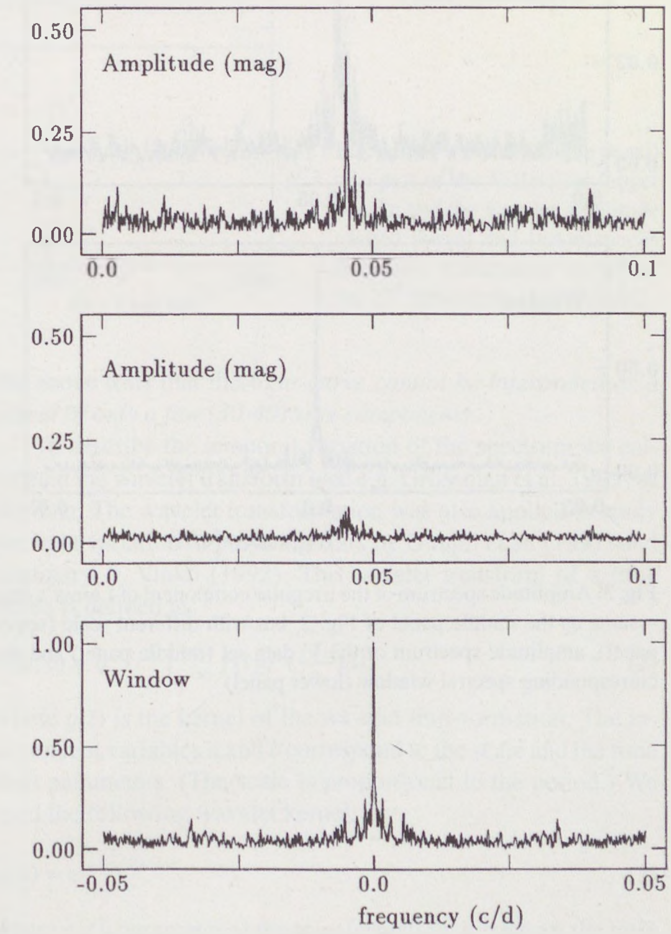

Fig. 2. Amplitude spectrum of the Leiner's data (upper panel), that after prewhitening (middle panel) and the corresponding spectral window (lower panel)

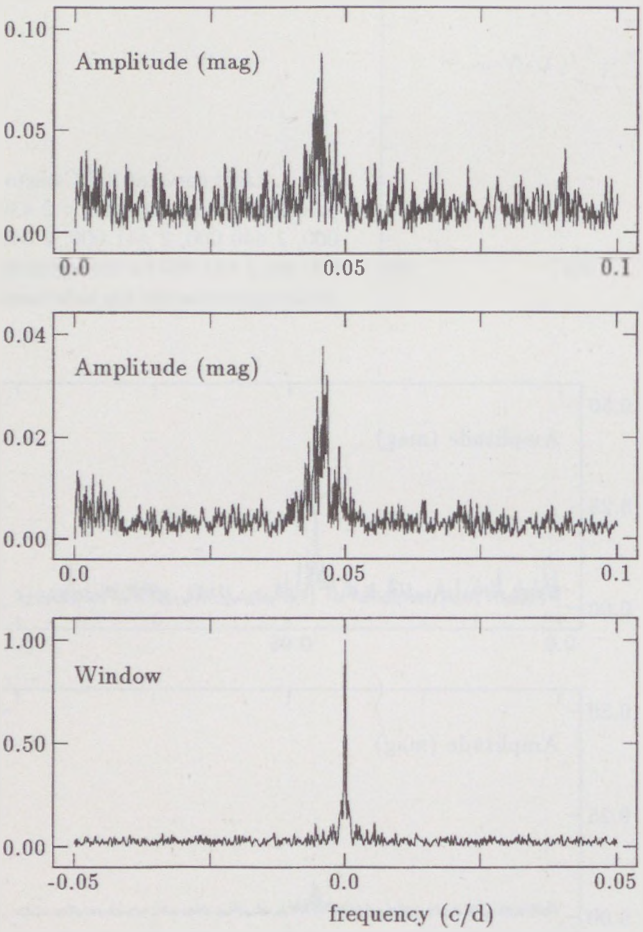

Fig. 3. Amplitude spectrum of the irregular component of Leiner's data – same as the middle panel of Fig. 2. but with different scale (upper panel), amplitude spectrum of the V data set (middle panel) and the corresponding spectral window (lower panel)

authors according to the early observations (Leiner 1923; Edelberg 1932), but these peculiarities were less striking than the recent irregular pulsation. In the amplitude spectra of Leiner's observations (Fig. 2.) only a few frequencies are dominant: the fundamental period, its harmonics and the peak of the systematic observational error due to the yearly azimuthal effect. Figure 2. also shows the spectra of the data whitened by these frequencies.

The photometric observations after the change of the character of the light variation have much more complex structure than the visual observations before 1960. The amplitude spectrum of the whole V data and the corresponding spectral window are presented in Fig. 3. This spectrum exhibits the complexity of the light curve. The spectral windows have a very simple structure with small aliases, *so the complex structure of the Fourier spectra cannot be attributed to the data sampling*.

We can find a striking similarity between the spectra of the irregular variation (V data set) and the whitened data of Leiner. In Fig. 3. we show these two spectra normalized to its maxima. The spectrum calculated from the whitened data of Leiner has more noise according to the larger observational noise, but the basic asymmetric feature is the same in both cases. It seems that the earlier irregularities persisted with a slightly changed power in the sixties and the periodic component ceased. It is a very important fact: *the later randomness of the pulsation existed already in the observations of the twenties with the same character*. We have to note that there is no contradiction between the change of the light curve amplitude, and that of the Fourier-spectra. While the amplitude of a periodic signal is constant in the discrete Fourier spectra, the spectral power of a noise-like signal decreases as the data length increases. The ratio of the signal and spectral amplitudes suggests the stochastic nature of the irregular component in the light variation.

Szeidl et al. (1992) discussed the period changes of RU Cam by the investigation of $O-C$ diagram. They found 22.16 and 21.65 days for the mean period before and after 1964. The investigation of Fourier spectra explains this period change. The period of the regular component of Leiner's observation is 22.19 days *i.e.* the value calculated from the $O-C$ variation. The spectrum of the V data set has its largest (central) peak at 21.83 days, close to the mean period derived by Szeidl et al. During the time when the regular component existed and was dominant in the pulsation, it determined the period of the variation. After the disappearing of this mode, the irregular part became the determinative factor. The central peak of the cluster of lines in the spectra of the whitened observations of Leiner was found at the period of 22.05 days.

A simple measure over the power spectrum was defined by Crutchfield et al. (1980). We define the degree of freedom of the spectrum (w) as follows:

$$w = \left(\sum_{i=1}^{n} P_i\right)^2 / \left(n \sum_{i=1}^{n} P_i^2\right), \qquad (1)$$

where $P_i (i = 1, 2, ..., n)$ is the power at the ith frequency bin. This quantity decreases monotonically as the length of data increases for spectra of multiperiodic signals. In the case of

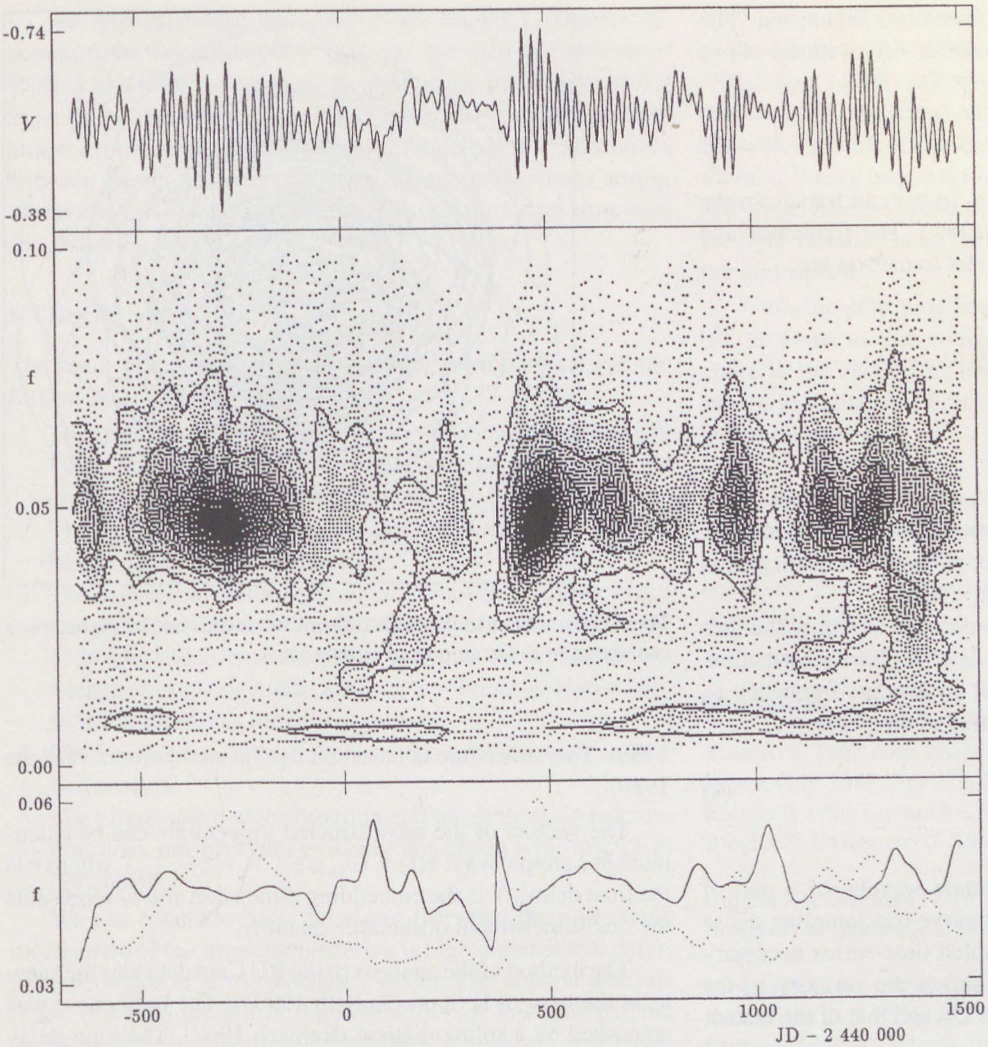

Fig. 4. The wavelet (middle panel) of a part of the V data set (upper panel); and the wavelet amplitude (dotted curve) and instantaneous frequency (continuous curve) of the 22^d periodicity (lower panel)

noise-like spectra, however, it is independent of the time coverage ($w = 0.5$ for white noise). For the spectrum of the whole V data set the degree of freedom of the spectrum is 0.09. For comparison, w was calculated from noisy multiperiodic signals with different number of sine components (corresponding to the highest peaks in the spectrum of the RU Cam data). The amplitudes of the sine components were chosen according to the spectrum. The time-span of the test data sets was the same as that of the observational data. The w values calculated from the artificial data sets are in the same order as the w value of the RU Cam data, with the following number of periods – noise amplitude pairs: $N = 10$, $\sigma = 0^{\mathrm{m}}.07$, $N = 20$, $\sigma = 0^{\mathrm{m}}.06$ and $N = 30$, $\sigma = 0^{\mathrm{m}}.05$. The quantity w is less than the one calculated from the RU Cam data even with 30 frequencies and a relatively high level of noise. We, however, could not fit the light curve even with 30 periods. We also take into consideration the variation of w as the data length changed. We got almost the same w values from the half and third parts of the observational data, but we detected the systematic increase of w calculated from the test data set with 30 frequencies and a large ($\sigma = 0.05$ mag) noise, as the amount of data decreased. It is clear from the above tests that the *light-curve cannot be interpreted as a signal of only a few (30-40) sine components.*

To describe the temporal variation of the spectrum we calculated the wavelet transform (see e.g. Grossman et al. 1989) of the data. The wavelet transformation was also applied to study the light variation of pulsating stars by Goupil et al. (1991) and Szatmáry & Vinkó (1992). The wavelet transform of a time series is defined as:

$$S(a,b) = a^{-1/2} \int_{-\infty}^{+\infty} f(t) g^{*}(\tfrac{t-b}{a}) dt, \tag{2}$$

where $g(t)$ is the kernel of the wavelet transformation. The independent variables a and b correspond to the scale and the time shift parameters. (The scale is proportional to the period.) We used the following wavelet kernel:

$$g(x) = e^{-x^2/2 + icx}, \tag{3}$$

where c is a parameter of the transformation, it controls the ratio of the time and frequency resolution. For pulsation analysis it is convenient to use a modified version of the wavelet transform:

$$T(b,\nu) = a^{-1/2} S(b, a(\nu)), \tag{4}$$

where ν is the frequency with 1/(time unit) dimension. The transform T of the function $x(t) = \cos(\Omega t + \Phi)$ with the above kernel is the following:

$$|T| = (\pi/2)^{1/2} e^{-1/2(a\Omega - c)^2} \tag{5}$$

It has its maximum value at $a\Omega = c$, so we can transform the frequency to the scale a as $a = c/(2\pi\nu)$. The frequency and time resolution of the modified wavelet transform are:

$$\Delta\nu = \frac{(8 \ln 2)^{1/2}\nu}{c} = 2.355...\frac{\nu}{c} \tag{6}$$

and

$$\Delta b = \frac{(2 \ln 2)^{1/2} c}{2\pi\nu} = 0.187...\frac{c}{\nu} \tag{7}$$

We can see that the frequency-resolution – time resolution product is the theoretical minimal limit for all time – frequency pairs and value of c *i.e.* $\Delta\nu\Delta T = 2\ln 2/\pi = 0.44...$.

It is important to take into consideration not only the amplitude or power of the wavelet but also the phase information. The instantaneous frequency around an average frequency ν_0 can be calculated as the time-derivative of the phase:

$$\nu_{inst} = \frac{\partial}{\partial b}\varphi(\nu_0, b), \tag{8}$$

where $\varphi(\nu, b) = \arg(T(\nu, b))$

In Fig. 4. we present the amplitude wavelet of a part of the V data set (this part of the light curve has sampling dense enough, to construct an evenly sampled time-series necessary to wavelet analysis). Figure 4. also shows the variation of the instantaneous frequency around .045 c/d and that of the instantaneous amplitude ($a_{inst}(b) = |T(\nu_{inst}(b), b)|$. On the top of the figure the corresponding part of the V data set is also plotted. We compared the Fourier-spectrum of the analyzed part of the light variation and that of a sine function modulated with the instantaneous amplitude a_{inst} and frequency ν_{inst}. The two spectra were exactly the same around .045 c/d, *i.e.* this amplitude and phase modulated signal is a good decomposition of the 22 day quasi periodicity. Although the correlation of the amplitude and period of the locally fitted sine was suspected by Broglia & Guerrero (1973) and Broglia et al. (1978) the investigation of the instantaneous amplitude and frequency does not confirm this hypothesis.

The wavelet analysis of the V data set supports the hypothesis that the irregularity of the variation is connected to strong amplitude and frequency modulation.

4. Phase space analysis

Low dimensional deterministic chaotic processes can be recognized by the geometry of the phase-space of the system. The structure of the trajectories in the case of low-dimensional chaos is usually simple and regular. For extracting this qualitative feature of dynamics from the one-dimensional observable *i.e.* the light-curve, the method of delays can be applied (Packard et al. 1980). This procedure is based on the Takens-theorem (Takens 1980).

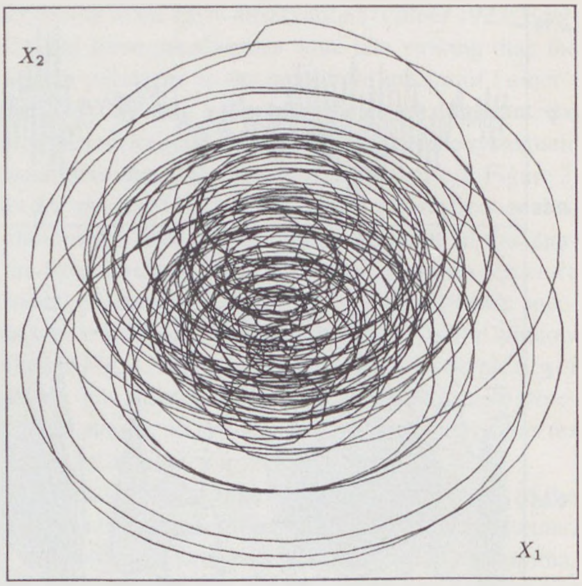

Fig. 5. Two dimensional projections of the reconstructed phase space calculated from the smoothed V data set

The vectors of the reconstructed trajectories can be calculated in a simple way: $\mathbf{x}(t) = (v_t, v_{t+\tau}, ..., v_{t+(N-1)\tau})$, where τ is the time delay, N is the embedding dimension and v_t represents the one-dimensional observable quantity.

The method of the analysis of the RU Cam data was the same as in the case of R Scuti (Kolláth 1990a). The light curve was smoothed by a spline method (Reinsch 1967). The time-delay of the reconstruction was 4 days and the embedding dimension was 5. The phase portrait of the RU Cam data after the transformation of singular value analysis (Bromhead & King 1986) are shown in Fig. 5. We could not find any regular structure in the phase-trajectories. The irregularity of the phase space is a general characteristic of the stochastic signals (e.g. AR random signal) and for high dimensional deterministic systems. The complex structure of the phase portrait of the RU Cam light variation suggests that *the light curve of this star cannot be modelled with low dimensional chaos.*

An important measure of the trajectories is the dimension of the space filled by them. If the phase space has more complex structure, its dimension is usually higher. In order to give some quantitative information about the phase portraits we calculated their correlation dimension (see. e.g. Grassberger & Procaccia 1984). For chaotic systems, as the embedding dimension is increased, the slope of the correlation integral converges to a value given by the correlation dimension. Here, however, we could not find any convergence but the saturation due to the finite amount of data. Unfortunately, according to the high complexity of the light variation and the limited length of data it is impossible to make satisfactory analysis of the dimension. We can conclude only that the dimension is higher than 3 – 4.

We also calculated the value of the largest Lyapunov exponent from the smoothed V data set. We used the method of Wolf et al. (1986) to estimate the rate of the divergency of the trajectories. We found $\lambda = 0.049\ \mathrm{day}^{-1}$ for the largest Lyapunov exponent of the V data set. This means that the distances between the neighboring trajectories increase by e times during 20 days, *i.e.* we cannot predict the variation for longer time than one pulsation period.

5. Conclusion

The main observational facts on RU Camelopardalis are the following:

1. The character of the light variation was changed abruptly in 1965. The regular part of the pulsation disappeared and the erratic component became dominant.
2. The irregular part of Leiner's light curve and the V data set has a very similar behaviour.
3. The irregularity of the light curve cannot be interpreted by a few stationary pulsation modes. The Fourier spectrum of the V data set has a dense noise-like structure. Assuming the existence of only one mode, a strong amplitude and frequency modulation of this mode was found.
4. The Fourier-spectrum of the data also contains power at low frequencies.
5. The phase-space reconstruction and the dimension calculation does not give any evidence for chaos with low (2-3) dimension.

We can conclude from the above facts that the underlying mechanism of the irregular pulsation is high dimensional deterministic chaos or some stochastic effect. The chaotic analysis (Fact 5) does not confirm the assumption of low dimensional chaos, unless the dimension of the underlying attractor is larger than 3-4. The observation of power at the low frequencies in the Fourier analysis (Fact 4) is not inconsistent with chaos, but this phenomenon is also quite common for other processes. The predicted dimension of chaotic pulsation in the W Virginis regime is extremely low (2-3) (see Kovács & Buchler 1988). A previous investigation has shown that the underlying mechanism of the irregular pulsation of the RV Tauri star R Scuti is deterministic chaos, but the dimension was found probably higher than four (Kolláth 1990a). To conclude, a discrepancy exists between the theory and the observations even accepting the chaotic origin of irregular pulsation. The observed light curve, however, may be more complicated than the underlying physical oscillations, if we take into consideration the effect of dynamic atmosphere on the transfer mechanism from the envelope pulsation to light variation. (Kolláth 1993a).

We calculated the transition of waves generated by chaotic processes through an isothermal atmosphere by a hydrocode similar to that of Bowen (1988) using chaotic signals as the lower boundary condition (for the detailed discussion see Kolláth 1993b). We found that: (a) The oscillation of the upper regions of the atmosphere have more complicated phase space structure and higher dimension. (b) If the lower boundary has broad-band-noise like spectrum (frequency clustering), power appears at low frequencies at upper atmospheric regions due to the nonlinearity of the dynamics. These results are correlated with Facts 4 and 5. Wallerstein (1968) suggested that the unique behaviour of RU Cam may be associated with the process of mass loss and mixing which modify the pulsation conditions of the outer layers of the star. Taking into consideration also that RU Cam is a carbon star these facts increase the possibility that the irregularity of the light variation has a strong connection with the dynamics of the atmosphere.

Unfortunately even these reasonably long observational data do not allow to find convincing evidence for the existence of chaos as the underlying mechanism. At the same time we cannot exclude the possibility that the underlying mechanism is a stochastic process.

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